I-C-Th-v1: Three Ledgers for Destructive Readout Access, specified reset, and reservoir heat in finite Enter implementations on a prescribed conical host

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Abstract

A destructive readout channel does not specify a thermodynamic implementation. We expand the thermal ledger of the shredder interface into three explicitly related accounts: complete energy and work, specified reset and resource restoration, and complete residual leakage. The framework does not replace established thermodynamics or introduce a new law. The principal result is a constructive non-identifiability statement. Even after the visible channel and spectator Hamiltonians are fixed, energy-conserving implementations can have different designated-reservoir energy gains and different resource-return states. A separate driven spectator changes external work without changing that visible channel. These examples establish underdetermination, not a violation of Landauer's principle and not an optimum cyclic erasure cost. Local proofs connect this statement to the finite Gibbs-reservoir identity $\beta Q_B = \Delta S_M + I(M: B) + D(\omega_B \| \tau_B)$. We give its initial-correlation correction, an archive-sensitive approximate-reset bound, and a quantitative finite-full-rank obstruction to exact pure reset. Positive complete entropy decrease is sufficient, but not necessary, for positive deposited heat. Zero heat does not imply retained quantum access. Append-only record entropy, quantum residual entropy, and hybrid output entropy are kept separate. A finite revival model has zero bath-energy change and exact recovery error $(1 - |\cos 2\theta|)/2$, while conserving finite constructions can share selected entropy histories and disagree completely on endpoint recovery. Temperature, a KMS prescription, a zero-temperature limit, and a geometrical thermal factor are different inputs or conclusions, not readings of a channel name. The contribution is an integrated, submission-oriented set of identities, counterexamples, and applicability tests. Its potential use is to make heat, reset, and recovery claims independently auditable without treating them as independent unconstrained variables. The underlying thermodynamic and quantum-information principles are attributed rather than claimed as new. No cyclic machine, Carnot efficiency, third-law theorem, continuum detector, gravitational endpoint, Page curve, island, Einstein dynamics, or spacetime emergence is constructed. O.1 on the cone remains texttt NOT_CLOSED.

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Three ledgers: one channel is not one implementation

Enter, spacetime Enter, and the shredder philosophy

Cheung's Enter calculus treats runtime quantum names as access rights to tensor factors. Destructive

computational-basis readout consumes the measured names and returns classical outcomes, together with every explicitly retained quantum residual. Its operational characterization belongs to a specified strategy class; the present paper assumes the usual Born-channel semantics and does not rederive that characterization [CheungEnter].

The *Philosophy of Enter* provides an introductory map: destructive readout is not coherent recovery, a classical record is not a quantum-return decoder, a channel is not a thermal machine, and an analogy is not a physical identification [CheungPhil]. I-B installs finite operational interfaces on a prescribed spacetime [CheungIB]. I-C makes the ``shredder'' comparison an access statement rather than a microscopic information-annihilation or compulsory-heating claim [CheungIC].

The present expansion concerns the third separation. It is not based on the assertion that established thermodynamics uses only one entropy or fails to distinguish heat from work. Modern quantum thermodynamics already treats subsystem entropy, correlations, reservoirs, side information, and resources explicitly [Landauer,ReebWolf,DelRio,Gallego]. Enter contributes an interface discipline for applying those accounts to its named outputs. It does not found the laws.

The prescribed host is not a thermal machine

Use $c = \hbar = 1$, retaining k_B. The host remains

$$\begin{aligned} M_\alpha &= \mathbb{R}_t \times (0,\infty)_r \times S^1_\varphi \times \mathbb{R}_z, \\ g_\alpha &= -\mathrm{d}t^2 + \mathrm{d}r^2 + \alpha^2 r^2 \mathrm{d}\varphi^2 + \mathrm{d}z^2, \qquad 0 < \alpha \le 1, \end{aligned}$$

with angular period 2π. The completed defect is a worldsheet; the regular laboratory excludes its apex.

The regular spacetime is not globally hyperbolic. Indeed, for $T > r_0 > 0$, the causal diamond between $(-T, r_0, \varphi_0, 0)$ and $(T, r_0, \varphi_0, 0)$ contains $(0, r_n, \varphi_0, 0)$ for $r_n \downarrow 0$, with no convergent subsequence in the regular host. This reproduces the local noncompact-diamond argument of I-B-G and I-C-Ex [CheungIBG,CheungEx].

A field realization, self-adjoint domain, state, and kernel must be specified when used. None is needed for the finite thermal proofs below. Placing finite systems at regular locations does not select their Hamiltonians, temperatures, couplings, or material properties.

Notation and three distinct questions

All channels and thermodynamic systems in the local theorems are finite-dimensional unless a spectral comparison is explicitly labeled otherwise. A channel is completely positive and trace preserving. We use

$$d_{\mathrm{tr}}(\rho,\sigma) = \frac{1}{2}\|\rho - \sigma\|_1, \qquad d_\diamond(\Phi,\Psi) = \frac{1}{2}\|\Phi - \Psi\|_\diamond.$$

The latter includes untouched finite references [Watrous].

All Shannon and von Neumann entropies use natural logarithms:

$$H(p) = -\sum_x p_x \ln p_x, \qquad S(\rho) = -\mathrm{Tr}\,(\rho \ln \rho).$$

Physical entropy is $k_B S$; one unbiased classical bit has dimensionless entropy $\ln 2$. Define

$$D(\rho\|\sigma) = \mathrm{Tr}\,[\rho(\ln\rho - \ln\sigma)], \qquad I(A:B) = S(A) + S(B) - S(AB).$$

Relative entropy is $+\infty$ outside its support domain. Finite-temperature finite Gibbs states are full rank, so the relative entropies used in the thermal identities are finite.

A thermal claim must specify:

- the physical memory, its input state or ensemble, the accessible outputs, and all message-bearing residuals;
- system, reservoir, battery, and interaction Hamiltonians, including endpoint and switching conventions;
- the reservoir preparation, inverse temperature, and initial correlations;
- controllers, archives, usable side information, blanks, and other nonequilibrium resources;
- the complete dynamics and relevant final correlations;

• the definitions of heat and work and the required accuracy and restoration of reusable resources. The three questions are:

*ll*energy/work:	what energy changes under the declared dynamics?
reset/restoration:	what factor is standardized, and what must be reusable?
leakage:	how does the complete inaccessible output depend on the code?

These ledgers are different kinds of data, not a claim that their numerical values can never coincide or that their feasibility constraints are independent.

A specified reduced channel $\mathcal{N}$ *does* determine $\mathcal{M}(\rho)$ and its entropy for a specified input ρ. With fixed input/output local Hamiltonians it also determines the corresponding local energy change. A reset property can likewise be a channel property. What is not determined by this reduced map alone is its complete physical heat/work and resource-restoration account.

This is the careful form of the separation documented in En07-A/B/C [CheungEn07A,CheungEn07B,CheungEn07C].

Readout, discard, and quantum-return obstruction

Let C have orthonormal basis $\{|\tau_j\rangle\}$, and let A contain all returned quantum factors. Put $D_j = \langle\tau_j| \otimes I_A$.

Proposition. The map

$$\mathcal{M}_C(X) = \sum_j \mathrm{proj}\, j_R \otimes D_j X D_j^\dagger$$

is a channel and satisfies

$$\mathrm{Tr}_R\, \mathcal{M}_C(X) = \mathrm{Tr}_C X.$$

The identities hold with every untouched reference. The returned quantum interface contains A, not C.

Proof. The recorded Kraus maps are $|j\rangle_R \otimes D_j$, with squared moduli summing to I_{CA}. Writing $X = \sum_{j,\ell} |\tau_j\rangle\langle\tau_\ell| \otimes X_{j\ell}$ identifies the output blocks as X_{jj}. Their sum is $\mathrm{Tr}_C X$. The calculation includes a reference by enlarging each block.

The name-consumption statement is an interface rule; it is not the evaluation of an energy observable. Equation is the I-B/En09-A comparison [CheungIB,CheungEn09], locally proved here.

For $Q \cong \mathbb{C}^d$, write

$$|\Phi_d\rangle = \frac{1}{\sqrt{d}} \sum_{i=0}^{d-1} |i\rangle_E |i\rangle_Q, \quad \Phi_d = |\Phi_d\rangle\langle\Phi_d|, \quad F_{\mathrm{e}}(T) = \mathrm{Tr}\,[\Phi_d (\mathrm{id}_E \otimes T)(\Phi_d)].$$

Set

$$e_{\mathrm{rec}}(\mathcal{N}) = \inf_{\mathcal{D}} d_\diamond(\mathcal{D}\mathcal{N}, \mathrm{id}_Q),$$

over deterministic decoders on the declared accessible output.

Theorem. Suppose the complete accessible output is

$$\mathcal{N}(X) = \mathrm{bigoplus}_r \mathcal{N}_r(X), \qquad \mathcal{N}_r : \mathcal{B}(Q) \to \mathcal{B}(K_r), \qquad \dim K_r \le k \le d.$$

Then every decoder satisfies

$$F_{\mathrm{e}}(\mathcal{D}\mathcal{N}) \le \frac{k}{d}, \qquad d_\diamond(\mathcal{D}\mathcal{N}, \mathrm{id}_Q) \ge 1 - \frac{k}{d}.$$

A complete implementation allowance δ changes the distance lower bound to $[1 - k/d - \delta]_+$. For classical-only output, $k = 1$.

Proof. A normalized pure bipartite vector of Schmidt rank at most k has coefficient matrix A with $\|A\|_2 = 1$, $\mathrm{rank}\, A \le k$, and Bell overlap

$$\frac{|\mathrm{Tr}\, A|^2}{d} \le \frac{\|A\|_1^2}{d} \le \frac{k}{d}.$$

Convexity extends the bound to Schmidt number at most k. Local Kraus maps cannot increase this number. Each

Bell-output block lives on $E \otimes K_r$; its decoded Bell overlap is at most its trace times k/d. Sum the blocks and test the difference from Φ_d with the Bell projector. The implementation statement is the reverse triangle inequality and channel contractivity.

This is the established Schmidt-number/entanglement-breaking mechanism [TerhalHorodecki,HSR]. The sharp finite constructions are printed in I-C-Ex and I-C-Ex2 [CheungEx,CheungEx2]; sharpness is not needed for the thermal conclusions. At the classical endpoint, fixed-basis dephasing attains $1 - 1/d$, as follows by writing it as the uniform mixture of d phase conjugations.

An untouched reference, receiver memory, previously transmitted quantum factor, or shared entanglement available for decoding cannot be omitted from the access contract. A quantum reservoir does not become available merely because its energy is reported classically.

A replacement benchmark

For a fixed state σ, let $\mathcal{T}_{\sigma}(X) = Tr(X)\sigma$.

Lemma. For a constant accessible channel on a d-dimensional code,

$$e_{\mathrm{rec}} = a_d, \qquad a_d := 1 - d^{-2}.$$

Proof. Any decoded output is a replacement state. On Φ_d, its Bell overlap is $1/d^2$, independently of that state, giving the lower bound a_d. The maximally mixed replacement is the uniform mixture of the d^2 Weyl conjugations. Separating its identity term gives half-diamond distance at most $1 - d^{-2}$.

Fixed-channel thermodynamic non-identifiability

Theorem. Fix any finite visible implementation and its selected input–output channel. Appending independent finite spectators can leave that channel unchanged while changing:

• designated-reservoir energy gain, even with fixed spectator Hamiltonians and energy-conserving spectator dynamics;

• resource-return states;

• externally supplied work, under a separate specified driven implementation. Consequently the visible channel alone does not specify a unique full heat/work or reusable-resource reset ledger. This is not a statement about an optimum over a fully fixed cyclic implementation class.

Proof. Append independent qubits W, L with fixed Hamiltonians

$$H_W = H_L = \varepsilon \mathrm{proj}\, 1, \qquad \varepsilon > 0.$$

Prepare W excited and L in its Gibbs state

$$\tau_L = (1 - p)\mathrm{proj}\, 0 + p\,\mathrm{proj}\, 1, \qquad p = (1 + e^{\beta\varepsilon})^{-1}.$$

Compare identity on WL with $SWAP_{WL}$. Both commute with $H_W + H_L$, and neither changes the visible channel. Identity gives $Q_L = 0$. SWAP gives

$$Q_L = \varepsilon(1 - p) > 0, \qquad \Delta E_W = -\varepsilon(1 - p),$$

with final states $\tau_W \otimes proj\ 1_L$. Thus both the designated bath energy and the state of a consumed resource differ at fixed Hamiltonians. A smooth pulse proportional to SWAP, vanishing at the endpoints and having area $\pi/2$, implements the swap up to commuting free phases. Its net switching work is zero, since the SWAP expectation is constant.

For external work, append an independent isolated qubit F, initially in $|0\rangle$, with the same initial and final Hamiltonian $H_F = \varepsilon proj\ 1$. Doing nothing costs zero external work. A driven inversion changes its energy by ε. An explicit pulse is

$$H_F(t) = H_F + f(t)e^{-\mathrm{i}H_F t}Xe^{\mathrm{i}H_F t}, \qquad \int f(t)\,\mathrm{d}t = \pi/2,$$

with f smooth and zero at the endpoints. Its interaction-picture propagator is $-iX$. The unitary energy balance proved in Proposition gives $W_{on} = \varepsilon$. Again the visible channel is unchanged.

The first spectator is often called a battery, but its final entropy increases from zero to $h(p)$. Its energy decrease is

therefore not automatically entropy-free mechanical work. The theorem concerns an explicit energy/resource account. It is compatible with more restrictive operational notions of work [Gallego].

The examples locally reproduce the counterexamples in En07-A/B/C [CheungEn07A,CheungEn07B,CheungEn07C]. They prove that implementation data are missing; they do not prove different optimal costs for the same completely specified optimization problem.

Example. For an independent blank A in $|0\rangle$,

$$\mathrm{SWAP}_{MA}(\rho_{EM} \otimes \mathrm{proj}\, 0_A)\mathrm{SWAP}_{MA}^{\dagger} = \mathrm{proj}\, 0_M \otimes \rho_{EA}.$$

The local memory channel is exact reset for every input, including references. The unknown state has moved to A.

With $H_M = H_A = \varepsilon proj\ 1$ and initial excited probability p, the energy changes are $\Delta E_M = -\varepsilon p$, $\Delta E_A = \varepsilon p$. Changing the gap changes those transfers without changing the reduced reset channel. Restoring the used blank is an additional task.

It would be inaccurate to say that no erasure has occurred in *any* sense: the selected memory has been logically reset. What has not occurred is global deletion of the unknown code or a demonstrated cyclic thermal erasure of all consumed resources. Likewise, SWAP relocates the logical state, not a syntactic runtime name.

Non-inference. A name count is not an energy; a channel distance is not a heat capacity. Reduced channel equality does not price a reusable machine. Conversely, the existence of several implementations does not invalidate a bound for one fully specified implementation class.

Specified reset: the finite thermal identity and its limits

The one-step Gibbs-reservoir identity

Fix a reservoir Hamiltonian H_B, temperature $T > 0$, and inverse energy

$$\beta = (k_{\mathrm{B}}T)^{-1}, \qquad \tau_B = \frac{e^{-\beta H_B}}{Z_B}.$$

Throughout this section, heat deposited in that reservoir means its energy gain for this fixed Hamiltonian:

$$Q_B = \mathrm{Tr}\,[H_B(\omega_B - \tau_B)].$$

This convention is part of the implementation package. A general quantum energy transfer need not admit a unique heat/work interpretation without such a convention.

Theorem. Let the complete modeled non-reservoir system M and the reservoir B initially have product state $\rho_M \otimes \tau_B$. For a specified unitary step,

$$\omega_{MB} = U(\rho_M \otimes \tau_B)U^{\dagger}, \qquad \Delta S_M = S(\rho_M) - S(\omega_M),$$

one has

$$\text{boxed}\ \ \beta Q_B = \Delta S_M + I(M:B)_{\omega} + D(\omega_B \| \tau_B).$$

In particular,

$$Q_B \mathrm{gek}_{\mathrm{B}}\, T\, \Delta S_M.$$

The identity requires neither a cyclic machine nor energy conservation of the step.

Proof. The Gibbs logarithm gives

$$D(\omega_B \| \tau_B) = \beta Q_B - S(\omega_B) + S(\tau_B).$$

Unitarity and the product preparation give

$$I(M:B)_{\omega} = S(\omega_M) + S(\omega_B) - S(\rho_M) - S(\tau_B).$$

Eliminate the bath entropy difference. Nonnegativity of mutual information and relative entropy gives .

This is the finite-reservoir Landauer equality of Reeb and Wolf, with its hypotheses exposed [ReebWolf]. Landauer's original account concerns physical standardization, not a heat assignment to arbitrary channel notation [Landauer]. The identity is also the thermal content of I-C Theorem C and the supplied I-C-Ex and Hawk05-ST

comparisons [CheungIC,CheungEx,CheungHawkST].

Cyclic restoration is *not* a prerequisite for this one-step identity. It becomes essential when a claim concerns repeatable use, total cost per cycle, or restoration of blanks, batteries, and controllers.

Heat positivity has only the stated direction

Corollary. Under Theorem ,

$$\Delta S_M > 0 \quad \Rightarrow \quad Q_B > 0.$$

If $\Delta S_M > 0$, equality in $Q_B \geq \Delta S_M/\beta$ is impossible for this finite step. The converse implication $Q_B > 0 \Rightarrow \Delta S_M > 0$ is false.

Proof. The implication follows from . Equality would force both nonnegative correction terms to vanish. In particular $\omega_B = \tau_B$, hence $Q_B = 0$ and therefore $\Delta S_M = 0$, a contradiction.

For the converse, take matched qubit Hamiltonians $Eproj\ 1$, a Gibbs bath of excited probability $p_{\beta} = (1 + e^{\beta E})^{-1}$, and memory probability $1 - p_{\beta}$. SWAP gives

$$Q_B = E(1 - 2p_\beta) > 0, \qquad \Delta S_M = h(1 - p_\beta) - h(p_\beta) = 0.$$

Here $h(x) = -x\ln x - (1 - x)\ln(1 - x)$. Dropping the correction terms gives an entropy-decrease inequality for the declared bath. It is not by itself a cycle theorem or a derivation of Carnot efficiency.

Example. Take $H_M = H_B = 0$, $dim\ M = dim\ B = d$, and $\tau_B = I/d$. SWAP gives

$$\rho_M \otimes I_B/d \text{longmapsto} I_M/d \otimes \rho_B, \qquad Q_B = 0.$$

The visible M-channel is replacement and has recovery error a_d. A subsequent classical readout of M does not improve it. Coherent access to B returns the code exactly, including references.

The thermal equality remains exact:

$$\Delta S_M = S(\rho_M) - \ln d, \quad I(M:B)_\omega = 0, \quad D(\rho_B \| I/d) = \ln d - S(\rho_M).$$

The two entropy terms cancel at zero energy gain.

This is the I-C zero-heat witness, now with its entropy ledger displayed [CheungIC]. The bath's departure from a Gibbs state can change entropy without changing energy when its Hamiltonian is degenerate.

Initial correlations and nonequilibrium resources

Proposition. Let the initial joint state be ρ_{MB}, not necessarily product, and let $\tau_B = e^{-\beta H_B}/Z_B$ be a fixed full-rank reference Gibbs state. Define

$$Q_B = \mathrm{Tr}\,[H_B(\omega_B - \rho_B)].$$

For unitary evolution,

$$\begin{aligned}\beta Q_B = \quad & \Delta S_M + I(M:B)_\omega - I(M:B)_\rho \\ & + D(\omega_B \| \tau_B) - D(\rho_B \| \tau_B).\end{aligned}$$

Proof. Unitary entropy preservation implies

$$S(\omega_B) - S(\rho_B) = \Delta S_M + I(M:B)_\omega - I(M:B)_\rho.$$

Subtract the two Gibbs-relative-entropy expressions and substitute this equality.

If the initial bath marginal is Gibbs but correlated with the memory, the initial relative entropy term vanishes, but the subtraction of initial mutual information remains. An apparent violation obtained by dropping that term is a missing-resource calculation, not an exception to .

A mathematical purification used to define a complete complement need not be a controlled thermal apparatus. If its purifier is instead included as a physical non-reservoir resource, its initial correlations must be accounted for through . One cannot simultaneously include it in M and retain the false product assumption $\rho_M \otimes \tau_B$.

Approximate reset with an archive

Theorem. Apply Theorem with $M = YR$, where $dim\ Y = m \geq 2$, the archive R has unchanged marginal state, and the final memory satisfies

$$\langle 0|\omega_Y|0\rangle \geq 1-\varepsilon, \qquad 0 \leq \varepsilon \leq 1 - 1/m.$$

Then

$$\beta Q_B \geq S(Y \mid R)_\rho - h(\varepsilon) - \varepsilon \ln(m-1),$$

where $S(Y \mid R) = S(YR) - S(R)$.

If YR is initially classical, the first term is the ordinary conditional Shannon entropy in natural units. For an unbiased bit without side information, the bound is

$$Q_B \text{gek}_\text{B} T[\ln 2 - h(\varepsilon)].$$

Proof. Let $t = 1 - \langle 0|\omega_Y|0\rangle \leq \varepsilon$. Dephase in a basis containing $|0\rangle$. This cannot decrease entropy; distributing the remaining mass uniformly maximizes it. Therefore

$$S(\omega_Y) \leq h(t) + t\ln(m-1) \leq h(\varepsilon) + \varepsilon \ln(m-1).$$

Subadditivity and the unchanged archive give

$$\begin{aligned} S(\rho_{YR}) - S(\omega_{YR}) &\geq S(\rho_{YR}) - S(\rho_R) - S(\omega_Y) \\ &\geq S(Y \mid R)_\rho - h(\varepsilon) - \varepsilon \ln(m-1). \end{aligned}$$

Apply .

This bound does not prove existence or optimal work cost of a reset. Quantum conditional entropy can be negative; a negative right-hand side supplies no guarantee of cooling or work extraction. Quantum side information has a separately developed thermodynamic theory [DelRio]. No attainability theorem from that theory is imported here.

The familiar $k_BTln\ 2$ follows when the complete non-reservoir entropy decreases by $ln\ 2$ under the thermal assumptions. A pure-reset statement at $\varepsilon = 0$ is conditional: the next proposition shows why the finite full-rank setting can make that exact task unavailable.

Proposition. Let $dim\ M = m \geq 2$, $dim\ B = b$, with both ρ_M and τ_B full rank. Set

$$\lambda = \lambda_{\min}(\rho_M)\lambda_{\min}(\tau_B) > 0.$$

Every unitary output satisfies

$$\omega_M \geq b\lambda I_M.$$

Consequently, for every target $|0\rangle$,

$$1 - \langle 0|\omega_M|0\rangle \geq (m-1)b\lambda > 0.$$

Exact pure reset is impossible under these hypotheses.

Proof. The initial joint state is at least λI_MB. Unitary conjugation preserves this inequality. Partial trace gives the marginal lower bound. The trace on the $(m-1)$-dimensional orthogonal complement of $|0\rangle$ gives .

Introducing a pure blank removes a full-rank hypothesis, but consumes a resource, as Example shows. A limit of increasingly accurate protocols requires its own growing-reservoir, resource, or other implementation analysis. It is not constructed by writing $\varepsilon downarrow 0$ in .

An exact archive-corrected SWAP account

Let Y and B be matched qubits with Hamiltonian $Eproj\ 1$. The archive R may initially be correlated with Y, but RY is independent of the Gibbs bath. After SWAP,

$$\omega_{RYB} = \tau_Y \otimes \rho_{RB},$$

up to commuting free phases.

Proposition. For this process,

$$\Delta S_{RY} = S(Y \mid R)_\rho - S(\tau_B), \qquad I(RY:B)_\omega = I(R:Y)_\rho.$$

If ρ_Y is diagonal with excited probability p,

$$Q_B = E(p - p_\beta), \qquad D(\omega_B \| \tau_B) = D_{\text{bin}}(p \| p_\beta).$$

These terms satisfy on the complete non-reservoir system *RY*.

Proof. The final *Y*-factor is independent of *RB*, so *S(ω_RY) = S(ρ_R) + S(τ_B)*. The original *RY*-correlation is transferred to *RB*. The bath marginal is the original *ρ_Y*, giving the energy and relative-entropy formulas. Their sum agrees with the thermal identity.

For a perfect classical archive of a uniform bit, *S(Y | R) = 0*. The local memory entropy decreases from *ln 2* to *h(p_{β})*, but the complete non-reservoir entropy decrease is *−h(p_{β})*, not *ln 2 − h(p_{β})*. The final archive–bath mutual information is *ln 2*. Counting the local entropy decrease while omitting the surviving archive would count the same information incorrectly.

Reusable restoration is a further specification

A reusable apparatus claim must identify which factors return to their initial states, which correlations must be removed, and which resources are replenished. Restoring a controller marginal is weaker than restoring its joint state independently of the logical input. Replacing a blank or fuel cell by a fresh one is an allowed operation only if the fresh supply is included in the resource account.

Reversible uncomputation is also a different task: *U^{†}U* restores a complete coherent input when all required factors are available. It does not erase the unknown input. Bennett's compute–copy–uncompute account is the established comparison [Bennett]. It does not grant the joint inverse after the necessary quantum factors have been excluded from access.

Non-inference. The one-step equality is not a cost optimization, a cycle theorem, or a physical resetting machine. The spectator constructions consume resources. They do not demonstrate evasion of a specified thermal erasure bound.

Record entropy, residual entropy, and zero-heat revival

The entropy of a retained report

Fix one input state and its unconditioned law. Let *R_n = (X_1, …, X_n)* contain every retained outcome, failure, and stop symbol. An append-only continuation preserves old values and is trace preserving when all new outcomes are summed.

Theorem. For every finite append-only report,

$$H(R_{n+1}) - H(R_n) = H(X_{n+1} \mid R_n) \geq 0.$$

Equality holds exactly when the new symbol is almost surely a function of the previous history.

For a classical–quantum output

$$\omega_{RK} = \mathrm{bigoplus}_r p_r \rho_{K_r \mid r}, \qquad \dim K_r \leq k,$$

$$\begin{aligned} S(RK) &= H(R) + \textstyle\sum_r p_r S(\rho_{K_r \mid r}), \\ H(R) &\leq S(RK) \leq H(R) + \ln k. \end{aligned}$$

Neither the residual contribution nor total hybrid entropy is generally monotone.

Proof. Trace preservation preserves the old-history marginal. Expand *p(r, x) = p(r)p(x | r)* in Shannon entropy. Finite classical conditional entropy is an average of nonnegative entropies, zero precisely for point-mass conditional laws.

If *λ_{r, j}* are residual eigenvalues, the block eigenvalues are *p_rλ_{r, j}*. Substitution gives . For nonmonotonicity, keep a nontrivial classical record and reset an independent mixed residual to a pure state. Its hybrid entropy decreases. That last example is a channel statement, not a free thermal implementation.

This locally establishes I-C Theorem B and the I-C-Ex hybrid refinement [CheungIC,CheungEx]. The supplied Hawk05-A extract states the same identity; its title-page authorship is not inferred here [Hawk05ARecord].

Equation is a chain-rule consequence of retaining a particular classical register. It is not a new thermodynamic second law. A lossy summary, a postselected ensemble, or an observer's conditional uncertainty about another message is a different object. For continuous records, the statement applies to declared finite digitizations, not to unrestricted differential entropy.

Once *H(R) > 0*, the raw classical–quantum output cannot be pure while that same record is retained. This does not imply that the complete microscopic dilation has become mixed or classical.

Classical entropy growth without compulsory bath heat

For a finite basis measurement, the isometry

$$V|\psi\rangle = \sum_j \langle j \mid \psi\rangle |j\rangle_R |j\rangle_A$$

has the desired dephased record as its *R*-reduction. It can be extended to a unitary with suitable blank apparatus factors.

Take all participating apparatus Hamiltonians zero and leave a separate Gibbs reservoir uncoupled. Its heat is exactly zero, although a superposed input can produce a positive-entropy classical record. Globally the isometry preserves the input spectrum and reference correlations.

Fresh fair-bit records can similarly be appended using fresh pure blanks and finite unitary recording dilations. Then *H(R_n) = nln 2* while the designated reservoir remains unchanged. The blank supply and unreturned correlations are not cyclically restored by this construction. There is no contradiction with Section .

A Bell pair gives a second elementary distinction:

$$S(B_1) = \ln 2, \qquad S(B_1B_2) = 0.$$

After measuring both in the correlated basis,

$$H(X_1) = \ln 2, \qquad H(X_1, X_2) = \ln 2.$$

Quantum conditional entropy can be negative; classical conditional entropy in cannot.

A finite revival with zero bath-energy change

Let *S, B* be qubits with bare Hamiltonians zero and initial state

$$\rho_{SB}(0) = \rho_S \otimes I_B/2.$$

Install the smooth pulsed interaction

$$H_{\text{int}}(t) = g(t) Z_S \otimes Z_B, \qquad \theta(t) = \int_0^t g(s)\,\mathrm{d}s.$$

Its unitary is *U_t = e^{−iθ(t)Z_SZ_B}*, and its reduced channel is

$$\Lambda_t(\rho) = \begin{pmatrix} \rho_{00} & c(t)\rho_{01} \\ c(t)\rho_{10} & \rho_{11} \end{pmatrix}, \qquad c(t) = \cos(2\theta(t)).$$

Theorem. For *ρ_{±} = |±⟩⟨±|*,

$$d_{\text{tr}}(\Lambda_t(\rho_+), \Lambda_t(\rho_-)) = |c(t)|.$$

For input *ρ_{+}*,

$$S(\Lambda_t(\rho_+)) = h\left(\frac{1 + |c(t)|}{2}\right).$$

The optimal deterministic quantum-return error is

$$e_{\text{rec}}(\Lambda_t) = \frac{1 - |c(t)|}{2}.$$

When *θ* passes from *0* through *π/4* to *π/2*, distinguishability falls to zero and revives.

The bath remains *I/2*, and *Q_B(t) = 0*. For pure input *ρ_{+}*,

$$\Delta S_S = -S(\omega_S), \quad I(S:B)_\omega = S(\omega_S), \quad D(\omega_B \| I/2) = 0.$$

Thus holds exactly at zero bath heat.

Proof. Condition on the two bath *Z*-eigenvalues to obtain . The difference of the two system outputs has eigenvalues *±c(t)*, and the single output has eigenvalues *(1 ± |c(t)|)/2*.

Correcting the sign of *c*, if needed, by *Z*, gives

$$\frac{1+|c|}{2}\mathrm{id}+\frac{1-|c|}{2}\mathrm{Ad}_Z.$$

This has error at most *(1 − |c|)/2*. For any decoder of error *e*, contraction and the two orthogonal target inputs give *1 ≤ 2e + |c|*, proving optimality.

The bath populations and marginal are unchanged. The joint entropy is initially and finally *ln 2*; with *S(B) = ln 2*, its mutual information is *S(S)*. The thermal statements follow.

If *g* vanishes at the endpoints, the interaction energy also vanishes there. Moreover ⟨*Z_SZ_B*⟩ *= 0* initially and is conserved, so the switching-work expectation is zero: ∫ *ġ* ⟨*Z_SZ_B*⟩ *dt = 0*. This is an explicit finite ledger, not an assertion that all reservoir engineering or control preparation is free.

The revival is a finite non-Markovian witness: no CPTP intermediate map can turn the coincident outputs at *c = 0* into distinguishable outputs later. Its relation to information-flow diagnostics is standard [BreuerLainePiilo]. It is the model used in I-C-Ex2, locally derived here [CheungEx2].

A constant selected bath is not a constant complete complement

Purify the initial *I_B/2* by a fixed Bell vector |*Φ_2*⟩*_BP*. For basis input |*z*⟩*_S*, the complementary *BP*-state is

$$|\chi_z\rangle = (e^{-\mathrm{i}(-1)^z\theta Z_B}\otimes I_P)|\Phi_2\rangle.$$

Both selected bath marginals are *I_B/2*, but

$$|\langle\chi_0\mid\chi_1\rangle| = |c(t)|.$$

They are distinct whenever |*c(t)*| < *1*. Thus constancy of *B* does not establish constancy of the complete complement *BP*.

The thermal equality above concerns the initially product reduced *SB*-experiment. If the purifier is physically included in the non-bath system *SP*, initial *SP: B* correlations are present and Proposition , not the product version, applies. There is no permission to omit a purification from a recovery theorem while using it as a physical resource elsewhere.

Appending noninformative finite system instruments *L_0 = L_1 = I/√(2)* adds fair bits without changing the revival channel. The transcript entropy can then rise at every attempt while the residual entropy oscillates. Recovering at a revival uses a live quantum factor. It is not decoding from those classical bits.

Entropy histories and selected spectra do not decide recovery

Proposition. Let *D = 2^N ≥ d*, let *J: Q → (ℂ^2)^{⊗N}* be an isometry, let *U ∈ U(D)*, and let *L ≅ Q* be inaccessible. Compare

$$V_{\mathrm{g}}|\psi\rangle = UJ|\psi\rangle|0\rangle_L, \qquad V_{\mathrm{b}}|\psi\rangle = UJ|0\rangle|\psi\rangle_L.$$

Release the same output factors in the same order. Then:

• for input |*0*⟩, all accessible prefix states agree;

• under the additional Haar hypothesis for *U*, every fixed pure input independent of *U* has the same joint distribution of prefix entropies in the two ensembles;

• endpoint recovery errors are *0* and *a_d*. The construction can conserve a declared energy by taking the logical labels in a degenerate energy sector.

Proof. For the fixed probe the complete vectors coincide. For any fixed unit vector *Jψ*, *UJψ* has the same uniform distribution as *UJ0*; every prefix entropy is a function of that vector. The good endpoint is isometric and invertible on its range. The bad endpoint is replacement, so Lemma gives *a_d*. Unitaries acting within a degenerate sector commute with its scalar energy.

This is the finite mechanism of the Hawk05-B and Hawk05-ST comparisons, not a physical radiation history [CheungHawk05B,CheungHawkST]. The Haar hypothesis is input data, not a thermal or gravitational law.

Selected thermal spectra are equally insufficient. If *σ_K* has purification |*χ*⟩*_KP*, the two isometries

$$|\psi\rangle \mapsto |\psi\rangle_{Q_R}|\chi\rangle_{KP}|0\rangle_L, \qquad |\psi\rangle \mapsto |0\rangle_{Q_R}|\chi\rangle_{KP}|\psi\rangle_L$$

have identical accessible spectra for every pure input on *Q_RK*, but errors *0, a_d*. For *H_R = I_Q_R ⊗ H_K*, their selected energy laws agree. One may choose *σ_K* Gibbs. The selected thermal factor is not the complete code-to-output channel.

Non-inference. Classical transcript monotonicity is not radiation entropy, a Page curve, or a generalized gravitational second law. A shared entropy history is not a recovery certificate. A memory outside the declared algebra does not increase the accessible quantum block dimension in Theorem .

Energy and temperature require their own data

A specified unitary energy balance

Proposition. Let a finite system obey

$$\dot{\rho}(t) = -\mathrm{i}[H_{\mathrm{tot}}(t), \rho(t)]$$

with differentiable Hamiltonian. Define external work on the complete modeled system by

$$W_{\mathrm{on}} = \int_0^{t_f} \mathrm{Tr}\,[\rho(t)\dot{H}_{\mathrm{tot}}(t)]\,\mathrm{d}t.$$

Then

$$W_{\mathrm{on}} = \mathrm{Tr}\,[\rho(t_f)H_{\mathrm{tot}}(t_f)] - \mathrm{Tr}\,[\rho(0)H_{\mathrm{tot}}(0)].$$

If

$$H_{\mathrm{tot}}(t) = H_M(t) + H_B + H_{\mathrm{int}}(t)$$

with fixed *H_B*, and the interaction vanishes at both endpoints, then

$$W_{\mathrm{on}} = \Delta E_M + Q_B.$$

With nonzero endpoint interaction energies, their expectation change must be added. A separately excluded battery contributes its own *ΔE_W*.

Proof. Differentiate *Tr (ρH_tot)*. The commutator term vanishes by cyclicity. Integrate and split the endpoint Hamiltonians into their declared terms.

Equation is energy bookkeeping under a specified dynamics and work convention. It is not derived from Enter. With heat into *M* denoted *Q_M = −Q_B*, the same convention reads *ΔE_M = Q_M + W_on*.

For fixed *H_tot*, exact conservation for every input means

$$U^{\dagger}H_{\mathrm{tot}}U = H_{\mathrm{tot}}.$$

For an input/output isometry, an energy-intertwining hypothesis is *H_outV = VH_in*. Neither follows from unitarity alone: *X* is unitary but does not commute with *Eproj 1*.

Energy conservation does not imply outgoing recovery. SWAP with an inaccessible matched blank conserves the sum Hamiltonian while leaving the selected memory in a constant state. This is the En07-C energy separation [CheungEn07C].

Free Hamiltonian is not thermodynamic free energy

For fixed *H*, define the nonequilibrium Helmholtz functional

$$\mathscr{F}_\beta(\rho) = \mathrm{Tr}\,(H\rho) - \beta^{-1}S(\rho).$$

A direct Gibbs-logarithm calculation gives

$$\mathscr{F}_\beta(\rho) - \mathscr{F}_\beta(\tau_\beta) = \beta^{-1}D(\rho\|\tau_\beta).$$

Conservation of a bare or ``free" Hamiltonian *H_0* does not imply conservation of this functional: a marginal can change entropy while preserving its energy. Nor is a battery energy change automatically a statement about extractable work or cyclic restoration.

For the bath in ,

$$\Delta\mathscr{F}_\beta(B) = \beta^{-1} D(\omega_B \| \tau_B) \geq 0.$$

The correction term records the final bath's departure from its initial Gibbs state. Calling all such finite-bath energy gain ideal macroscopic heat is a convention that must not conceal the state change.

Temperature can be inferred only with calibration

A finite Gibbs preparation has

$$\tau_\beta(H) = \frac{e^{-\beta H}}{\operatorname{Tr} e^{-\beta H}}.$$

If H is not specified, the state alone has the energy-scale ambiguity

$$\tau_\beta(H) = \tau_{\beta/s}(sH), \qquad s > 0.$$

For $H = 0$, the state is maximally mixed for every positive temperature.

This does not prohibit thermometry. For a known qubit gap $E > 0$, a Gibbs occupation $p = (1 + e^{\beta E})^{-1}$ determines

$$\beta = \frac{1}{E} \ln \frac{1-p}{p}.$$

An actual estimate requires a preparation, measurement, sampling assumptions, and error account. A finite dictionary that reports a supplied temperature constructs neither that preparation nor such an acquisition protocol [CheungVP8].

A finite KMS example and a zero-temperature limit

For finite H, fix dynamics $\tau_t(A) = e^{itH} A e^{-itH}$.

Proposition. For every $\beta > 0$, the Gibbs state has the KMS strip property

$$F_{A,B}(t) = \operatorname{Tr}[\tau_\beta A \tau_t(B)], \qquad F_{A,B}(t + i\beta) = \operatorname{Tr}[\tau_\beta \tau_t(B) A].$$

It is invariant under the fixed dynamics.

Proof. Set $F_{A,B}(z) = \operatorname{Tr}[\tau_\beta A e^{izH} B e^{-izH}]$. In finite dimension it is entire, continuous and bounded on the closed strip $0 \leq \operatorname{Im} z \leq \beta$. Use $e^{-\beta H} = Z\tau_\beta$ and cyclicity to obtain the reversed upper boundary. Commutation of τ_β with H gives invariance.

The general algebraic equilibrium framework is established KMS theory [HHW]. This finite proof does not construct the continuum Friedrichs state of I-B-VP-8.

Proposition. Let the ground energy of a finite H have projection P_0 of rank g. Then

$$\tau_\beta \to P_0/g \quad \text{in trace norm as} \beta \to \infty, \qquad S(\tau_\beta) \to \ln g.$$

If excited levels exist with smallest gap $\Delta > 0$ and total dimension m, then

$$d_{\mathrm{tr}}(\tau_\beta, P_0/g) \leq \frac{m-g}{g} e^{-\beta\Delta}.$$

Proof. Subtract the ground energy. The total excited Gibbs weight is

$$\frac{\sum_{\mathrm{exc}} e^{-\beta E_j}}{g + \sum_{\mathrm{exc}} e^{-\beta E_j}},$$

which equals the displayed trace distance and satisfies the bound. Entropy continuity in finite dimension gives the limit. If there are no excited levels, the state is already I/g.

A limit indexed by β is not a process reaching zero temperature in finite time. No cooling dynamics or time resource is specified. The limit does not establish unattainability, a Nernst theorem, or universal vanishing ground-state entropy.

The field comparison: thermal infrared control is stronger

Write A_F for a supplied nonnegative spatial Friedrichs operator and $h = \sqrt{A_F}$. A scalar Bose occupation is

$$n_\beta(\omega) = \frac{1}{e^{\beta\omega} - 1}.$$

For spectral vectors in the indicated domains,

$$C_\beta = \frac{1 + 2n_\beta(h)}{2h}, \qquad C_0 = \frac{1}{2h}.$$

The elementary inequality $e^x - 1 \geq x$ gives

$$0 \leq \langle v, (C_\beta - C_0)v \rangle \leq \beta^{-1} \|h^{-1}v\|^2.$$

Thus this sufficient thermal estimate uses $v \in dom(A_F^{-1/2})$, whereas ground-state quadratic finiteness requires only $dom(A_F^{-1/4})$. Absence of an L^2 zero mode does not alone prove either required infrared estimate.

The supplied I-B-VP-8 record provides the stronger compact-test control and its Friedrichs KMS construction under a fixed massless product, holonomy, and centered zero-chemical-potential prescription [CheungVP8]. The ground-state estimate in I-B-VP is not substituted for that thermal theorem [CheungVP]. The source's weak zero-temperature limit is not promoted to a cooling or third-law result.

The analytic heat semigroup e^{-sA_F} has a ``heat kernel." Its parameter s and kernel are spectral analysis objects. They are not the reservoir energy gain Q_B. Even if a particular heat trace can be used to form a partition function, no energy-transfer history follows merely from that name.

A geometrical Bose factor is not a bath implementation

In the source-qualified Em03-A horizon-current comparison, with its prescribed surface gravity and Killing-time normalization,

$$n_{\mathrm{H}}(\omega) = \frac{1}{e^{\beta_{\mathrm{H}}\omega} - 1}, \qquad \beta_{\mathrm{H}} = \frac{2\pi}{\kappa_{\mathrm{H}}}.$$

Its two-point spectral expression contains the factors $1 + n_H$ and n_H, and satisfies the stated detailed-balance relation [CheungEm03A]. The bulk state existence and the physical Hawking comparison remain external field-theoretic inputs [Hawking].

The symbol κ_H here is surface gravity, not a gravitational coupling constant. A source-qualified horizon two-point KMS relation does not make every greybody-weighted outgoing state, switched detector, or selected exterior observable a KMS system at that temperature. It also does not identify that field with the independent finite Gibbs reservoir of Theorem .

Heating and material failure require another model

Proposition. Suppose a device has a temperature $\Theta(t)$ on an interval where $C_{dev}(\Theta) = U_{dev}'(\Theta) > 0$, and an independently prescribed net power:

$$\frac{\mathrm{d}}{\mathrm{d}t} U_{\mathrm{dev}}(\Theta(t)) = P_{\mathrm{dep}}(t) - P_{\mathrm{out}}(t) = P_{\mathrm{net}}(t).$$

Then

$$\dot{\Theta} = P_{\mathrm{net}}/C_{\mathrm{dev}}.$$

For a higher threshold Θ_{fail}, the device is below threshold at time t precisely when

$$\int_0^t P_{\mathrm{net}}(s)\,\mathrm{d}s < U_{\mathrm{dev}}(\Theta_{\mathrm{fail}}) - U_{\mathrm{dev}}(\Theta(0)).$$

To exclude any crossing on $[0, T]$, the strict inequality must hold for every $t \in [0, T]$, not merely at T.

Proof. Use the chain rule and integrate the energy balance. Strict increase of U_{dev} gives the threshold comparison at each time.

Readout does not provide the power, heat capacity, cooling law, or material failure model. Positive heat deposited in an external reservoir need not raise the computer's temperature. A later low temperature does not exclude an earlier threshold crossing.

Finally, the prescribed cone is not sourced by these finite ledgers. A constant tensor $t_{\mu\nu} = diag(3e, e, e, e)$, $e > 0$, in a flat chart is symmetric, conserved, and traceless, but does not satisfy an Einstein source equation with $\Lambda = 0$ on that fixed flat metric. This is the I-B prop:not-source comparator [CheungIB].

Non-inference. A thermal state prescription is not a reservoir reset; a thermal factor is not a complete emission channel; an analytic heat kernel is not Q_B. None derives the metric, a material explosion, or Einstein dynamics.

Endpoints, applicability tests, and finite release

The last instruction does not determine the last access

Proposition. For a recorded prefix

$$\mathscr{R}(X) = \mathrm{bigoplus}_r \mathscr{P}_r(X)_A,$$

the following terminal specifications preserve the old record law:

• keep A inaccessible;

• destructively measure A and retain all new outcomes;

• grant coherent access to A. They can differ only in ownership or can also differ in the complete output map. Granting A need not by itself enable code recovery.

Proof. Any trace-preserving continuation on A, including the sum of all branches of a measurement, leaves its traced output equal to the old record marginal. Ownership alone need not change the unrestricted joint map.

Finite-program termination is also not equilibrium. A schedule can stop on a qubit in $|+\rangle$ with $H = Eproj\ 1$; its subsequent free state is nonstationary. No equilibration follows from the absence of additional instructions.

A vanishing energy label does not imply a one-dimensional remaining Hilbert space. A zero-energy degeneracy can retain an arbitrary code. These are the endpoint distinctions in I-C, En07-B/C, and Hawk01-B [CheungIC,CheungEn07B,CheungEn07C,CheungHawk01B].

Complete complements constrain leakage

For an isometry $V: Q \to A \otimes B$, define

$$\mathscr{N} = \mathrm{Tr}_B \mathrm{Ad}_V, \qquad \mathscr{E} = \mathrm{Tr}_A \mathrm{Ad}_V, \qquad \delta_{\mathrm{leak}}(\mathscr{E}) = \inf_{\mathrm{sigmad}} _\Diamond(\mathscr{E}, \mathscr{T}_\sigma).$$

Theorem. Exact whole-code recovery from A is equivalent to

$$\mathscr{E}(X) = \mathrm{Tr}\,(X)\sigma_B \quad \text{for every} X \in \mathscr{B}(Q).$$

If $s = rank\ \sigma_B$, then

$$\mathscr{N}(X) = J(X \otimes \sigma_S)J^\dagger, \qquad J : Q \otimes \mathbb{C}^s \to A \text{ isometric}, \qquad \dim A \geq ds.$$

The two complementary outputs cannot independently recover a nontrivial whole code.

Proof. Diagonalize $\sigma_B = \sum_\{a = 1\}\^s p_a|b_a\rangle\langle b_a|$, $p_a > 0$. Write $V|i\rangle = \sum_a \sqrt{(p_a)}|u_ia\rangle|b_a\rangle$. The complementary matrix-unit identities imply $\langle u_jb \mid u_ia\rangle = \delta_ij\delta_ab$. This gives J and the normal form. An everywhere-defined decoder is

$$\mathscr{D}(Y) = \mathrm{Tr}_S\,(J^\dagger Y J) + \mathrm{Tr}\,[(I - JJ^\dagger)Y]\tau_Q.$$

Conversely, dilate an exact decoder by $L: A \to Q \otimes G$. For a basis input, purity of the decoded state gives $(L \otimes I_B)V|i\rangle = |i\rangle|\xi_i\rangle$. Exactness on all off-diagonal matrix units gives $\langle \xi_j \mid \xi_i\rangle = 1$; all these vectors coincide. Tracing QG makes the complement constant. Interchanging A, B proves the last assertion.

This is the standard exact-correction/no-hiding structure [KnillLaflamme,BraunsteinPati]. It is a full-code theorem, not a universal subsystem criterion. The rank is that of the constant complete complement, not that of a selected bath marginal.

Proposition. For pure fixed-ancilla dilations of the same finite visible channel, their complete complementary channels agree up to an isometry between their effective environment supports. Their optimized replacement distances δ_leak agree.

Proof. Let *V_i: Q → A ⊗ B_i* dilate the same channel. In each *B_i*, take the span of vectors *(⟨a| ⊗ I)V_i|q⟩*. The inner products of all such vectors are matrix elements of the common visible channel applied to *kb qq^{'}*. Their Gram matrices agree. Sending one generating family to the other therefore defines an isometry between the spans, and *V_2 = (I_A ⊗ W)V_1* on the code.

Embedding a replacement state gives one inequality for *δ_leak*. For the opposite inequality use the CPTP left inverse

$$L_W(Y) = W^\dagger Y W + \mathrm{Tr}\,[(I - WW^\dagger)Y]\tau$$

on the effective support and channel contractivity.

Thus the three ledgers must not be advertised as three arbitrarily independent variables. In the complete pure dilation convention, the visible channel constrains complementary information. The heat can still vary with implementation Hamiltonians and independent resources, as Theorem proves.

A sufficient endpoint package, not an endpoint derivation

Supply:

• a finite code and complete preparation, with required purifiers retained;

• justified complete unitary evolution, hence isometries *V_t: Q → A_t ⊗ B_t*;

• access-preserving isometries *a_t: A_t → A_{*}, b_t: B_t → B_{*}* and a proved comparison

$$\|\widehat{V}_t - V_*\|_\infty \le \nu_t, \qquad \widehat{V}_t = (a_t \otimes b_t)V_t, \qquad \nu_t \to 0;$$

• a constant *complete* endpoint complement *Tr_{A_{*}} Ad_{V_{*}} = 𝒯_{σ_{*}}*. The embeddings cannot rename a still-inaccessible logical register as accessible output.

Proposition. Under Specification , *V_{*}* is an isometry and there is a fixed endpoint decoder *𝒟_{*}* such that

$$\mathcal{D}_* \mathrm{Tr}_{B_*} \mathrm{Ad}_{V_*} = \mathrm{id}_Q.$$

The decoder *𝒟_{*}Ad_a_t* on the actual accessible output has error at most *min (1, ν_t)*. A further complete receiver allowance *α_t* adds at most *α_t*.

More generally, if a specified endpoint decoder has a proved error *e_{*}*, the same argument gives *min (1, e_{*} + ν_t + α_t)*.

Proof. The operator-norm limit of isometries is isometric. Theorem gives the exact decoder. For isometries *V, W*, expansion of *VXV^{†} − WXW^{†}*, including references, gives half-diamond distance at most *‖V − W‖_{∞}*. Partial trace, decoding, and the triangle inequality give the stated budgets.

This locally proves the exact-complement endpoint implication. The broader approximate-complement bounds in the supplied En07-B/C records need their own information–disturbance input; no unproved approximate-complement constant is used in this paper [CheungEn07B,CheungEn07C].

For example, if the complete accessible channel is also within *η_t* of replacement, then the exact-endpoint package requires

$$a_d \le \eta_t + \nu_t.$$

Indeed, every decoder after a replacement has error at least *a_d*; contraction gives the lower bound *e_rec ≥ a_d − η_t*, whereas the endpoint decoder gives the upper bound *ν_t*. Featurelessness and accurate recovery cannot be asserted with mutually inconsistent allowances on one interface.

Applicability map: corrected shortcuts, not rejected laws

begingroup toprule Shortcut to avoid Correct finite statement Local witness

midrule endfirsthead toprule Shortcut to avoid Correct finite statement Local witness

midrule endhead bottomrule endfoot

Every consumed name costs *k_BTln 2*. A specified entropy-decreasing reset under the Gibbs hypotheses has a lower bound. Name consumption alone does not. Theorem ; Example .

One entropy determines the process. Classical record, residual, hybrid, and complete-system entropies refer to different objects. Theorem ; Theorem .

Loss of access implies positive heat. A declared visible replacement can coexist with zero bath energy gain and exact recovery elsewhere. Example .

A channel fixes its full heat/work bill. It fixes selected output states; implementation energy and restoration require further data. Theorem ; Proposition .

Termination is equilibrium or evaporation. The terminal schedule and terminal ownership are additional specifications. Proposition ; Specification .

Temperature is a label read off an arbitrary channel. It requires a Hamiltonian, state/dynamics convention, and, for inference, calibration. Propositions –.

A thermal entropy history certifies quantum return. Selected histories and spectra can agree while recovery errors are *0* and *a_d*. Proposition .

endgroup

The first column is not presented as a faithful statement of established thermodynamics. It lists shortcuts that can arise when its hypotheses are compressed into interface language.

Resolution of the four proposed development steps

• The fixed-visible-channel heat non-identifiability question is proved in Theorem with fixed spectator Hamiltonians and conserving dynamics.

• The resource-restoration issue is resolved as a specification requirement, not an optimum-cost theorem. Example identifies the consumed blank; the spectator theorem identifies consumed fuel and changed battery entropy.

• Equality with partial trace or a selected reset is shown insufficient for a full implementation bill. Proposition and Example exhibit the channel equalities; the thermal and work counterexamples leave them unchanged.

• Independent unobserved operations change heat or work even after the visible memory Hamiltonians are fixed. Both independent constructions are printed in Theorem .

These questions were already accompanied by source counterexamples in En07-A/B/C. They are not reclassified as previously unsolved theorems. The local contribution is a coherent derivation and audit of their scope.

Mathematical release and unchanged programme boundary

Theorem. Under the printed hypotheses:

• complete destructive readout and discard have the channel identity , but no heat assignment;

• Theorem establishes constructive heat/work/resource non-identifiability from a selected visible channel;

• the finite Gibbs identity, its correlation correction, and the archive-sensitive approximate-reset bound are , , and ;

• positive complete entropy decrease is sufficient, not necessary, for positive bath heat, and excludes exact pure reset in the specified finite full-rank setting;

• record, residual, and hybrid entropies have the separations and exact revival formulas proved in Section ;

• energy balance, KMS input, zero-temperature limit, and lumped heating have their separate hypotheses in Section ;

• terminal ownership is not selected by a prefix, while the exact endpoint package implies the locally proved recovery budget. No new thermodynamic law, optimum cyclic cost, Carnot efficiency, unattainability theorem, physical endpoint, or gravitational interpretation is released.

Proof. Items (i)–(iv) are Proposition , Theorem , Theorem , Proposition , Theorem , Corollary , and Proposition . Theorems and , with Proposition , prove (v). The named local energy and temperature proofs give (vi). Propositions and give (vii). None constructs an excluded physical model.

The five source-named Hawk05-B gravitational obligations remain open here [CheungHawk05B,CheungHawkST]. The four finite endpoint obligations are not identified with those five gravitational obligations.

The historical H2 stop remains version-qualified to XIV-H; XIV-I's narrower constructed I2 amplitude is not withdrawn. The required full I3/J2 characteristic maps, subsequent joint-section, transverse, contact, output, and admission obligations are not supplied by any thermal identity in this paper [CheungAnalytic]. The existence of technical XV-B lifting results is not an opened downstream quantum-admission exit.

par **Standing conclusion** par The cone, its field realizations, the reservoir, its Hamiltonian, and β remain inputs where used.

Readout is not automatically reset. Local reset is not global deletion. Record entropy is not a universal thermodynamic entropy. Heat is not leakage. A heat kernel is not reservoir heat. Termination is not equilibrium. A thermal spectrum is not a quantum-return channel.

O.1 on the cone remains *texttt NOT_CLOSED*. *texttt bulk_symmetry_restored = texttt false*. XVI-N and Route B are not discharged. The downstream XV-B quantum-admission exit is not opened. No inherited characteristic quantum coefficient is admitted.

No spacetime emergence, Einstein dynamics, continuum detector, Page curve, island, or physical evaporation endpoint is constructed. par

Declarations

Nature and novelty of the work. This is a finite analytical expansion of an implementation ledger, with source-qualified continuum comparisons. Its proposed significance is methodological and integrative. The thermal identity, Schmidt-number mechanism, error-correction criterion, KMS framework, and reversible computation principles are established results. No priority claim is made for them.

Authorship and submission. The author field follows the supplied I-C manuscript series. Submission requires the author's approval of the mathematics, attribution, source versions, and final text. Submission format and written local proofs do not constitute independent technical review or a guarantee of acceptance.

Source consultation and version status. The attached I-C-v2 and I-C-Ex-v2 sources, the earlier I-C-Ex2 text in the supplied conversation, and the supplied version-qualified excerpts were available for this preparation. The public philosophy and I-B pages were consulted. The attempted I-C, En07-C, and I-B-VP-8 browse endpoints did not return usable pages through the consulted interface. Those references therefore rely on the supplied text or excerpts, not a claimed independent retrieval of their full registered sources.

The Frontiers publisher record for [CheungEnter] was consulted; it reports acceptance on 28 September 2026 and that final formatting is forthcoming. No later final-format status or page range is invented.

Companion citations are not hidden premises. Every finite result needed for the release has its local proof here. Companion citations identify provenance, comparisons, and stated limitations. They do not supply unprinted field acquisition, microscopic gravitational dynamics, restoration protocols, or admitted characteristic coefficients.

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Physical and optimization scope. No complete reusable erasure machine, work-extraction cycle, reservoir-size optimum, physical thermometer, cooling protocol, continuum detector, or gravitational endpoint is constructed. The independent spectator examples establish missing implementation data; they are not cyclic cost optima. No quantum conditional-entropy attainability theorem is asserted.

Execution. No successful pdfLaTeX execution, numerical calculation, symbolic-program run, gate compilation, laboratory implementation, proof-assistant check, independent rebuild, or independent technical review is asserted. The mathematical evidence consists of the written statements and proofs on their declared domains.

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N. Cheung, *Hawk05-B*, version 1, supplied excerpts, Enter Calculus Companion. thm:retain: conserving finite entropy-history/recovery separation. The source-named gravitational obligations remain outside the present construction.

N. Cheung, *Hawk05-ST*, version 1, supplied excerpts, Enter Calculus Companion. thm:thermal, thm:retain, and the finite Enter/Page assembly. Not a gravitational Page curve or endpoint.

N. Cheung, *Hawk01-B*, version 1, supplied excerpts, Enter Calculus Companion. thm:endpoint and prop:endpointtest: a declared endpoint completion and its separate no-hidden-factor conditions. Not semiclassical endpoint dynamics.

N. Cheung, *I-B-VP: Vacuum polarization of the Friedrichs ground state on a prescribed conical spacetime*, version 3, supplied excerpts, Enter Calculus Companion. prop:heat, lem:infrared, and the distinguished spectral ground-state prescription. The heat kernel is not a reservoir heat transfer.

N. Cheung, *I-B-VP-8*, version 1, supplied excerpts, Enter Calculus Companion. thm:heat, thm:KMS,

thm:zerotemperature, and prop:instrument. Friedrichs massless product KMS construction with specified temperature and stronger thermal infrared control. No thermal acquisition apparatus or third-law conclusion is transferred here.

N. Cheung, *Em03-A*, version 1, supplied excerpts, Enter Calculus Companion. lem:fourier, eq:thermalspectral, and eq:detailedbalance: source-qualified horizon-current spectral comparison. Bulk Unruh-state existence remains an external input.

N. Cheung, *XIV-H*, *XIV-I*, and *XIV-J*, version 1; *XV-B*, version 2; and supplied *XVI-N*, revision r12, and *XVI-O*, revision o70-sync scope records, Enter Calculus Companion. Historical H2 stop, narrower I2 internal construction, unresolved full I3/J2 characteristic target, technical lifting interfaces, and unchanged programme admission boundaries. Documentary comparisons only.

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