# I-C-Ex-v2: Sharp Access Bounds, Residual Capacity, and Nine Extensions of Destructive Enter Classical trajectories, named observables, thermal ledgers, and prescribed-spacetime comparisons

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**Abstract**

An informative classical record need not return an unknown quantum code. We develop this distinction into a common access framework for nine extensions of the destructive Enter interface on the prescribed conical host of I-B. The central quantitative result is a sharp resource law: if the complete message-bearing output consists of an arbitrary classical record and a quantum factor of dimension at most k, every deterministic decoder for a d-dimensional code satisfies F_e ≤ k/d, 1/2‖𝒟𝒩 − id ‖_{◇} ≥ 1 − k/d, 1 ≤ k ≤ d. A finite-outcome construction attains both constants for every d, k. Branch-dependent dimensions, implementation allowances, and genuine path-space instruments are included. Version 2 strengthens the framework in four directions. Exact recovery requires accessible dimension at least d times the rank of the constant complete complement. Hybrid entropy separates monotone classical accumulation from nonmonotone quantum residual entropy. Continuous programming of one fixed finite reporting instrument yields a complete-channel limit without enlarging its declared quantum access. Finally, the distance between two binary-effect channels equals the operator-norm difference of their effects, giving a numerical, correctly directed comparison between observable reconstruction and whole-code recovery. Explicit weak, diffusive, jump, monitored-circuit, detector, packet-transfer, environment, reset, transport, and endpoint models instantiate the nine extensions. A conserving finite construction has identical entropy-history statistics for fixed pure probes but endpoint recovery errors 0 and 1 − d^{−2}. These results may support sharper audits of what records, residuals, and resources actually certify. The contribution is the explicit sharp synthesis and its application-specific separations, not a priority claim for Schmidt-number theory, continuous measurement, exact error correction, or Landauer balance. No continuum detector, measurement-induced transition, physical cosmological horizon, evaporation endpoint, Page curve, Einstein dynamics, or spacetime emergence is derived. O.1 on the cone remains texttt NOT_CLOSED.

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## Introduction and revision scope

### Enter, spacetime Enter, and the shredder philosophy

Cheung's Enter calculus treats runtime quantum names as access rights to tensor factors. Destructive computational-basis readout consumes the measured names and returns a classical record. Its operational characterization of measurement belongs to a specified strategy class; it is not a record-free derivation of the Born rule or a construction of a continuum field instrument [CheungEnter].

The *Philosophy of Enter* organizes four separations: destructive readout is not quantum return; a record is not a decoder; a channel is not a thermal machine; and an analogy is not a physical identification [CheungPhilosophy]. Spacetime Enter, in I-B, places a finite operational interface on an independently prescribed conical host [CheungIB]. The shredder philosophy of I-C is an access statement: the observer may receive useful classical information without receiving the coherence and reference correlations needed to return an arbitrary unknown code [CheungIC].

Consumption at this interface does not assert physical annihilation of every apparatus or environmental degree of freedom. Nor does it exclude an earlier logical transfer into a separately returned quantum factor. These distinctions are enforced mathematically below.

### What this revision adds

The nine finite models of version 1 are retained:

• weak and continuous trajectories;

• monitored circuits;

• a regulated accelerated scalar detector;

• a late-observer access surrogate;

• an analogue packet-transfer interface;

• environmental monitoring and decoherence;

• a two-zone record and an explicit thermal reset;

• a scheduled quantum transport protocol;

• terminal residual-ownership comparisons.

Version 2 adds proofs of complement-rank capacity, hybrid entropy, fixed-interface report continuity, and the binary-effect distance formula. It also adds a finite photonic sibling, a window-to-inference budget, a flagged packet model, an algebraic dependency-cut protocol, and an entropy-history/recovery separation.

### Contribution and attribution

The sharp access law is an application of established Schmidt-number methods [TerhalHorodecki,ChruscinskiKossakowski]. Its classical endpoint is the entanglement-breaking mechanism [HSR]. The complete-complement criterion is the Knill–Laflamme/no-hiding structure [KnillLaflamme,BraunsteinPati]. Continuous measurement and finite-reservoir thermal balance are established frameworks [JacobsSteck,ReebWolf].

The contribution of this manuscript is their explicit combination with one declared access algebra, a finite sharpness construction, reference-preserving error accounts, and nonempty models for all nine extensions. The potential usefulness is operational: record precision, entropy, spectra, residual dimension, and physical interpretation can be assessed without substituting one for another.

No priority claim is made for the underlying mathematical frameworks. No unquoted companion theorem is silently required for a local finite proof. Companion field formulas are either explicitly declared analytical inputs or source-qualified comparisons.

## The prescribed conical host and field contract

### Regular host and missing apex

Use units $c = \hbar = k_B = 1$. The prescribed regular spacetime is

$$M_\alpha = \mathbb{R}_t \times (0,\infty)_r \times S^1_\varphi \times \mathbb{R}_z,$$
$$g_\alpha = -\mathrm{d}t^2 + \mathrm{d}r^2 + \alpha^2 r^2 \mathrm{d}\varphi^2 + \mathrm{d}z^2, \qquad 0 < \alpha \le 1,$$

with angular period $2\pi$. The missing apex worldsheet is denoted by $\mathscr{W} = \{r = 0\}$; it is not part of $M_{\alpha}$. The metric and deficit $2\pi(1 - \alpha)$ are inputs [CheungIB,CheungIBG].

**Proposition.** The regular host is not globally hyperbolic. This remains true at $\alpha = 1$ when the axis is removed.

*Proof.* Fix $r_0 > 0$, $\varphi_0$, $z = 0$, and choose $T > r_0$. Let $p = (-T, r_0, \varphi_0, 0)$ and $q = (T, r_0, \varphi_0, 0)$. For $r_n \downarrow 0$, the points $x_n = (0, r_n, \varphi_0, 0)$ belong to $J_g^{+}(p) \cap J_g^{-}(q)$: radial travel takes less than $T$. They have no convergent subsequence in the regular host. This causal diamond is noncompact.

A wave problem on this incomplete spatial geometry therefore requires a specified operator realization. The Friedrichs realization is an input, not a consequence of destructive readout. This does not assert that every individual angular channel is nonunique [KayStuder,CheungIBG].

### Field species and domains are not interchangeable

The source-qualified continuum contract is:

**Scalar.** I-B-G supplies extension-selected scalar Green operators, holonomy conventions, and a Friedrichs finite-propagation theorem [CheungIBG].

**Free photon.** The cited free-photon comparison is I-B-G-S thm:spin1, with compatible vector and scalar-ghost Friedrichs domains:

$$C_1 G^{\pm}_{(1)} = G^{\pm}_{(0)} C_1, \qquad \nabla^a_x W^{(1)}_{ab'}(x,x') = -\nabla_{b'} W^{(0)}(x,x').$$

These are free source-test identities, not interacting detector or interacting Ward theorems [CheungIBGS].

**Tensor core.** I-B-G-S-B proves positive-core selection for its specified nonnegative tensor forms and excludes the stated exceptional curvature-regular branch from that core class. This is a domain-selection result for those operators. It is neither a photon identification nor a decoder [CheungTensor].

**One-point observables and frozen response.** I-B-VP supplies state- and prescription-qualified Wick squares, stress tensors, and currents. I-B-VP-4 treats a restricted frozen-current Maxwell response. Neither is automatically a coupled quantum Maxwell system [CheungVP,CheungVP4].

**Echo windows.** I-B-ECHO supplies specified window functionals, propagation comparisons, and error bounds. Reporting a supplied window does not acquire an unknown continuum amplitude [CheungEcho].

No field at the completed apex, physical gravitational Hessian, interacting gauge identity, or backreaction equation is constructed.

### Three support qualifications

First, finite propagation applies to the full specified wave realization and its admitted source/data class. It does not automatically apply to a projection onto finitely many positive-frequency modes. Such packets need not have compact spatial support, and a mode truncation can destroy exact locality.

Second, let $d_h$ be the spatial length distance on the regular host. The distance-defined propagation envelope

$$\mathrm{C}^{+}_{d}(S) = \{(t,x) : t - s \ge d_h(x,y) \text{for some} (s,y) \in S\}$$

must not automatically be identified with a curve-generated causal relation at unattained null endpoints. For example, on the $\alpha = 1$ punctured plane, opposite points at radius $r_0$ have infimum spatial distance $2r_0$, but the straight minimizing segment meets the removed axis. At elapsed time exactly $2r_0$, no regular causal route joins those endpoints. At larger elapsed times, detours exist. We use the cited propagation theorem only in its declared conventions, and require actual regular routes for finite communication permissions.

Third, a scheduled dependency set is not a field support theorem and is not automatically a physical causal past. These distinctions will be used in Extension VIII.

## Complete instruments and access cuts

### Channels, distances, and references

All code spaces and all quantum blocks in the finite access theorems are finite-dimensional. A channel is a completely positive trace-preserving linear map. For states and channels, use

$$d_{\mathrm{tr}}(\rho,\sigma)=\frac{1}{2}\|\rho-\sigma\|_1, \qquad d_\diamond(\Phi,\Psi)=\frac{1}{2}\|\Phi-\Psi\|_\diamond.$$

The diamond norm is the supremum of the amplified trace norm over unit trace-norm input operators. It controls every input together with every untouched finite reference [Watrous].

Fix $Q^{\prime}\cong Q\cong\mathbb{C}^d$, $d\ge 2$, and a reference $E\cong Q$. Put

$$|\Phi_d\rangle=\frac{1}{\sqrt{d}}\sum_{j=0}^{d-1}|j\rangle_E|j\rangle_Q, \qquad \Phi_d=|\Phi_d\rangle\langle\Phi_d|.$$

For $T: Q\to Q^{\prime}$, define

$$F_{\mathrm{e}}(T)=\mathrm{Tr}\,[\Phi_d(\mathrm{id}_E\otimes T)(\Phi_d)], \qquad e_{\mathrm{rec}}(\mathcal{N})=\inf_{\mathscr{D}} d_\diamond(\mathscr{D}\mathcal{N},\mathrm{id}_Q).$$

The infimum ranges over deterministic channels on the complete declared output algebra.

### The destructive clock

**Proposition.** Let $C$ have orthonormal basis $\{|\tau_j\rangle\}$, and let $A$ contain every surviving finite quantum register. Define

$$D_j=\langle\tau_j|_C\otimes I_A, \qquad \mathscr{M}_C(X)=\sum_j \mathrm{kb}\,jj_R\otimes D_jXD_j^\dagger.$$

Then

$$\sum_j D_j^\dagger D_j=I_{CA}, \qquad \mathrm{Tr}_R\,\mathscr{M}_C(X)=\mathrm{Tr}_C X.$$

These are complete channel identities. The returned quantum context contains $A$, not $C$. For $\dim C\ge 2$, $\mathscr{M}_C$ has no channel left inverse on the full $CA$ input space. The record marginal does not determine the residual $A$-state.

*Proof.* The recorded Kraus operators are $|j\rangle_R\otimes D_j$; their squared moduli sum to $I_{CA}$. For

$$X=\sum_{j,\ell}|\tau_j\rangle\langle\tau_\ell|\otimes X_{j\ell},$$

the returned blocks are $X_{jj}$, whose sum is $\mathrm{Tr}_C X$. The calculation remains valid with a reference factor.

The distinct clock vectors

$$(|\tau_0\rangle+|\tau_1\rangle)/\sqrt{2}, \qquad (|\tau_0\rangle-|\tau_1\rangle)/\sqrt{2}$$

give identical outputs with a fixed $A$-preparation. Thus no left inverse exists. Finally, $|\tau_0\rangle\langle\tau_0|\otimes\rho_A$ and $|\tau_0\rangle\langle\tau_0|\otimes\sigma_A$ have the same deterministic record for arbitrary distinct $\rho_A, \sigma_A$.

This locally proves the finite interface compared in I-B and En09-A [CheungIB,CheungEn09]. It consumes clock access names, not the geometric time axis.

### Finite adaptive experiments

**Definition.** An instrument is a family of completely positive maps $\{\mathscr{I}_x\}$ whose sum is trace preserving. Its recorded channel is

$$\mathscr{I}(\rho)=\mathrm{bigoplus}_x\,\mathscr{I}_x(\rho).$$

For $p_x=\mathrm{Tr}\,\mathscr{I}_x(\rho)>0$, its conditional residual is $\mathscr{I}_x(\rho)/p_x$. No normalized state is assigned to a zero-probability branch.

A finite Kraus representation satisfies

$$\mathscr{I}_x(X)=\sum_a K_{x,a}XK_{x,a}^\dagger, \qquad \sum_{x,a}K_{x,a}^\dagger K_{x,a}=I.$$

The isometry

$$V|\psi\rangle = \sum_{x,a} |x,a\rangle \otimes K_{x,a}|\psi\rangle$$

is a mathematical dilation. Its existence is not a laboratory gate, timing, or locality certificate. Instruments are established operational objects [DaviesLewis].

**Lemma.** A finite adaptive experiment with only classical accessible output has the form

$$\mathscr{C}(X) = \sum_h \mathrm{Tr}\,(E_h X)\mathrm{kb}\,hh, \qquad E_h \geq 0, \quad \sum_h E_h = I.$$

The complete history includes every exported setting, guard, failure, retained seed, stopping flag, capacity-test result, and heat-meter outcome required by the declared interface.

*Proof.* Compose the linear Kraus maps along each history into *K_{h, a}*, including unobserved Kraus indices. Completeness at each node gives *∑_{h, a} K_{h, a}^{†}K_{h, a} = I*. Set *E_h = ∑_a K_{h, a}^{†}K_{h, a}*. Conditional normalization factors cancel against branch probabilities and do not change this linear channel.

### The complete classical–quantum cut

**Definition.** A finite cut has quantum block dimension at most *k* if its complete message-bearing channel is

$$\mathscr{M}(X) = \mathrm{bigoplus}_{r \in \mathscr{R}} \mathscr{N}_r(X), \qquad \mathscr{N}_r : \mathscr{B}(Q) \to \mathscr{B}(K_r), \qquad \dim K_r \leq k,$$

where each *𝒩_r* is completely positive and *∑_r Tr 𝒩_r(X) = Tr X*. The direct-sum label is classical.

The decoder owns precisely these blocks and labels. No later message-bearing quantum transmission may bypass the cut. Previously transmitted quantum systems retained by the receiver belong to the cut. Shared sender–receiver entanglement and its receiver-side quantum factor must be declared.

The case *k = 1* is classical-only. An arbitrary number of classical bits does not increase *k*. A bound on this output dimension is not a bound on an encoder's working memory or on every streaming architecture.

## Sharp recovery, complete complements, and capacity

### The sharp dimension law

**Lemma.** If a state on $\mathbb{C}^d \otimes \mathbb{C}^d$ is a convex combination of pure states of Schmidt rank at most *k*, then

$$\mathrm{Tr}\,(\Phi_d \omega) \leq k/d.$$

Local completely positive operations, including selected branches, preserve the corresponding Schmidt-number bound after normalization.

*Proof.* A normalized pure state has coefficient matrix *A* with *‖A‖_2 = 1* and *rank A ≤ k*. Its Bell overlap is

$$\frac{1}{d}|\mathrm{Tr}\,A|^2 \leq \frac{1}{d}\|A\|_1^2 \leq \frac{k}{d}\|A\|_2^2.$$

Convexity proves the mixed-state assertion. Each local Kraus map multiplies the coefficient matrix on one side and cannot increase its rank.

This is the Schmidt-number mechanism of [TerhalHorodecki,ChruscinskiKossakowski], with the normalization fixed explicitly.

**Theorem.** Let *𝒩* satisfy Definition , with *1 ≤ k ≤ d*. For every deterministic decoder,

$$F_{\mathrm{e}}(\mathscr{D}\mathscr{N}) \leq k/d,$$
$$d_{\diamond}(\mathscr{D}\mathscr{N}, \mathrm{id}_Q) \geq 1 - k/d.$$

A finite-outcome instrument and decoder attain both constants for every *d*, *k*.

More precisely, if *k_r = dim K_r ≤ d* and *p_r = Tr 𝒩_r(I_Q/d)*, then

$$F_{\mathrm{e}}(\mathscr{D}\mathscr{N}) \leq \frac{1}{d}\sum_r p_r k_r, \qquad d_{\diamond}(\mathscr{D}\mathscr{N}, \mathrm{id}_Q) \geq 1 - \frac{1}{d}\sum_r p_r k_r.$$

*Proof.* The *r*-block of *(id_E ⊗𝒩)(Φ_d)* has trace *p_r*. Its normalized state, when nonzero, lives on *E ⊗ K_r* and has Schmidt number at most *k_r*. The decoder acts locally on that block. Lemma bounds its decoded Bell overlap by *p_rk_r/d*. Summing proves the fidelity bound.

The Bell projector is an effect, so

$$\begin{aligned} d_{\diamond}(\mathscr{D}_{\mathcal{M}}, \mathrm{id}_Q) &\geq d_{\mathrm{tr}}((\mathrm{id}_E \otimes \mathscr{D}_{\mathcal{M}})(\Phi_d), \Phi_d) \\ &\geq 1 - F_{\mathrm{e}}(\mathscr{D}_{\mathcal{M}}). \end{aligned}$$

This proves both distance bounds.

*Proof.* Fix a rank-*k* projection *P_0* on ℂ^*d*, and set *P_U = UP_0U^{†}*. Define

$$T_{d,k}(X) = \frac{d}{k}\int P_U X P_U \, \mathrm{d}U$$

with normalized Haar measure. Since ∫ *P_U dU = (k/d)I*, this is a channel. The Kraus trace formula gives

$$F_{\mathrm{e}}(T_{d,k}) = \frac{1}{d^2}\frac{d}{k}\int |\mathrm{Tr}\, P_U|^2 \, \mathrm{d}U = \frac{k}{d}.$$

The map is covariant under every unitary conjugation. A conjugation-covariant map is scalar on the identity and on the traceless matrix subspace. This can also be verified directly by applying diagonal phases, basis permutations, and rotations in each two-dimensional coordinate plane to matrix units. Consequently its normalized Choi state is isotropic:

$$J_{\mathrm{n}}(T_{d,k}) = \frac{k}{d}\Phi_d + \frac{1 - k/d}{d^2 - 1}(I - \Phi_d).$$

For the Weyl unitaries *W_ab = X^aZ^b*, the vectors *(I ⊗ W_ab)|Φ_d⟩* form an orthonormal Bell basis. Thus

$$T_{d,k} = \frac{k}{d}\mathrm{id} + \frac{1 - k/d}{d^2 - 1}\sum_{(a,b) \neq (0,0)} \mathrm{Ad}_{W_{ab}}.$$

The triangle inequality gives *d_{◇}(T_{d, k}, id) ≤ 1 − k/d*. The Bell test gives equality.

To obtain finitely many encoder outcomes, regard $X \mapsto P_U X P_U$ as a point of the real vector space of Hermiticity-preserving maps, of dimension *d^4*. The convex hull of its compact range is compact. For completeness, any finite convex combination with more than *d^4 + 1* terms has an affine dependence; changing its weights along that dependence removes one term while preserving the combination. Limits of the reduced combinations remain reduced combinations by compactness. Hence the Haar average is

$$\int P_U(\,\cdot\,)P_U \, \mathrm{d}U = \sum_{j=1}^{m} w_j P_j(\,\cdot\,)P_j, \qquad m \leq d^4 + 1.$$

Write *P_j = V_jV_j^{†}*, where *V_j:* ℂ^*k* → ℂ^*d* is an isometry. The encoder maps

$$A_j = \sqrt{\frac{d w_j}{k}}\, V_j^{\dagger}$$

satisfy ∑_*j A_j^{†}A_j = I*. Return *j* classically and *A_jXA_j^{†}* on ℂ^*k*. The block decoder *Y ↦ V_jYV_j^{†}* realizes *T_{d, k}*.

This is a finite exact construction, not an efficient gate-synthesis claim. Its *k* prices returned quantum access, not total working memory.

**Corollary.** For a complete classical-output channel,

$$F_{\mathrm{e}}(\mathscr{D}_{\mathscr{C}}) \leq \frac{1}{d}, \qquad d_{\diamond}(\mathscr{D}_{\mathscr{C}}, \mathrm{id}) \geq 1 - \frac{1}{d}.$$

Equivalently,

$$\|\mathscr{D}_{\mathscr{C}} - \mathrm{id}\|_{\diamond} \geq 2(1 - 1/d).$$

The Haar-average pure-state fidelity satisfies

$$F_{\mathrm{av}}(\mathscr{D}_{\mathscr{C}}) \leq \frac{2}{d+1}.$$

All three endpoint constants are attained.

*Proof.* Take *k = 1*. For a channel *T(X) = ∑_a L_aXL_a^{†}*, the Haar second moment gives

$$F_{\mathrm{av}}(T) = \frac{\sum_a \mathrm{Tr}\,(L_a^\dagger L_a) + \sum_a |\mathrm{Tr}\,L_a|^2}{d(d+1)} = \frac{1 + dF_{\mathrm{e}}(T)}{d+1}.$$

The second-moment identity follows from unitary invariance on the symmetric subspace and its trace normalization [NielsenAverage].

Fixed-basis dephasing attains the fidelity bound. Writing it as *Δ_Z = d^{−1}∑_{j = 0}^{d − 1} Ad_{Z^j}* gives the upper distance *1 − 1/d*, matching the Bell lower bound.

**Implementation allowances and heralded branches**

**Proposition.** Suppose $\mathscr{N}, \mathscr{M}$ have the same output type and $d_{\diamond}(\mathscr{N}, \mathscr{M}) \le \delta$. If $\mathscr{M}$ has access dimension $k$, then

$$d_{\diamond}(\widetilde{\mathscr{D}\mathscr{N}}, \mathrm{id}) \ge [1 - k/d - \delta]_+.$$

In particular, error at most $\varepsilon$ requires $k \ge d(1 - \varepsilon - \delta)$ when the right side is positive.

For compatible complete stage channels with allowances $\delta_j$, the composite allowance is at most

$$\min\{1, \sum_j \delta_j\}.$$

Moreover,

$$|e_{\mathrm{rec}}(\widetilde{\mathscr{N}}) - e_{\mathrm{rec}}(\mathscr{N})| \le \delta.$$

*Proof.* Channel pre- and postcomposition contract diamond distance. The reverse triangle inequality proves . Replacing one stage at a time gives the telescoping estimate. For any fixed decoder the two recovery errors differ by at most $\delta$; taking infima in both directions proves the last claim.

**Proposition.** For a trace-nonincreasing successful decoder branch after a $k$-dimensional cut, let $p_s$ be its probability on the Bell input and $f_s$ its subnormalized Bell overlap. Then

$$f_{\mathrm{s}} \le (k/d)p_{\mathrm{s}}.$$

If $p_s > 0$, the normalized successful Bell overlap is at most $k/d$.

*Proof.* The selected local operation preserves Schmidt number. Apply Lemma to the normalized nonzero successful output.

A normalized postselected map is not thereby a deterministic linear channel on all code inputs.

**Exact recovery and the complete complement**

**Theorem.** Let $V: Q \to A \otimes B$ be a finite isometry and define

$$\mathscr{N}(X) = \mathrm{Tr}_B\,(VXV^\dagger), \qquad \mathscr{E}(X) = \mathrm{Tr}_A\,(VXV^\dagger).$$

A deterministic decoder on $A$ satisfies $\mathscr{D}\mathscr{N} = id_Q$ if and only if

$$\mathscr{E}(X) = \mathrm{Tr}\,(X)\sigma_B \quad \text{for every} X \in \mathscr{B}(Q)$$

for one state $\sigma_B$.

When this holds, there are a state $\sigma_K$ and an isometry $J: Q \otimes K \to A$ such that

$$\mathscr{N}(X) = J(X \otimes \sigma_K)J^\dagger.$$

The complement includes every inaccessible loss system and every purification of a fixed mixed implementation environment.

*Proof.* Assume . Diagonalize $\sigma_B = \sum_{a = 1}^{s} p_a|b_a\rangle\langle b_a|$, $p_a > 0$. For a code basis write

$$V|i\rangle = \sum_a \sqrt{p_a}|u_{ia}\rangle_A|b_a\rangle_B.$$

The complementary matrix-unit identities imply

$$\langle u_{jb} \mid u_{ia}\rangle = \delta_{ij}\delta_{ab}.$$

Thus $J(|i\rangle|a\rangle) = |u_ia\rangle$ is an isometry. With $\sigma_K = \sum_a p_a proj\ a$, tracing $B$ gives . For any state $\tau_Q$,

$$\mathscr{D}(Y) = \mathrm{Tr}_K (J^\dagger Y J) + \mathrm{Tr}\,[(I - JJ^\dagger)Y]\tau_Q$$

is completely positive and trace preserving and recovers every matrix unit.

Conversely, dilate an exact decoder to $L: A \to Q \otimes G$. The isometry $(L \otimes I_B)V$ dilates the identity. Purity of its decoded output on basis input $|i\rangle$ gives

$$(L \otimes I_B)V|i\rangle = |i\rangle|\xi_i\rangle_{GB}.$$

Exactness on *kb ij* gives $\langle \xi_j \mid \xi_i \rangle = 1$. All normalized $\xi_i$ coincide. Tracing *QG* proves .

This is the established exact-correction structure [KnillLaflamme,BraunsteinPati], locally restated in the complete-output convention of En07-A/B and Hawk05-A [CheungEn07,CheungHawk05A]. It is a full-code criterion, not the universal criterion for every subsystem or observable task.

**Corollary.** If exact recovery from $A$ holds and $s = \mathrm{rank}\,\sigma_B$, then

$$\dim A \geq ds.$$

The bound is attained by $A = Q \otimes K$ and a fixed purification of $\sigma_K$.

If either complementary channel is constant on the entire code, the other output exactly recovers the code. For $d \geq 2$, the two complementary outputs cannot both be constant and cannot both independently recover the entire code.

*Proof.* The proof of Theorem exhibits *ds* orthonormal vectors $u_{ia}$ in $A$, proving the rank bound. The syndrome construction attains it. Interchanging $A, B$ proves the constant-output assertion. An exactly recoverable channel is not constant on two distinct inputs. An exactly recoverable output has constant complement, so that complement cannot itself recover the code.

The qualification ``complementary outputs'' is essential. It concerns a pure complete two-output dilation, not two selected marginals of a larger mixed or three-output interface.

**Corollary.** If a channel of the form is exactly recoverable, every nonzero branch has input-independent probability $p_r$, and its normalized channel $\mathscr{N}_r/p_r$ is exactly recoverable. Consequently $k_r \geq d$ on every nonzero branch.

*Proof.* Write the decoder restriction in block $r$ as $\mathscr{D}_r$. The positive Choi matrices of $\mathscr{D}_r\mathscr{N}_r$ sum to the rank-one Choi matrix of the identity. Each summand is therefore $p_r$ times that matrix: $\mathscr{D}_r\mathscr{N}_r = p_r id$. Trace preservation of $\mathscr{D}_r$ gives $\mathrm{Tr}\,\mathscr{N}_r(X) = p_r \mathrm{Tr}\,X$. For $p_r > 0$, $\mathscr{N}_r/p_r$ is a channel with an exact decoder. Apply .

The total Hilbert dimension used to store classical labels is not a replacement for this branchwise quantum capacity. Teleportation changes the access contract by supplying a receiver quantum factor from shared entanglement [Teleportation].

## The six I-C statements and the hybrid entropy refinement

toprule Label Controlled statement Boundary

midrule endfirsthead toprule Label Controlled statement Boundary

midrule endhead bottomrule endfoot A A classical-only cut has $F_e \leq 1/d$, $d_{\diamond} \geq 1 - 1/d$, sharply. No undeclared quantum side resource. Raw diamond norm has twice the distance constant.

B For an append-only finite transcript, $H(R_{n+1}) - H(R_n) = H(X_{n+1} \mid R_n) \geq 0$. Not differential entropy, selected residual entropy, or a physical Page history.

C An independent Gibbs reservoir and a unitary step give $\beta Q_B = \Delta S_M + I(M:B) + D(\omega_B \| \tau_B)$. Strict entropy decrease is sufficient, not necessary, for positive heat.

D A applies to a complete joint classical exterior report, including heat labels and postprocessing. A complete allowance $\delta$ weakens the bound by at most $\delta$. The access cut is declared, not inferred from a horizon name or a spectrum.

E A recorded prefix permits hiding, consuming, or granting its residual while preserving the old-record law. No terminal gravitational scenario is selected.

F A classical-only persistent accessible algebra is commutative; consumed names are absent from its quantum

output. A returned residual makes the output classical–quantum and changes the applicable access bound.

A and D are proved in Section . The remaining statements are proved here.

### Classical and hybrid entropy

**Proposition.** For finite $R_n = (X_1, \ldots, X_n)$,

$$H(R_{n+1}) = H(R_n) + H(X_{n+1} \mid R_n) \geq H(R_n).$$

Natural logarithms are used throughout. No independence or Markov hypothesis is needed.

*Proof.* Use $p(r, x) = p(r)p(x \mid r)$ on positive-probability histories. Expanding the logarithm gives the chain rule. Conditional Shannon entropy is an average of nonnegative finite-outcome entropies.

**Proposition.** For a classical–quantum state

$$\omega_{RK} = \mathrm{bigoplus}_r\, p_r \rho_{K_r \mid r}, \qquad \dim K_r \leq k,$$

one has

$$S(RK) = H(R) + \sum_r p_r S(\rho_{K_r \mid r}),$$

and therefore

$$H(R) \leq S(RK) \leq H(R) + \sum_r p_r \log k_r \leq H(R) + \log k.$$

For an append-only history, the classical term is monotone. The average conditional residual entropy, and the total hybrid entropy, need not be monotone. Once the retained classical record has positive entropy, the raw hybrid output cannot be pure while that record remains.

*Proof.* If $\lambda_{r, j}$ are the eigenvalues of $\rho_{K_r \mid r}$, the eigenvalues of $\omega_{RK}$ are $p_r \lambda_{r, j}$. Substitution into $-Tr\ \omega log\ \omega$ gives . The entropy of a $k_r$-dimensional state lies between $0$ and $log\ k_r$. The other conclusions follow.

For nonmonotonicity of the total hybrid entropy, keep a fixed nontrivial classical record and replace an independent maximally mixed residual by a pure residual. The record law is unchanged while the total entropy decreases. This is a channel example, not a thermodynamic cost account.

The hybrid identity is the finite result compared in Hawk05-A thm:recordentropy [CheungHawk05A]. It does not identify this entropy with radiation entropy.

For continuous records, B applies to finite digitizations. No difference of infinite Shannon entropies and no differential entropy monotonicity is asserted.

### Thermal balance

**Theorem.** Let $M$ be the declared complete non-reservoir system. Let $B$ have Hamiltonian $H_B$ and Gibbs state

$$\tau_B = Z_B^{-1} e^{-\beta H_B}, \qquad 0 < \beta < \infty.$$

Assume the initial state is $\rho_M \otimes \tau_B$, and let

$$\omega_{MB} = U(\rho_M \otimes \tau_B)U^\dagger.$$

Define heat deposited in the reservoir and non-reservoir entropy decrease by

$$Q_B = \mathrm{Tr}\,[H_B(\omega_B - \tau_B)], \qquad \Delta S_M = S(\rho_M) - S(\omega_M).$$

Then

$$\beta Q_B = \Delta S_M + I(M : B)_\omega + D(\omega_B \| \tau_B).$$

In particular, $Q_B \geq \Delta S_M/\beta$. A strict $\Delta S_M > 0$ implies $Q_B > 0$; the converse is false.

*Proof.* Initial independence and unitary entropy preservation give

$$S(\omega_B) - S(\tau_B) = \Delta S_M + I(M : B)_\omega.$$

The Gibbs logarithm gives

$$D(\omega_B\|\tau_B) = \beta Q_B - [S(\omega_B) - S(\tau_B)].$$

Combine these equations and use nonnegativity of mutual information and relative entropy.

This is the finite-reservoir Landauer equality [ReebWolf]. The equality applies whether or not entropy decreases. An entropy decrease is needed only to obtain the stated strictly positive lower bound from its first term.

**Corollary.** If *dim M = m ≥ 2*, *d_tr(ω_M, proj 0) ≤ ε*, and *0 ≤ ε ≤ 1 − 1/m*, then

$$\beta Q_B \geq S(\rho_M) - h_2(\varepsilon) - \varepsilon \log (m - 1),$$

where *h_2(x) = −xlog x − (1 − x)log (1 − x)*.

*Proof.* Set *t = 1 − ⟨0|ω_M|0⟩ ≤ ε*. Dephasing in a basis containing $|0\rangle$ cannot decrease entropy. The resulting distribution has one weight *1 − t* and remaining total weight *t*, so

$$S(\omega_M) \leq h_2(t) + t \log (m - 1).$$

The right side increases on *0 ≤ t ≤ 1 − 1/m*. Apply Theorem .

**Proposition.** If *ρ_M* and the finite-temperature *τ_B* are full rank, a unitary on *MB* cannot make the *M*-marginal exactly pure when *dim M ≥ 2*.

*Proof.* The joint input and unitary output are strictly positive. Their partial trace is strictly positive on every nonzero *M*-vector, hence is full rank.

The channel identity fixes no Hamiltonian, Gibbs preparation, control work, or reset. An independent driven reservoir operation can be appended without changing that readout channel while changing the reservoir energy account.

**Three thermodynamic ledgers.** The separation used in En07-A and Hawk05-A is: toprule Ledger Required data

midrule Energy and work Hamiltonians, interaction and switching conventions, complete systems, and measured energy changes.

Reset resources Initial reservoir, correlations, memory target, controller, archive, and any repetition or recycling step.

Residual leakage The complete inaccessible channel and its dependence on every code input.

bottomrule None of these is determined by the other two [CheungEn07,CheungHawk05A].

### Terminal ownership and persistence

**Proposition.** For a recorded prefix

$$\mathscr{R}(X) = \sum_r \mathrm{kb}\, rr_R \otimes \mathscr{P}_r(X)_A,$$

the following all preserve the old *R*-law: retain *A* but make it inaccessible; destructively measure *A* and retain all new outcomes; or grant quantum access to *A*.

For *dim A ≥ 2*, these are different ownership or output-type specifications. They need not define distinct unrestricted state maps, and granting *A* need not enable code recovery.

*Proof.* Every trace-preserving local continuation satisfies

$$\mathrm{Tr}_{A'}\,[(\mathrm{id}_R \otimes \Lambda_A)\mathscr{R}(X)] = \mathrm{Tr}_A\, \mathscr{R}(X).$$

For a measurement, sum over every new outcome. Changing ownership alone can leave the joint state map unchanged while changing decoder access.

**Proposition.** The retained classical observable algebra is

$$\mathscr{A}_R = \left\{ \sum_r f_r \mathrm{kb}\, rr \right\},$$

which is commutative. For returned quantum blocks it becomes

$$\mathscr{A}_{RK} = \mathrm{bigoplus}_r\, \mathscr{B}(K_r),$$

which is generally noncommutative. Consumption removes the measured names from the returned quantum context, not a distinct residual or an earlier transfer.

*Proof.* Diagonal matrices commute. A nontrivial matrix block contains noncommuting observables. The remaining assertion is the input/output typing of Proposition .

## Named observables, continuum reports, and statistical inference

### One observable is not an unknown code

**Proposition.** For a classical channel with effects $E_r$, a real statistic $f(r)$ reproduces the expectation of a Hermitian $A$ for every input state if and only if

$$A = \sum_r f(r)E_r.$$

If $A$ is an effect and $0 \le f(r) \le 1$, this is effect-valued classical postprocessing. Signed coefficients establish an unbiased statistic, not necessarily an effect-valued postprocessing.

*Proof.* The statistic's expectation is $Tr\,[\rho\sum_r f(r)E_r]$. Equality against all density matrices is operator equality.

**Example.** Let $n_j$, $j = 1, \ldots, 4$, be unit tetrahedral vectors with $\sum_j n_j = 0$ and $\sum_j n_j n_j^T = (4/3)I_3$. The effects $E_j = (I + n_j \cdot \sigma)/4$ are informationally complete, and

$$v \cdot \sigma = \sum_j 3(v \cdot n_j)E_j.$$

Repeated supplied preparations permit estimation of every Bloch component. Nevertheless a single-use decoder from the classical result has $d_{\diamond} \ge 1/2$. The linear inverse of a probability representation is not a CPTP inverse returning the unknown input specimen.

### An exact effect-distance formula

For an effect $A$, define the binary record channel

$$\mathcal{M}_A(X) = \mathrm{Tr}\,[(I - A)X]\mathrm{proj}\,0 + \mathrm{Tr}\,(AX)\mathrm{proj}\,1.$$

For $\mathcal{N}: Q \to Z$, define optimal one-effect reconstruction error

$$e_A(\mathcal{N}) = \inf_{0 \le F \le I_Z} \|\mathcal{N}^*(F) - A\|_\infty.$$

**Theorem.** For effects $A, B$ on the same finite space,

$$d_\diamond(\mathcal{M}_A, \mathcal{M}_B) = \|A - B\|_\infty.$$

Consequently,

$$e_A(\mathcal{N}) = \inf_{\mathcal{R}\,:\,Z \to \{0,1\}} d_\diamond(\mathcal{R}\mathcal{N}, \mathcal{M}_A),$$

and

$$e_A(\mathcal{N})\mathrm{le}_{\mathrm{rec}}\,(\mathcal{N}).$$

For a fixed quantum-return decoder $\mathcal{D}$,

$$\|\mathcal{N}^*\mathcal{D}^*(A) - A\|_\infty \mathrm{led}_\diamond\,(\mathcal{D}\mathcal{N}, \mathrm{id}).$$

*Proof.* Put $H = A - B$. For an amplified input operator $X$, the difference output has two blocks $T, -T$, where $T = Tr_Q\,[(H \otimes I)X]$. Trace-norm duality gives

$$\|T\|_1 = \sup_{\|Y\|_\infty \le 1} |\mathrm{Tr}\,[(H \otimes Y)X]| \le \|H\|_\infty \|X\|_1.$$

The two-block norm is $2\|T\|_1$. An eigenstate of $H$ with eigenvalue of maximal absolute value attains equality. This proves .

Every binary record channel on $Z$ is determined by an effect $F$, proving the infimum identity. For a quantum decoder, choose $F = \mathcal{D}^{*}(A)$; this is an effect. Postcomposition by $\mathcal{M}_A$ contracts diamond distance. Apply and then take infima.

**Remark.** A blanket assertion that no lower bound can transfer between these tasks would be false. A lower bound on the *optimal* effect error does lower-bound whole-code recovery under the matched channel and decoder class,

by .

The invalid transfers run in the other direction or change the task: a large whole-code recovery lower bound does not imply loss of a named effect, and failure of one chosen effect implementation does not prove intrinsic loss of the effect.

For example, computational dephasing has *e_rec = 1 − 1/d* while every diagonal effect is reconstructed exactly. For qubit dephasing, the effect *proj +* instead has optimal error *1/2*: its off-diagonal entry cannot be supplied by a diagonal pullback, and *F = I/2* attains that error.

This numerical separation is consistent with Oai290-A's code/observable/model discipline and XVII-A's reconstruction of one bounded sine effect [CheungOai290,CheungXVII]. Their geometric correspondences are not inferred here.

**Continuum windows programmed into a fixed finite instrument**

**Lemma.** Let *w_{ε} → w_0* be finite vectors of real window values. Let *P* be a fixed finite program space, and let *w ↦ τ(w)* be trace-norm continuous. Fix one complete channel

$$\mathscr{J}: \mathscr{B}(Q \otimes P) \to \operatorname{bigoplus}_{r=1}^{m} \mathscr{B}(K_r), \qquad \dim K_r \le k,$$

and define

$$\mathscr{N}_w(X) = \mathscr{J}(X \otimes \tau(w)).$$

Then

$$d_\diamond(\mathscr{N}_{w_\varepsilon}, \mathscr{N}_{w_0}) \le d_{\mathrm{tr}}(\tau(w_\varepsilon), \tau(w_0)) \to 0.$$

For every fixed input, its complete classical marginal converges in total variation. For *n* independent uses, the allowance is at most *n* times the single-use allowance.

Every limiting channel retains the same declared classical–quantum output algebra and block bound *k*. It therefore obeys Theorem .

*Proof.* For every reference-amplified operator *X*, the difference before $\mathscr{J}$ is *X ⊗ [τ(w_{ε}) − τ(w_0)]*. Its trace norm is the product of the two trace norms. Apply complete trace-norm contractivity and divide by two. Measurement and partial trace give the TV assertion. Tensor-product telescoping proves the repetition estimate. The limiting map is explicitly *𝒩_w_0* with the same fixed output type.

**Remark.** The program state is supplied from the declared classical vector. The lemma does not build an operation that acquires an unknown continuum functional. If *w* were defined as an arbitrary nonlinear functional of an unknown input density matrix and used to prepare a new state, the resulting prescription would not automatically be a channel.

The complete joint program preparation must be continuous. Continuity of separately specified marginal preparations does not constrain arbitrary joint correlations. A discontinuous rounding rule requires a margin from its thresholds.

For product programs

$$\tau(w) = \operatorname{bigotimes}_{j=1}^{m} [(1 - q_j(w_j))\operatorname{proj} 0 + q_j(w_j)\operatorname{proj} 1], \quad q_j(u) = \frac{1 + \tanh(\gamma_j u)}{2},$$

one has the explicit estimate

$$d_\diamond(\mathscr{N}_w, \mathscr{N}_{w'}) \le \frac{1}{2}\sum_j |\gamma_j|\,|w_j - w_j'|.$$

Indeed, *|q_j(u) − q_j(v)| ≤ |γ_j||u − v|/2*, and product states telescope.

This proves the finite-interface bridge used in the I-C/I-B-ECHO comparisons [CheungIC,CheungEcho]. It is stronger than TV convergence for a fixed probe but weaker than a continuum-detector construction: the latter is not supplied.

### A fixed-shot inference theorem

**Theorem.** Let $\Theta = [\theta_{-}, \theta_{+}]$. Suppose $q: \Theta \to [0, 1]$ is continuous, strictly monotone, and satisfies

$$|q(\theta) - q(\theta')| \geq b|\theta - \theta'|, \qquad b > 0.$$

In $n$ independently prepared fresh trials, chosen before data collection, suppose the actual click probability obeys

$$|\tilde{q}(\theta) - q(\theta)| \leq \eta$$

uniformly. Project the empirical mean $\bar{X}$ onto the interval $q(\Theta)$, then apply $q^{-1}$. For $0 < \zeta < 1$, with probability at least $1 - \zeta$,

$$|\hat{\theta} - \theta| \leq \frac{1}{b}\left[\sqrt{\frac{\log(2/\zeta)}{2n}} + \eta\right].$$

The right side may be capped by $\theta_{+} - \theta_{-}$.

*Proof.* Projection onto an interval cannot increase distance to $q(\theta)$. Thus

$$b|\hat{\theta} - \theta| \leq |\bar{X} - \tilde{q}(\theta)| + \eta.$$

For a centered Bernoulli variable, the second derivative of its log moment-generating function is a tilted Bernoulli variance, at most $1/4$. Integrating twice gives the bound $u^2/8$. Independence and exponential Markov inequalities yield

$$\Pr\{|\bar{X} - \tilde{q}| > v\} \leq 2e^{-2nv^2}.$$

Choose $v = \sqrt{(\log(2/\zeta)/(2n))}$.

This is the fixed-shot concentration mechanism compared in En10-A/B [CheungEn10]. It is not a guarantee for undeclared stopping, correlated preparations, or deleted failures. Repeated supplied probes are not a free channel $\rho \mapsto \rho^{\otimes n}$ acting on one unknown specimen.

### A window-to-inference budget

**Corollary.** Suppose a real window $y(\theta)$ is differentiable on $\Theta$, has derivative of one sign with $|y'(\theta)| \geq m > 0$, and $|y(\theta)| \leq Y$. Install the nominal law

$$q(\theta) = \frac{1 + \tanh(\gamma y(\theta))}{2}, \qquad \gamma > 0.$$

If $|\tilde{y}(\theta) - y(\theta)| \leq \eta_{win}$ and the implemented probability has an additional error at most $\eta_{dev}$, then one may take

$$b = \frac{\gamma m}{2}\operatorname{sech}^2(\gamma Y), \qquad \eta = \frac{\gamma}{2}\eta_{\mathrm{win}} + \eta_{\mathrm{dev}}$$

in .

*Proof.* Differentiate $q$ and bound its derivative away from zero. The mean-value theorem proves the inverse-Lipschitz estimate. The derivative of $u \mapsto (1 + \tanh \gamma u)/2$ has modulus at most $\gamma/2$, giving the probability error.

This is a worked inference certificate once $m$, $Y$ and the window allowance are supplied. It does not assert that every echo window or accelerated-detector response is injective.

For the fixed smeared profiles of the supplied I-B-ECHO comparison, write

$$y^{\varepsilon}[\chi] = \int_0^T \chi(t) Y^{\varepsilon}(t)\,\mathrm{d}t, \qquad t_{\varepsilon} = r_e + r_r - 2\varepsilon - 2s.$$

Its source-qualified error bound is

$$|y^{\varepsilon}[\chi] - y^{0}[\chi]| \leq 2T\|f\|_{L^1}\|h_r\|\|h_e\| \int_{t \geq t_{\varepsilon}} |\chi(t)|\,\mathrm{d}t.$$

The same source gives uniform response convergence for its declared neutral core family [CheungEcho]. Substitution of into Corollary prices that model mismatch when its parameter-sector hypotheses have separately been proved. Neither point-source constants nor acceleration identifiability are transferred to a spatially smeared

experiment by this substitution.

**A completely explicit prescribed-deficit report**

Let

$$c_0 = \frac{1}{48\pi^2 r_0^2}, \qquad w_\alpha = c_0(\alpha^{-2} - 1), \qquad \alpha \in [\alpha_-, \alpha_+] \subset (0, 1].$$

The identification of *w_{α}* with a neutral Friedrichs Wick square is the supplied one-point input of I-B-VP [CheungVP]. For the installed law

$$q(\alpha) = \frac{1 + \tanh(\gamma w_\alpha)}{2}, \qquad \gamma > 0,$$

the derivative is negative and

$$|q'(\alpha)| = \gamma c_0 \alpha^{-3} \operatorname{sech}^2(\gamma w_\alpha) \geq \gamma c_0 \alpha_+^{-3} \operatorname{sech}^2(\gamma c_0(\alpha_-^{-2} - 1)) =: b_\alpha > 0.$$

Thus gives a fully explicit deficit confidence radius with *b = b_{α}*. For *q ∈ q([α_{−}, α_{+}])*, the nominal inverse is

$$\alpha(q) = \left[1 + \frac{\operatorname{artanh}(2q - 1)}{\gamma c_0}\right]^{-1/2}.$$

This is a finite report prepared from a supplied parameter. It is not an apparatus acquiring a Wick square and is not the accelerated-detector click law. The echo comparison supplies a methodology for controlled windows; the one-point formula supplies a different named functional. They remain different objects.

**Apex coefficients as named spectral functionals**

The supplied description of XVI-O1-CONE-v6 names a normalized apex constant $\mathscr{C}_b$, a weighted coefficient $\mathscr{D}_b$, and a closure result for one constant-profile *hh/Kc* row on the declared CR and F branches [CheungCone]. The full coefficient definitions and the complete row calculation are not supplied as analytical premises here. Accordingly, no coefficient value, normalization formula, or new row proof is invented.

The appropriate placement is in the named-functional and model ledgers:

• $\mathscr{C}_b$, $\mathscr{D}_b$ are source-named spectral functionals of an installed cone/domain prescription. They are not automatically bounded effects on the finite code.

• A supplied numerical description can program a finite report under Lemma . This is not acquisition of the underlying spectral quantity.

• A finite report is not whole-code recovery and does not select CR rather than F. Proposition concerns residual ownership; it is not an operator-domain selection theorem. The absence of a domain-selection map here must not be replaced by that proposition.

• Positive tensor-core exclusion is the separately scoped result of I-B-G-S-B for its declared positive core class [CheungTensor]. It is not a universal physical core-selection principle.

• The free Maxwell identity is not a proof of the interacting Ward combinations named in the separate loop ledger. A reported single-row closure is not closure of O.1.

**The three output ledgers**

toprule Ledger Required mathematical claim

midrule Unknown code One complete channel and a reference-preserving quantum-return target, quantified over every code input.

Named effect or parameter An adjoint identity, an effect-channel comparison, or a calibrated statistical model with explicit error and sampling assumptions.

Geometry or physical model An independently specified host, domain, state, apparatus, trajectory, and matching theorem.

bottomrule

The distinction is compared with Oai290-A and XVII-A [CheungOai290,CheungXVII]. It does not identify their objects with the present models.

## Extension I: trajectories, consumed clocks, and a photonic sibling

### An explicit weak family

Place a finite qubit at a declared regular laboratory location. This qubit is not initially identified with a field. Let $Z = diag\,(1, -1)$ and

$$M_x^{(\eta)} = \sqrt{\frac{I + x\eta Z}{2}}, \qquad x = \pm 1, \quad 0 \le \eta < 1.$$

Then $\sum_x (M_x^{(\eta)})^{\dagger} M_x^{(\eta)} = I$. A finite meter dilation followed by destructive meter readout therefore defines a complete instrument.

For $x = (x_1, \ldots, x_n)$, set

$$K_{\mathbf{x}} = M_{x_n}^{(\eta)} \cdots M_{x_1}^{(\eta)}, \qquad L_z(\mathbf{x}) = \prod_{j=1}^{n} \frac{1 + z\eta x_j}{2}.$$

In the $Z$-basis,

$$\Pr_{\rho}(\mathbf{x}) = \rho_{++} L_+(\mathbf{x}) + \rho_{--} L_-(\mathbf{x}),$$

$$(K_{\mathbf{x}} \rho K_{\mathbf{x}}^{\dagger})_{zz'} = \rho_{zz'} \sqrt{L_z(\mathbf{x}) L_{z'}(\mathbf{x})}.$$

The record law has a useful classical-mixture representation; the conditional residual still contains its actual coherences. No ontology of preexisting measured values is asserted.

The complete output is record plus qubit. A/D apply only after restricting to record-only access. If the residual is returned, the cut has $k = 2$.

### A complete diffusive instrument

**Theorem.** Fix $T, \gamma > 0$, let $\Delta = T/n$, and take $\eta = \sqrt{(\gamma\Delta)} < 1$. At mesh times define

$$Y_{j\Delta}^{\Delta} = \sqrt{\Delta} \sum_{\ell=1}^{j} x_{\ell}.$$

Linearly interpolate these values and the corresponding normalized conditional residual states.

For each fixed input state, the joint record–residual process converges in distribution in the continuous-path topology to the following process. With standard Wiener measure $\mathbb{W}$ on $C([0, T], \mathbb{R})$, define

$$K_t(y) = \exp\left(\frac{\sqrt{\gamma}}{2} Z y_t - \frac{\gamma t}{4} I\right).$$

Then

$$\mathcal{I}_T(B)(\rho) = \int_B K_T(y) \rho K_T(y)^{\dagger} \, d\mathbb{W}(y)$$

is a complete instrument.

Its record law is the mixture

$$Y_t = z\sqrt{\gamma}\, t + W_t, \qquad \Pr(z = \pm 1) = \rho_{\pm\pm},$$

and its conditional residual density is

$$\rho_t(y) = \frac{K_t(y) \rho K_t(y)^{\dagger}}{\mathrm{Tr}\,[K_t(y) \rho K_t(y)^{\dagger}]}.$$

This is a regular conditional description, not normalization by the probability of a singleton path.

Writing $z_t = Tr\,(Z\rho_t)$, its physical innovation

$$dW_t^{\mathrm{in}} = dY_t - \sqrt{\gamma}\, z_t dt$$

is Wiener, and

$$\mathrm{d}\rho_t = \frac{\gamma}{4}(Z\rho_t Z - \rho_t)\mathrm{d}t + \frac{\sqrt{\gamma}}{2}(Z\rho_t + \rho_t Z - 2z_t\rho_t)\mathrm{d}W_t^{\mathrm{in}}.$$

No diamond-norm convergence between unspecified encodings of the finite and continuous path spaces is asserted.

*Proof.* Conditional on $z$ in , the outcomes are independent with mean $z\sqrt{(\gamma\Delta)}$. The centered record increments have variance $\Delta + O(\Delta\hat{}2)$, maximal size $O(\sqrt{(\Delta)})$, and characteristic functions whose logarithms converge to those of independent Gaussian increments. This gives finite-dimensional convergence to the stated drifted Wiener processes.

For tightness, expand the fourth moment of sums of independent centered bounded increments. For intervals containing full mesh steps the bound is $C_T|t - s|\hat{}2$. For partial steps, linear interpolation contributes at most $C|t - s|\hat{}4/\Delta\hat{}2 \le C|t - s|\hat{}2$ when $|t - s| \le \Delta$. Splitting an arbitrary interval into full increments and at most two partial increments proves the same uniform bound. The moment criterion gives tightness.

Relative to the symmetric word law, uniformly in words and in partial sums,

$$\sum_{\ell=1}^{j} \log(1 + z\sqrt{\gamma\Delta}\, x_\ell) = z\sqrt{\gamma}\, Y_{j\Delta}^{\Delta} - \frac{\gamma j\Delta}{2} + O_T(\sqrt{\Delta}).$$

Taking square roots identifies the limiting Kraus density . On bounded path sets, normalization is continuous because its denominator is strictly positive. The same expansion at all mesh times, followed by interpolation, therefore proves joint residual convergence. The argument also works for a fixed untouched reference by replacing the Kraus operators with $K_t \otimes I$.

Gaussian exponential moments give

$$\int K_T^{\dagger} K_T \,\mathrm{d}\mathbb{W} = \mathrm{diag}\left(\mathbb{E}e^{\sqrt{\gamma}W_T - \gamma T/2}, \mathbb{E}e^{-\sqrt{\gamma}W_T - \gamma T/2}\right) = I.$$

This proves completeness and the Gaussian drift likelihoods. Applying Itô's formula to the unnormalized state and its trace gives . The conditional record drift is $\sqrt{(\gamma)}\ \mathbb{E}[z \mid Y_\{[0,\ t]\}] = \sqrt{(\gamma)}\ z_t$. Subtracting it gives a continuous martingale of quadratic variation $t$, hence Wiener innovation.

The stochastic equation is within the established continuous-measurement framework [JacobsSteck]. The theorem supplies this particular instrument and limit, not a general photodetector realization.

### A continuous jump instrument

For $\kappa$, $T > 0$, define

$$K_{\varnothing} = \mathrm{kb}\,00 + e^{-\kappa T/2}\mathrm{kb}\,11, \qquad K_t = \sqrt{\kappa}\, e^{-\kappa t/2}\mathrm{kb}\,01, \quad 0 \le t \le T.$$

They satisfy

$$K_{\varnothing}^{\dagger}K_{\varnothing} + \int_0^T K_t^{\dagger}K_t \,\mathrm{d}t = I.$$

The no-jump probability is $\rho_00 + e\hat{}\{-\kappa T\}\rho_11$, and the jump-time density is $\kappa e\hat{}\{-\kappa t\}\rho_11$. These formulas define a complete instrument. Conditional states are assigned through nonzero densities or regular conditional distributions, not by treating a precise jump time as an event of positive probability.

### The complete path remains a classical cut when no residual is granted

**Theorem.** Let $\Omega$ be a standard Borel space. Suppose a finite-input instrument has a measurable CP density $\mathscr{I}_y: \mathscr{B}(Q) \to \mathscr{B}(K)$ with $\dim K \le k$ and

$$\int_{\Omega} \mathrm{Tr}\,\mathscr{I}_y(X)\,\mu(\mathrm{d}y) = \mathrm{Tr}\,X.$$

For every measurable family of deterministic decoders $\mathscr{D}_y: K \to Q\hat{}\{'\}$,

$$T(X) = \int_{\Omega} \mathscr{D}_y \mathscr{I}_y(X)\,\mu(\mathrm{d}y)$$

satisfies

$$F_{\mathrm{e}}(T) \le k/d, \qquad d_{\diamond}(T, \mathrm{id}) \ge 1 - k/d.$$

In particular, a path-space POVM followed by measurable state preparation has the classical bounds *1/d* and *1 − 1/d*.

Every countably additive instrument with finite input and fixed finite quantum output admits such a CP density with respect to a scalar dominating measure.

*Proof.* For the Bell input, each nonzero normalized state *(id_E ⊗𝒥_y)(Φ_d)* has Schmidt number at most *k*. Apply Lemma , integrate its subnormalized bound, and use the Bell test.

For the final assertion, let *J(B)* be the normalized Choi matrix of the instrument map assigned to *B*. It is a positive matrix-valued measure. The scalar measure *μ(B) = Tr J(B)* dominates every matrix entry. Entrywise Radon–Nikodym differentiation gives *J(B) = ∫_B J_y μ(dy)*. Positivity of all quadratic forms implies *J_y ≥ 0* almost everywhere; its trace is one almost everywhere. The Choi correspondence gives a measurable CP density. Completeness of the original instrument gives its integrated trace identity.

For record-only access, the observable algebra is *L^{∞}(Ω, μ)*, which is commutative. Finite digitizations obey B. The theorem does not rely on a presumed diamond-norm limit of finite-prefix decoders.

### Consumed clocks and relational conditioning

Proposition gives exactly the Born clock law and the unnormalized conditional surviving block used in En09-A's Page–Wootters extraction [CheungEn09]. For completeness, if

$$(U_C(t) \otimes U_S(t))\rho_{CS}(U_C(t)^{\dagger} \otimes U_S(t)^{\dagger}) = \rho_{CS}$$

and *P_j = U_C(t_j)P_0U_C(t_j)^{†}*, then

$$\sigma_{S,j} = \mathrm{Tr}_C\,[(P_j \otimes I)\rho_{CS}] = U_S(t_j)\sigma_{S,0}U_S(t_j)^{\dagger}.$$

All traces coincide; in a complete *D*-outcome covariant basis they equal *1/D*. This dynamical interpretation needs both stationarity and basis covariance. Stationarity of a marginal does not imply stationarity of every reference extension.

Fresh successive clocks compose as complete instruments. Their preparation, names, records, controllers, and storage are separately priced, as in X-K-ST [CheungXK]. Successive clocks are not thereby quantized geometric time.

### A declared finite photonic sibling

Choose two orthonormal physical transverse packets from an installed free Maxwell–Friedrichs one-particle realization. Their one-photon vectors span a code

$$\mathscr{H}_{\gamma} = \mathrm{span}\,\{|1_{h_0}\rangle, |1_{h_1}\rangle\} \cong \mathbb{C}^2.$$

On this declared finite code, install

$$Z_{\gamma} = |1_{h_0}\rangle\langle 1_{h_0}| - |1_{h_1}\rangle\langle 1_{h_1}|, \qquad M^{(\eta)}_{x,\gamma} = \sqrt{(I + x\eta Z_{\gamma})/2}.$$

These are exact finite Kraus operators. Measuring a finite meter consumes the meter names and returns the code residual; restricting to the record gives A/D. A complete destructive basis readout can instead consume the two-dimensional code encoding itself.

The field-domain comparison is , including the compatible ghost domain [CheungIBGS]. It does not supply physical addressability of these packets, a local photon–meter interaction, a homodyne limit, or a continuum photodetector. The ghost is not an extra measured physical-photon register. The tensor-core result of I-B-G-S-B is not this photon [CheungTensor].

The supplied-report sibling in X-J-ST-2 has the declared law

$$P_s(F) = r, \qquad P_s(\pm) = \frac{1 - r}{2}(1 \pm \tanh(vs/\sigma)),$$

and therefore

$$\mathrm{TV}\,(P_s, P_0) = \frac{1-r}{2}|\tanh\,(vs/\sigma)|.$$

This follows by summing the two opposite probability changes. With a common spectator–reference state, orthogonality of the classical blocks makes the full trace distance the same. These are the scoped thm:TV and thm:clock comparisons; prop:noacquisition retains the non-acquisition boundary [CheungXJ]. An informative TV gap is not a repair of a different convolution problem.

**Three-ledger output.** Record-only code return has error at least *1 − 1/d*; a granted finite residual uses the *k*-law. Named trajectory statistics can be informative. No field acquisition, collapse ontology, or photon apparatus is identified.

## Extension II: finite monitored circuits

Install *n* qubits at declared regular locations. At layer $\ell$, apply a fixed brickwork unitary *U_{ℓ}*. Independently mark sites with probability $p \in [0, 1]$. Retain the full masks and every outcome.

A marked old site is measured in the computational basis and consumed. Allocate a new qubit in $|0\rangle$ if a fixed-size presentation is desired. The marked-site maps are

$$L_x = |0\rangle_{\mathrm{new}}\langle x|_{\mathrm{old}}, \qquad x = 0, 1.$$

An unmarked site retains its name. A complete layer has Kraus operators

$$K_{\mathbf{m},\mathbf{x}} = \sqrt{p^{|\mathbf{m}|}(1-p)^{n-|\mathbf{m}|}}\,\big(\mathrm{bigotimes}_{j\,:\,m_j=1} L_{x_j} \otimes \mathrm{bigotimes}_{j\,:\,m_j=0} I_j\big) U_\ell.$$

Completeness follows by summing all outcomes and masks.

Fix an embedding *V: Q → (ℂ^2)^{⊗n}*, a finite depth *T*, and a residual bipartition. The live output algebra is the direct sum of its actual residual matrix algebras over complete histories.

**Proposition.** For every fixed *n*, *T*, *p*, *U_{ℓ}*, *V*, a record-only decoder obeys A/D. The transcript obeys B. A complete residual of block dimension at most *k* obeys Theorem .

For a fixed pure probe, fixed gates, and fixed residual bipartition, the average conditioned bipartite entropy is a polynomial in *p* at finite *n, T*. No nonanalytic thermodynamic transition follows from this finite statement.

*Proof.* Lemma identifies the complete record channel. Apply the access and entropy theorems.

For a full mask/outcome history *h*, its probability is *p^a_h(1 − p)^b_hq_h*, where *q_h* is independent of *p*. Whenever *q_h > 0*, its normalized residual and its entropy *S_h* are also independent of *p*. The finite sum

$$\sum_h p^{a_h}(1-p)^{b_h} q_h S_h$$

is a polynomial.

A Bell-pair creation followed by measurement of one member gives conditioned entanglement values *0, log 2, 0*, while its retained outcome record does not lose entropy.

Controls, guards, capacity tests, settings, and retained storage are part of the complete network whenever accessible or causally used. A decoder cannot be hidden in the word ``circuit.'' These are the En01-A recorded-circuit and En03-A observation-boundary comparisons [CheungRecords]. A fixed-probe entropy average has a different quantifier from whole-code recovery, which ranges over every code–reference input [CheungHawk05B].

Measurement-induced transitions are separate large-system results with their own ensembles and limiting procedures [SkinnerRuhmanNahum]. Oai212-A is retained as a transport comparison, not a transition theorem [CheungNegative].

**Three-ledger output.** The record-only recovery obstruction is uniform in the circuit. Conditioned entanglement and named statistics remain separately computable. No area-law phase, critical exponent, or geometric causal structure is derived.

## Extension III: a regulated accelerated scalar detector

### Declared field, trajectory, and coupling

Choose finitely many smooth positive-frequency packets *u_j(x)* from an installed scalar Friedrichs spectral realization on *M_{α}*, and a finite occupation cutoff *P_N*. Define

$$\varphi_{m,N}(x) = P_N \sum_{j=1}^{m} \left( u_j(x) a_j + \overline{u_j(x)} a_j^\dagger \right) P_N.$$

The state, packets, and cutoff are inputs. This is a bounded finite-matrix field, not a proof that the truncated mode span is local or invariant under continuum dynamics.

At *r = r_0 > 0*, *φ = φ_0*, prescribe

$$t_a(\tau) = \frac{\sinh(a\tau)}{a}, \qquad z_a(\tau) = \frac{\cosh(a\tau) - 1}{a},$$

with continuous values *t_0 = τ*, *z_0 = 0*. Its tangent has squared norm *−1*, and its acceleration has squared norm *a^2*.

Let a two-level detector start in $|g\rangle$, with gap *Ω > 0*, real *λ*, and real *χ ∈ C_c^{∞}(ℝ)*. Set

$$H_I(\tau) = \lambda \chi(\tau) \left( \sigma_+ e^{i\Omega\tau} + \sigma_- e^{-i\Omega\tau} \right) \otimes \varphi_{m,N}(x_a(\tau)).$$

**Theorem.** The finite Hamiltonian has a unique unitary propagator *U_a*. The maps

$$K_s(a) = \langle s | U_a | g \rangle, \qquad \mathcal{I}_s^{(a)}(X) = K_s(a) X K_s(a)^\dagger, \quad s = g, e,$$

form a complete instrument. The excitation probability is

$$q_a(\rho) = \lambda^2 \mathcal{F}_a(\Omega; \rho) + R_a,$$

where

$$\mathcal{F}_a = \iint \chi(\tau) \chi(\tau') e^{-i\Omega(\tau - \tau')} W_{m,N,\rho}(x_a(\tau), x_a(\tau')) \, d\tau \, d\tau',$$

$$W_{m,N,\rho}(x, x') = \mathrm{Tr}\left[ \rho \varphi_{m,N}(x) \varphi_{m,N}(x') \right],$$

and, with

$$L_a = \int |\chi(\tau)| \| \varphi_{m,N}(x_a(\tau)) \|_\infty \, d\tau,$$

one has

$$|R_a| \le \sinh^2(|\lambda| L_a) - (|\lambda| L_a)^2.$$

Its complete click record alone obeys A/D on every installed unknown finite field code.

*Proof.* The compactly supported continuous finite Hermitian Hamiltonian has a norm-convergent Dyson series. Its propagator is unitary and *∑_s K_s^{†}K_s = I*.

Every interaction flips the detector, so *K_e* contains only odd orders. Its first term is

$$-i\lambda \int \chi(\tau) e^{i\Omega\tau} \varphi_{m,N}(x_a(\tau)) \, d\tau.$$

Its squared norm in *ρ* gives *λ^2ℱ_a*. The remaining odd terms have operator norm at most *r_a = sinh (|λ|L_a) − |λ|L_a*. The probability error is at most *2|λ|L_ar_a + r_a^2*, which is . The record-only restriction is a classical channel.

### Support, inference, and energy are separate

I-B-G's finite-speed result constrains the full scalar Friedrichs response for its admitted compact sources [CheungIBG]. It does not prove strict support containment for the finite positive-frequency approximation . No such claim is needed in Theorem .

For independently prepared known probes, an acceleration interval with a separately certified inverse-Lipschitz click law has the inference guarantee . If a perturbative response is used as a nominal law, it must first be verified

to lie in *[0, 1]*; its error budget then includes the uniform remainder in , numerical error if certified, and apparatus calibration error. No monotonicity follows just from ``uniform acceleration."

I-B-ECHO gives the useful pattern of an injective sector window with a controlled report error [CheungEcho]. Corollary shows the exact statistical consequence of such a certificate; it does not supply a missing detector calibration.

The scalar Unruh effect and smeared detector regularization have their own primary literature [Unruh,LoukoSatz]. A Planckian stationary response in an appropriate uncut Minkowski model does not make every switched finite-mode response exactly thermal. A temperature inferred from a response is not the Gibbs reservoir declaration of Theorem .

As a separate one-point ledger, the neutral conformal Friedrichs state in the supplied I-B-VP convention has

$$\langle T^t{}_t\rangle_{\mathrm{ren}} = \frac{\alpha^{-4}-1}{1440\pi^2 r^4}, \qquad \varepsilon_{\mathrm{vac}} = -\langle T^t{}_t\rangle_{\mathrm{ren}}.$$

This is not the click law and not a reset heat cost [CheungVP]. Em03-A/B concern a different fixed-background scalar emission and packet-readout construction, not a continuum Unruh detector on the present cone [CheungEmission].

**Three-ledger output.** The click-only channel cannot return an unrestricted unknown code. Acceleration or deficit inference requires its own calibration. The trajectory, field realization, and any continuum apparatus interpretation remain independently supplied.

## Extension IV: the late-observer access surrogate

The host is ultrastatic. No expanding scale factor, de Sitter state, or cosmological horizon is produced by relabeling a readout as ``late time." QdS in the supplied XVI record is an external-leg prescription, not a de Sitter state [CheungStanding]. De Sitter comparisons in a prescribed-cone ledger remain comparisons.

Define the finite isometry

$$V_\eta|0\rangle = |00\rangle_{LB}, \qquad V_\eta|1\rangle = \sqrt{\eta}\,|10\rangle_{LB} + \sqrt{1-\eta}\,|01\rangle_{LB}, \quad 0 \le \eta \le 1.$$

Its visible number record has

$$\Pr(R=1) = \eta\rho_{11}.$$

Declare the late observer's accessible algebra to be the record algebra, optionally enlarged by a complete named quantum factor $K$.

**Proposition.** Record-only recovery obeys A/D for every $\eta$, including $\eta = 1$. If the complete quantum enlargement has dimension at most $k$, its recovery error is at least $1 - k/d$. The old record law does not decide whether $B$ is hidden, consumed, or later granted.

*Proof.* The visible number measurement is a complete classical channel. The enlarged interface has the declared direct-sum block type. Apply Theorem and Proposition .

XVII-A is the negative control: exact reconstruction of one finite-matrix bounded sine effect from one declared region is not an arbitrary bulk-code decoder, an entanglement wedge, or holographic emergence [CheungXVII]. Theorem gives the corresponding numerical task separation.

**Three-ledger output.** The access algebra determines the recovery theorem. Number statistics can estimate a declared transfer parameter. No wave-function of the universe, physical cosmological observer algebra, or expanding geometry is constructed.

## Extension V: two finite analogue packet models

### Vacuum–excitation exchange

Use two equal-frequency oscillator packets and restrict to the invariant vacuum-plus-one-excitation sector. For real $g$,

$$H_0 = \omega(n_L + n_B), \qquad H_{\mathrm{int}} = g(a_L^\dagger a_B + a_B^\dagger a_L).$$

A pulse of duration *t* realizes up to a known phase, with *η = cos^2 (gt)*.

**Proposition.** Known one-excitation probes give $q = \eta$, so $n$ independent trials estimate $\eta$ with radius

$$\sqrt{\frac{\log(2/\zeta)}{2n}}$$

at confidence at least $1 - \zeta$. Inferring $gt$ requires an injective inverse-Lipschitz sector.

For an unknown qubit in the vacuum/excitation span, the same number-only record has recovery error at least *1/2*. Furthermore,

$$[H_0, H_{\mathrm{int}}] = 0, \qquad \Delta E_B = \omega(1-\eta)\rho_{11}.$$

This is an energy-transfer identity, not a finite-temperature reset theorem.

*Proof.* The one-excitation calibration is Bernoulli with mean $\eta$. Apply Theorem with *b = 1, η_error = 0*. The unknown-code statement is A/D. The exchange Hamiltonian preserves total occupation, and the bath packet occupation is *(1 − η)ρ_11*.

### A flagged finite Friedrichs-type packet model

A second finite model carries a *d*-dimensional internal code in one excitation. Let the one-excitation location Hamiltonian be

$$h = \omega_0 \mathrm{proj}\, e + \sum_{j=1}^{m} \omega_j \mathrm{proj}\, j + \sum_{j=1}^{m} g_j(\mathrm{kb}\, ej + \mathrm{kb}\, je), \qquad g_j \in \mathbb{R}.$$

Its action on the internal labels is the identity. The vacuum is invariant. Write

$$e^{-ith}|e\rangle = a(t)|e\rangle + \sum_j b_j(t)|j\rangle, \qquad |a(t)|^2 + \sum_j |b_j(t)|^2 = 1.$$

The accessible packet *A* consists of location *e*; the other locations form *B*. For *r(t) = 1 − |a(t)|^2*, the complete code isometry has the form

$$V_t|\psi\rangle = a(t)J|\psi\rangle_A|\mathrm{vac}\rangle_B + \sqrt{r(t)}|\mathrm{vac}\rangle_A K_t|\psi\rangle_B,$$

where *K_t* is an isometry on every nonzero-loss branch. The vacuum flag is orthogonal to the occupied code ranges.

Set

$$a_d = 1 - d^{-2}, \qquad e_{\mathrm{leak}}(\mathscr{E}) = \inf_{\sigma} d_{\diamond}(\mathscr{E}, X \mapsto \mathrm{Tr}(X)\sigma).$$

**Lemma.** On $\mathbb{C}^d$,

$$\inf_{\mathrm{sigmad}} _{\diamond}(\mathrm{id}, X \mapsto \mathrm{Tr}(X)\sigma) = d_{\diamond}(\mathrm{id}, X \mapsto \mathrm{Tr}(X)I/d) = a_d.$$

*Proof.* The Bell overlap of a replacement output is *1/d^2*, independently of *σ*, giving the lower bound *a_d*. The completely depolarizing channel is the uniform mixture of all *d^2* Weyl conjugations. Separating the identity term gives the matching upper bound.

**Theorem.** For ,

$$e_{\mathrm{rec}}(\mathscr{N}_t) = e_{\mathrm{leak}}(\mathscr{E}_t) = r(t)a_d.$$

An optimal decoder returns the occupied code and prepares *I/d* on the vacuum flag. The number-only flag law is independent of the internal code, so its record-only channel is constant.

If *ω_j = ω_0 = ω*, then, with *G = (∑_j g_j^2)^{1/2}*,

$$r(t) = \sin^2(Gt), \qquad \Delta E_B = \omega r(t)$$

on the one-excitation input. The interaction commutes with the bare total-number energy.

*Proof.* Orthogonality of the vacuum and occupied sectors gives

$$\mathscr{N}_t(X) = (1-r)JXJ^{\dagger} + r\mathrm{Tr}(X)|\mathrm{vac}\rangle\langle\mathrm{vac}|,$$

$$\mathscr{E}_t(X) = rK_tXK_t^\dagger + (1-r)\mathrm{Tr}\,(X)|\mathrm{vac}\rangle\langle\mathrm{vac}|.$$

The stated decoder gives *(1 − r)id +r Tr ( · )I/d*, with error *ra_d*. For any decoder, the Bell overlap from the first branch is at most *1 − r*; the vacuum branch contributes exactly *r/d^2*. The Bell test proves optimality.

For leakage, choose

$$\sigma_B = rK_t(I/d)K_t^\dagger + (1-r)|\mathrm{vac}\rangle\langle\mathrm{vac}|.$$

Lemma gives the upper bound *ra_d*. For an arbitrary candidate state, apply a channel retaining the logical *K_t* -range and mapping its complement to one flag. Let *u* be the candidate's weight in that range. On the Bell input, the logical Bell effect has expectation difference at least *r − u/d^2*. The effect consisting of that Bell effect plus the full flag has difference at least *u(1 − d^{−2})*. For *u ≤ r*, the first is at least *ra_d*; for *u ≥ r*, the second is at least *ra_d*. This proves the leakage lower bound.

For equal frequencies, only the normalized bright vector *G^{−1}∑_j g_j|j⟩* couples to *|e⟩*. The resulting two-dimensional exchange has amplitude *cos (Gt)*, proving the final formulas.

This finite star model is a declared Friedrichs-type analogue. It is not asserted to be the continuum operator or complete model of Hawk02-C. The latter supplies separately scoped autonomous packet and loss comparisons [CheungHawkPackets]. No continuum spectral limit, Markov decay, or laboratory horizon is derived.

Controls, guards, locations, retained storage, energy meters, and all accessible packet outputs belong to the laboratory observation boundary [CheungRecords]. If a smoothed tensor core is installed, its positive-core selection theorem remains a domain comparison, not photon selection or code recovery [CheungTensor].

**Three-ledger output.** Transfer parameters and number records can be estimated. Coherent packet access has the exact error ; record-only access is more restricted. Neither a Hawking-like spectrum nor an energy-transfer curve constructs a sonic, optical, or gravitational horizon.

## Extension VI: environmental monitoring and decoherence

Let a system qubit *S* interact with a fresh environment qubit:

$$V_\theta|0\rangle = |0\rangle|0\rangle, \qquad V_\theta|1\rangle = |1\rangle(\cos\theta|0\rangle + \sin\theta|1\rangle), \quad 0 \le \theta \le \pi/2.$$

Destructive environment readout gives

$$K_0 = \mathrm{kb}\,00 + \cos\theta\,\mathrm{kb}\,11, \qquad K_1 = \sin\theta\,\mathrm{kb}\,11,$$

with record probability *Pr (R = 1) = sin^2 θ ρ_11*.

**Proposition.** The environment record alone obeys A/D. The unconditional system channel multiplies off-diagonal entries by *cos θ*.

At *θ = 0*, the returned system channel is the identity while the record is constant. At *θ = π/2*, even record plus system is entanglement breaking. No compulsory positive heat cost follows from this instrument.

*Proof.* Sum the two Kraus branches to obtain the dephasing formula. The record-only map is classical. At zero angle, *K_0 = I, K_1 = 0*. At *π/2*, both Kraus operators have rank one, so every reference–output state is separable even with the record flag. No thermal Hamiltonian or reservoir reset was specified.

### A zero-reservoir-energy decoherence realization

Take *H_B = 0, τ_B = I/2*, and a controlled *X_B* from *S*. Tracing *B* dephases *S*, because *Tr (τ_BX_B) = 0*, while *Q_B = 0*. For input *proj +*,

$$\Delta S_S = -\log 2, \qquad I(S:B)_\omega = \log 2, \qquad D(\omega_B\|\tau_B) = 0.$$

The thermal equality reads *0 = −log 2 + log 2*. This is decoherence without an entropy-decreasing reset.

### Spectator noninterference

**Proposition.** For a fixed local channel *Λ: S → Z*, the unconditioned *Z*-output depends only on the input *S* -reduction. The joint *ZE*-output with an untouched reference depends only on the joint *SE*-reduction. Additional inaccessible spectators do not change these laws when the operation and declared controls are unchanged.

*Proof.* Output effects pull back as $\Lambda^{*}(F) \otimes I$. Joint effects pull back as $(\Lambda^{*} \otimes id_E)(G) \otimes I$. Equality of the corresponding input reductions therefore gives equality of every output expectation.

Equality of the source marginal alone does not imply equality of the output together with an arbitrary reference. This is the scoped En03-A spectator discipline [CheungRecords]. Psy02-A/B remain documentary scalar/pruning and reduction-pilot comparisons, not a new density law, pointer ontology, consciousness model, or Orch-OR conclusion [CheungPsy].

**Three-ledger output.** Record-only access is classical; returned system access can retain coherence. Named dephasing factors are not universal code-recovery certificates. No physical annihilation or unconditional heating principle is inferred.

## Extension VII: two-zone records, recycling, and reset

### A complete two-zone sensing model

Append a fresh $|0\rangle$ to a source qubit and apply CNOT. For calibration use

$$\rho_\theta = (1-\theta)\mathrm{proj}\,0 + \theta\,\mathrm{proj}\,1.$$

Call the two zones $N, A$. Apply an independent bit flip to $A$ with probability $\nu \in [0, 1/2)$, then destructively measure both data names. The exported word has law

$$p_{00} = (1-\theta)(1-\nu),\ p_{01} = (1-\theta)\nu,$$
$$p_{10} = \theta\nu, \qquad p_{11} = \theta(1-\nu).$$

The coarsened $A$-click law is $q_{\theta} = \nu + (1-2\nu)\theta$, giving with $b = 1-2\nu$. For a general unknown quantum input this preparation is still a linear channel and does not clone it.

This is the explicit simplified model retained from version 1, not a verbatim reproduction of En10-A/B's full gate family [CheungEn10]. Fresh ancillas, selector purification, retained controllers, failures, and clocks must be counted separately.

### An explicit energy-conserving reset

Let memory $M$ and bath qubit $B$ have equal Hamiltonians

$$H_M = E\,\mathrm{proj}\,1, \qquad H_B = E\,\mathrm{proj}\,1, \qquad E > 0.$$

Set

$$\tau_B = (1-p_\beta)\mathrm{proj}\,0 + p_\beta\,\mathrm{proj}\,1, \qquad p_\beta = (1+e^{\beta E})^{-1}.$$

For a smooth pulse vanishing at both endpoints, choose

$$H(t) = H_M + H_B + g(t)\mathrm{SWAP}, \qquad \int g(t)\,\mathrm{d}t = \pi/2.$$

The SWAP operator commutes with $H_M + H_B$. The interaction-picture endpoint is $-iSWAP$.

**Proposition.** For $\rho_M = (1-p)\mathrm{proj}\,0 + p\,\mathrm{proj}\,1$, the final states are $\omega_M = \tau_B$, $\omega_B = \rho_M$, and

$$Q_B = E(p - p_\beta),$$
$$\Delta S_M = h_2(p) - h_2(p_\beta),$$
$$I(M:B)_\omega = 0,$$
$$D(\omega_B \| \tau_B) = D_{\mathrm{bin}}(p \| p_\beta).$$

They satisfy exactly. The memory has trace distance $p_{\beta}$ from its ground state.

For $p = 1 - p_{\beta}$, the entropy decrease is zero but $Q_B = E(1-2p_{\beta}) > 0$. Thus strict entropy decrease is sufficient, not necessary, for positive reservoir heat.

*Proof.* SWAP exchanges the two factors. They remain a product. The Gibbs ratio $\log[p_{\beta}/(1-p_{\beta})] = -\beta E$ gives

$$\beta E(p - p_\beta) = h_2(p) - h_2(p_\beta) + D_{\mathrm{bin}}(p \| p_\beta).$$

Binary entropy symmetry proves the witness.

The bare total energy is conserved in this pulse model. The net switching work of *g(t)SWAP* is zero: the expectation of SWAP is constant and the pulse begins and ends at zero. Preparation, sensing, physical pulse control, and resetting other controllers are not priced by that statement.

### A surviving archive changes the complete entropy ledger

**Proposition.** Let $R$ be an untouched archive initially correlated with $M$, with state $\rho_RM$, independent of $\tau_B$. After the same SWAP, up to irrelevant free phases,

$$\omega_{RMB} = \tau_M \otimes \rho_{RB},$$

where $\rho_RB$ is $\rho_RM$ with $M$ relabeled as $B$. For the complete non-bath system $RM$,

$$\Delta S_{RM} = S(M \mid R)_\rho - S(\tau_B),$$
$$I(RM : B)_\omega = I(R : M)_\rho.$$

The heat remains $E(p - p_{\beta})$ for a diagonal memory marginal of excitation probability $p$. The old archive law is unchanged.

*Proof.* SWAP transfers the entire $RM$-correlation to $RB$ and puts an independent $\tau_M$ in the memory. Hence

$$S(\omega_{RM}) = S(\rho_R) + S(\tau_B),$$

giving . The final independent $M$-factor does not contribute to mutual information with $B$, proving . The bath marginal is the original memory marginal. Trace preservation leaves $R$ unchanged.

For a perfect classical archive, $S(M \mid R) = 0$. Then the complete entropy decrease is $-h_2(p_{\beta})$, even when the local memory entropy decreases. The missing positive term reappears as final archive–bath mutual information. This prevents double-counting the same record as both erased and untouched.

### Recycling is not quantum access through a heat label

Complete unitary uncomputation can recycle a coherent computation when all required registers remain available. The exact recovery criterion is still constancy of the *complete* complement, not constancy of a selected bath marginal [CheungEn07]. A mixed constant complement gives a syndrome state; it does not mean every physical work register is a pure blank.

Appending a classical heat-meter outcome does not grant coherent bath access. A real meter must itself be included in the complete channel and, for a postmeasurement thermal claim, in the appropriate enlarged energy/entropy account. This is the energy/reset/leakage split of Section [CheungHawk05A].

**Three-ledger output.** The word estimates a source parameter but does not return the unknown source code. The SWAP model supplies a Hamiltonian, Gibbs state, reset map, and exact balance. It is not an entire work-extraction cycle, a universal heat per Enter operation, or a free reset of every controller.

## Extension VIII: transport and algebraic dependency cuts

### Protocol A: a reference-preserving SWAP route

Choose a finite chain of regular laboratory locations in one convex regular reserve, with declared communication operations and durations. Initialize equal payload buffers in independent blanks. Sequential SWAPs move one $d$ -dimensional payload to the receiver.

**Proposition.** The ideal SWAP route implements the identity on the payload, including every untouched reference. If an intermediate complete cut passes only a classical record, with no message-bearing quantum bypass, every end-to-end decoder obeys A/D. If its complete quantum access has dimension at most $k$, the error is at least $1 - k/d$ , modified by complete implementation allowances as in .

*Proof.* Each SWAP moves all payload matrix units and reference correlations into the next buffer. Everything following the modified cut is postprocessing of that cut's complete output. Apply Theorem .

A consumed sender buffer that has already transferred its logical content is not evidence that the receiver lost the code. The route permissions and speed are inputs, not a derived Hamiltonian of gravity. This is the scoped I-B transport comparison [CheungIB].

### Complete scheduled ancestry

For finite sites $X$, include records, controllers, and capacity wires at their actual sites. At tick $j$, let the complete channel factor over a declared partition $\mathscr{B}_j$. For terminal sites $Y$, set

$$N_{T,T}(Y) = Y, \qquad N_{j,T}(Y) = \bigcup \{B \in \mathscr{B}_{j+1} : B \cap N_{j+1,T}(Y) \neq \varnothing\}.$$

**Proposition.** The Heisenberg pullback satisfies

$$\Phi^{*}_{1:T}(\mathscr{A}(Y)) \subseteq \mathscr{A}(N_{0,T}(Y)).$$

Equal reductions on this region give equal terminal reductions. For joint reference outputs, equality is required on the joint ancestor–reference reduction. A complete trace-preserving intervention outside the backward region cannot change the terminal output with the schedule fixed.

If every dependency edge has an independently certified regular future-causal host route, scheduled ancestry is contained in the prescribed curve-generated metric causal relation.

*Proof.* An output observable is the identity on every excluded block. Channel adjoints are unital, so such blocks remain identities. Meeting blocks enlarge support only within themselves. Backward induction proves . Joint observables prove the reference assertion. The same identity argument permits excluded-block replacement. Concatenate the separately supplied causal routes for metric containment.

This is the local finite mechanism compared in I-B, En03-A, VII-J-ST, and X-K-ST [CheungIB,CheungRecords,CheungCausality,CheungXK]. It is an upper bound, not necessarily minimal influence: two SWAPs on the same pair have nontrivial scheduled paths but total identity channel.

### A tensor-free finite algebra lemma

**Lemma.** Let $\mathscr{A}_{in} \subseteq M_n(\mathbb{C})$ be a unital finite-dimensional $C^{*}$-subalgebra, and let $T: \mathscr{A}_{out} \to M_n(\mathbb{C})$ be *textup UCP*. The following are equivalent:

• equal input restrictions to $\mathscr{A}_{in}$ imply equal output restrictions through $T$;

• $T(\mathscr{A}_{out}) \subseteq \mathscr{A}_{in}$;

• $T$ factors through a unique *textup UCP* map $\mathscr{A}_{out} \to \mathscr{A}_{in}$ and the inclusion. No spatial complementary tensor factor is required.

*Proof.* Containment gives the unique factor and the state implication. The factor is completely positive because the subalgebra inherits the same positive cones at every matrix level.

Conversely, let $h = T(b)$ for self-adjoint $b$. If $h \notin \mathscr{A}_{in}$, its Hilbert–Schmidt component $H$ perpendicular to the self-adjoint part of $\mathscr{A}_{in}$ is nonzero. Since $I \in \mathscr{A}_{in}$, $\operatorname{Tr} H = 0$. For sufficiently small $u > 0$, the matrices $I/n \pm uH$ are states with equal restrictions to $\mathscr{A}_{in}$, but their values on $h$ differ by $2u\operatorname{Tr}(Hh) = 2u\|H\|_2^2 > 0$. This contradicts (i). Linear decomposition gives containment for every $b$.

This finite proof is the algebraic comparison pattern of En05-C and Oai282-A [CheungEn05C,CheungNegative]. It constructs no continuum local net or geometric region.

### Protocol B: a hybrid algebra at a dependency slice

Let $Q \cong \mathbb{C}^4$, let $U$ be a fixed unitary, and define

$$P_j = |j\rangle\langle j| \otimes I_2, \qquad \mathscr{A}_{\mathrm{cut}} = U^{\dagger}(M_2 \oplus M_2)U \subseteq M_4.$$

The input dependency slice is declared by this algebra, not by calling one geometric region a tensor factor. Install

$$\mathscr{N}(X) = \operatorname{bigoplus}_{j=0}^{1} (\langle j| \otimes I)UXU^{\dagger}(|j\rangle \otimes I).$$

A complete finite schedule can implement $U$, read the first label destructively, retain the second qubit, and deliver the classical label and qubit through declared wires. Every controller and wire belongs to its backward dependency network.

**Proposition.** Protocol B factors through the state restriction to $\mathscr{A}_{cut}$, has $d = 4$, $k = 2$, and

$$e_{\mathrm{rec}}(\mathscr{N}) = \frac{1}{2}.$$

If only its classical label is delivered, A/D applies with the weaker classical output type. The algebraic cut alone does not certify a physical causal embedding.

*Proof.* The Heisenberg output pullback is exactly the inclusion of $\mathscr{A}_{cut}$. Lemma proves factorization.

A decoder reembeds each $2 \times 2$ block and applies $U^{\dagger}$. Its composite is

$$T_U(X) = U^{\dagger}\Big(\sum_j P_j U X U^{\dagger} P_j\Big)U.$$

With $S = U^{\dagger}(Z \otimes I)U$,

$$T_U = \tfrac{1}{2}(\mathrm{id} + \mathrm{Ad}_S),$$

so $d_{\diamond}(T_U, id) \le 1/2$. Theorem gives the matching lower bound. The final statements follow from the output restriction and the absence of geometric permissions in an algebra inclusion.

I-B-G's full-field finite speed is a separate support constraint, not a source of finite gate permissions [CheungIBG]. If this protocol merely reports a continuum window, the X-Enter-ST supplied-evaluation identity does not acquire the field or repair a missing convolution construction [CheungCausality,CheungXJ].

An arrival-event probability is bounded by a complete-channel allowance only if its arrival flag belongs to the compared output. Logical-channel distance alone does not price an omitted clock or flag.

**Three-ledger output.** The ideal SWAP route returns the code; the hybrid algebra example has exact optimal error $1/2$. Named arrival or window statistics are separate tasks. No physical causal past, universal summoning protocol, continuum-field simulation, or Lorentz-cone selection is derived.

## Extension IX: endpoint ownership and entropy-history witnesses

### A private-channel prefix

Let $W_u$, $u = 1, \ldots, d^2$, be the Weyl unitaries. Define

$$\mathscr{P}_{\mathrm{pad}}(X) = \frac{1}{d^2}\sum_u \mathrm{kb}\, uu_R \otimes W_u X W_{uA}^{\dagger}.$$

**Theorem.** For every density matrix $\rho$,

$$(\mathscr{P}_{\mathrm{pad}}(\rho))_R = I_R/d^2, \qquad (\mathscr{P}_{\mathrm{pad}}(\rho))_A = I_A/d.$$

Nevertheless:

• record-only access is constant and has $e_{rec} = 1 - d^{-2}$;

• granted $RA$ access recovers the code exactly by applying $W_u^{\dagger}$ in block $u$;

• destructive measurement of $A$, with all new outcomes retained, gives a complete classical cut and hence A/D. All three preserve the old $R$-law.

*Proof.* The Weyl twirl obeys

$$d^{-2}\sum_u W_u X W_u^{\dagger} = \mathrm{Tr}\,(X) I/d.$$

This gives both marginals. Lemma gives (i). The block inverse gives (ii). Statement (iii) follows from its classical output type. Trace preservation gives the old-record statement.

This is the private-channel mechanism [PrivateChannels]. An explicit complete isometry is

$$V|\psi\rangle = \frac{1}{d}\sum_u |u\rangle_R |u\rangle_E \otimes W_u|\psi\rangle_A.$$

Its complement to $RA$ is the constant uniform $E$-channel, consistent with Theorem . The constant selected marginals $R$, $A$ are not the two complementary outputs of a pure bipartite dilation; therefore Corollary is not contradicted.

### A finite entropy-history/recovery separation

We now prove locally the mechanism of the supplied Hawk05-B thm:retain comparison [CheungHawk05B].

Fix $D = 2^N \geq d$, an isometry $J: Q \to (\mathbb{C}^2)^{\otimes N}$ with $J|0\rangle = |0\rangle^{\otimes N}$, a unitary $U \in U(D)$, and an inaccessible $L \cong Q$. At prefix $t$, declare the first $t$ output flavor registers accessible and the remaining $N - t$ registers inaccessible. Set

$$V_{\mathrm{return},U}|\psi\rangle = UJ|\psi\rangle \otimes |0\rangle_L, \qquad V_{\mathrm{retain},U}|\psi\rangle = UJ|0\rangle \otimes |\psi\rangle_L.$$

This is a sequential complete interface by releasing one factor at a time; explicit equal-energy SWAP buffers can implement the same release schedule.

**Theorem.** The two models have the same complete output spaces and satisfy:

• for input $|0\rangle$, their accessible states agree at every prefix for the same $U$;

• for Haar-distributed $U$, every fixed pure code input has the same distribution of each accessible-prefix entropy in the two models;

• at the endpoint,

$$e_{\mathrm{rec}}(\mathcal{N}_{\mathrm{return}}) = 0, \qquad e_{\mathrm{rec}}(\mathcal{N}_{\mathrm{retain}}) = a_d;$$

their optimal complete-complement leakages are respectively $0$ and $a_d$;

• the construction admits a conserving finite realization with degenerate logical labels. The first statement is a fixed-probe equality, not equality of the two channels on every input.

*Proof.* For $\psi = 0$, the two complete vectors coincide, so every prefix reduction coincides.

For a fixed normalized $J\psi$, Haar invariance makes $UJ\psi$ uniformly distributed on the unit sphere of $\mathbb{C}^D$, just as $UJ0$. Every prefix reduced-state distribution, and therefore every prefix entropy distribution and mean, agrees.

At $t = N$, the returning channel is the isometric channel $X \mapsto UJXJ^{\dagger}U^{\dagger}$, with constant complement. It has an exact decoder. The retaining accessible channel is replacement by $UJ\mathrm{proj}\ 0J^{\dagger}U^{\dagger}$, while $L$ receives the original code. Lemma , and a left inverse of the $L$-embedding, give the stated errors and leakages.

For conservation, realize each two-dimensional flavor register as the occupied sector of a site with a vacuum and two degenerate excitation labels. On the sector with every such site occupied, $H_0 = \Delta \sum_j n_j$ is scalar. Extend $U$ by the identity on other occupation sectors. Give the logical labels of $L$ equal energy. The encodings, storing operations, and equal-site SWAPs can then be extended within the corresponding equal-energy sectors. They commute with the bare total occupation energy. This is explicit finite sector conservation, not a gravitational mass-loss law.

For orientation, if the prefix split has dimensions $m, n$, the common Haar entropy mean is the usual Page–Sen mean. Writing $\ell = \min(m, n)$, $h = \max(m, n)$, its standard closed evaluation is

$$\mathrm{P}(m,n) = \sum_{j=h+1}^{mn} \frac{1}{j} - \frac{\ell - 1}{2h}.$$

The evaluation is an explicitly cited external identity [PageAverage,Sen]. The separation theorem does not depend on that evaluation: equality of the distributions was proved directly by Haar invariance.

**Proposition.** Let $\sigma_K$ have purification $|\chi\rangle_{KP}$, and let $L \cong Q$. The isometries

$$V_{\mathrm{good}}|\psi\rangle = |\psi\rangle_{Q_R}|\chi\rangle_{KP}|0\rangle_L, \qquad V_{\mathrm{bad}}|\psi\rangle = |0\rangle_{Q_R}|\chi\rangle_{KP}|\psi\rangle_L$$

have identical accessible spectra for every pure input when the accessible factor is $Q_{RK}$. Their recovery errors are $0$ and $a_d$. For $H_R = I_{Q_R} \otimes H_K$, their selected energy laws agree. One may choose $\sigma_K$ Gibbs.

*Proof.* The accessible channels are $X \mapsto X \otimes \sigma_K$ and $X \mapsto \mathrm{Tr}(X)\mathrm{proj}\ 0 \otimes \sigma_K$. For pure inputs the nonzero spectra coincide. The energy law depends only on $\sigma_K$. Exact recovery and the replacement benchmark give the errors.

### What an endpoint interpretation would still require

The Haar entropy proxy is not an evaporation theorem. In particular, it does not remove the hidden logical register in the retaining model. The further gravitational obligations discussed in Hawk05-B remain outside this construction [CheungHawk05B].

A physical identification would at least need:

• an identified physical code and encoding;

• one common physical dynamics and state prescription;

• a complete outgoing-access and inaccessible-output assignment, including any analogue of $L$;

• a physical endpoint and conservation account justifying what persists, disappears, or becomes accessible;

• a decoder/apparatus and observable matching certificate with its resources and errors. This is this manuscript's matching checklist, not a claimed verbatim reproduction or discharge of an unavailable source list.

Hawk03-A's encoded partner is not a smooth horizon. Hawk01-A/B's selected late thermal marginals are not recovery laws. Em03-H's remnant comparisons remain source-qualified [CheungEndpoints]. A finite label called ``soft hair'' is not thereby physical soft hair.

No final-state postselection rule is installed. A trace-nonincreasing final-state proposal would need its own probabilities, failure output, and reference-preserving task.

**Three-ledger output.** Residual ownership can change endpoint recovery while old records or selected entropy statistics agree. Named spectra remain meaningful within their models. No evaporation, remnant, burst, firewall resolution, or gravitational ending is selected.

## Resources, error types, and non-acquisition

A physical implementation claim must provide:

• the finite code, field/apparatus encoding, initial environment, and relevant initial correlations;

• the complete instrument, including failures, guards, settings, selectors, capacity tests, stopping labels, and returned quantum factors;

• the observation algebra and residual ownership before, at, and after the cut;

• an instruction schedule, permitted operations, fresh allocations, consumed names, storage, locations, and durations;

• purifications, shared entanglement, prior quantum transmissions, and controllers;

• separate complete-channel, continuum-approximation, calibration, finite-precision, and sampling allowances;

• Hamiltonians, Gibbs preparation, work convention, archive, and reset target for each thermal claim;

• an independent physical matching or causal-embedding theorem for every geometric interpretation.

A finite Kraus construction is not an assertion that these laboratory requirements have been met. The supplied X-K-ST, En03-A, and transport records retain this distinction [CheungXK,CheungRecords,CheungCausality].

### Finite probes and continuum functions

**Proposition.** A finite family of linear evaluations on an unrestricted infinite-dimensional smooth test space is not injective. On an independently established finite-dimensional admissible class, injectivity of the probe map is sufficient for determination within that class.

*Proof.* A linear map from an infinite-dimensional vector space to a finite-dimensional space has nonzero kernel. The second assertion is the definition of injectivity on the restricted class.

This is not a claim that finite experiments can never determine a finite parameter family. Nor does arbitrarily close finite-precision data prove equality of exact real functionals. Lemma concerns fixed continuous report preparation, not all imaginable enlarged apparatuses [CheungIC,CheungEcho,CheungNegative].

### Complete channel error is not an unbounded-energy estimate

**Proposition.** If $d_{tr}(\rho, \sigma) \le \delta$, every effect probability differs by at most $\delta$. For a bounded Hamiltonian with spectral width $E_{max} - E_{min}$,

$$|\mathrm{Tr}\,[H(\rho - \sigma)]| \le (E_{\max} - E_{\min})\delta.$$

No analogous universal energy bound follows from trace distance alone for an unrestricted unbounded Hamiltonian.

*Proof.* The effect bound is trace-distance duality. Subtract the midpoint of the Hamiltonian spectrum and use

trace-norm duality for the energy bound. For the last assertion, mix a state of weight $\delta$ at arbitrarily high energy into a fixed ground state. Its trace distance from the ground state is at most $\delta$ while its energy can be arbitrarily large.

A complete-channel allowance prices an arrival or heat-meter probability only when that record belongs to the compared output. Dimensionless probability errors must not be added to energy errors without a justified conversion.

## Completion, debugging, and mathematical release

### The nine finite extensions

begingroup toprule Task Constructed here Not inferred

midrule endfirsthead toprule Task Constructed here Not inferred

midrule endhead bottomrule endfoot I Weak Kraus family, complete diffusive instrument and limit, continuous jump instrument, path-space access theorem. A general photon detector, arbitrary homodyne apparatus, collapse ontology, or unrestricted continuum instrument limit.

II Finite monitored brickwork family, complete masks and outcomes, actual residual algebra, polynomial finite-size entropy average. A volume-law/area-law transition or critical exponent.

III Bounded scalar packet detector, prescribed accelerated trajectory, complete unitary instrument, response and remainder. A pointlike continuum detector, universal acceleration identifiability, or heat inferred from Unruh temperature.

IV Late-observer packet surrogate and declared classical or classical–quantum output algebra. An expanding geometry, de Sitter state, cosmological horizon, or wave-function of the universe.

V Energy-conserving two-packet exchange, number instrument, transfer inference, and energy ledger. A laboratory horizon, gravitational Hawking radiation, or quantum-return decoder from a spectrum.

VI System–environment instrument, record/residual distinction, and zero-reservoir-energy decoherence realization. Pointer ontology, compulsory heating, pruning density law, or physical annihilation.

VII Two-zone record law, fixed-shot inference, finite Gibbs bath, explicit SWAP reset, and exact thermal balance. Universal heat per readout, free controller reset, or net work extraction for an unspecified cycle.

VIII Reference-preserving SWAP route, classical-cut obstruction, scheduled ancestry, and complete error budget. Derived physical causality, universal summoning, or identification of scheduled ancestry with physical past.

IX Three terminal specifications and private-channel example with identical selected marginals but different access. Evaporation, a remnant, final burst, firewall resolution, or a selected gravitational endpoint.

endgroup

### Version-2 additions and retained exclusions

begingroup toprule Addition Established here Not inferred

midrule endfirsthead toprule Addition Established here Not inferred

midrule endhead bottomrule endfoot Complement rank $dim\ A \geq drank\ \sigma_B$, branchwise quantum capacity, and no independent duplicate recovery. A subsystem criterion, efficient decoder, or peak-memory theorem.

Hybrid entropy Exact block entropy and $H(R) \leq S(RK) \leq H(R) + log\ k$. Monotonic total hybrid entropy or radiation Page history.

Continuum reports Fixed-interface diamond continuity and fixed-budget TV convergence. Field acquisition, an arbitrary rounding limit, or convergence of undeclared enlarged apparatuses.

Named effects Exact binary-effect distance and $e_A(\mathcal{M})lee_rec\ (\mathcal{M})$. A reverse implication from code loss to loss of one effect.

Photonic sibling Finite code Kraus maps on supplied transverse packets. A local Maxwell detector, interacting Ward theorem, or tensor-core/photon identification.

Window inference Explicit propagation of window and calibration allowances into a fixed-shot radius. Unproved

window injectivity, point-source transfer, or universal acceleration identification.

Second packet model Finite star Hamiltonian and exact flagged recovery/leakage $r(t)(1 - d^{-2})$. Continuum Friedrichs dynamics, Markov decay, or a physical horizon.

Spectators and archive Correct joint-reduction and complete entropy accounts. Reference equality from source-marginal equality, or erasure of an untouched archive.

Algebraic dependency cut Finite UCP factorization and a sharp $d = 4$, $k = 2$ protocol. A continuum local net, spatial tensor factor, or geometric embedding.

Endpoint entropy witness Same fixed-probe prefix states and Haar entropy laws, with endpoint errors $0$, $a_d$. Evaporation, no hidden logical memory, or discharge of gravitational matching obligations.

Apex-function ledger Source-qualified placement of $\mathscr{C}_b$, $\mathscr{D}_b$. Their acquisition, branch selection, new row evaluation, or O.1 closure.

endgroup

### Seven principal corrections retained

• **Half-diamond normalization.** The sharp $1 - 1/d$ constant is a half-diamond-distance constant. The raw diamond-norm lower bound is twice it.

• **A residual is not a classical cut.** The complete output must be declared. A quantum residual requires the $k$-law or a more specific channel analysis.

• **Heat positivity.** Strict complete non-reservoir entropy decrease is sufficient, not necessary, for positive deposited heat. Proposition is the explicit witness.

• **Continuous records.** A path-space POVM or instrument must be defined. Finite-prefix formulas alone do not construct an unspecified continuum process. Differential entropy is not Theorem B.

• **The cone is not an expanding universe.** The unchanged ultrastatic host does not produce a de Sitter state, cosmological observer algebra, or horizon.

• **A cutoff is not a continuum detector.** A finite instrument does not remove field regulators, supply local apparatus dynamics, or acquire a continuum kernel. Full-field propagation does not automatically survive finite-mode truncation.

• **Topic labels are not source theorems.** Oai212-A and Oai282-A do not prove a monitored transition or the Unruh effect. Oai-prefixed Cheung manuscripts are not reassigned to the separate OpenAI catalogue.

### Additional version-2 debugging

• **Effect bounds have a direction.** An optimal one-effect lower bound can lower-bound code recovery under matched interfaces. A code-recovery lower bound does not imply failure of that effect; see Remark .

• **Complement rank is not label storage.** The total size of a classical label register cannot replace a deficient quantum branch. Corollary retains the branchwise condition.

• **Pure complete dilation is essential.** No independent duplicate recovery concerns complementary outputs of one complete isometry. Selected marginals of a larger mixed interface need not satisfy that two-output hypothesis.

• **Singleton paths do not have positive branch probability.** Continuous conditional states are described by densities or regular conditional distributions.

• **Ownership is not domain selection.** Theorem E does not choose CR, F, a self-adjoint extension, or a physical core. Those are independently installed domain data.

• **Thermal archives are not omitted.** If an archive survives, the complete entropy decrease is computed on the actual non-reservoir system, as in Proposition .

• **Exact entropy quantifiers are retained.** A fixed-probe prefix equality and equality of Haar means are not equality of channels for all inputs.

• **External programme gates remain distinct.** The historical H2 stop in XIV-H, the constructed internal I2 amplitude in XIV-I, and the unresolved full I3/J2 maps are not collapsed into one verdict. The technical XV-B

lifting results do not supply the missing physical characteristic coefficient [CheungAnalytic].

### Mathematical release

**Theorem.** For the definitions, access contracts, and models printed here:

• the classical–quantum access law is sharp for every $d \geq 2$, $1 \leq k \leq d$, with a finite attaining instrument and decoder;

• complete complements determine exact whole-code recovery and imply the stated rank-capacity and no-duplicate results;

• classical accumulation, hybrid entropy, terminal ownership, and finite thermal balance have the stated proofs;

• continuous programming of one fixed finite reporting interface preserves its access type and has the complete error bound ;

• binary-effect reconstruction has the exact distance and the one-way comparison ;

• the weak family has the declared diffusive limit, the jump and path-space instruments are complete, and the photonic sibling is a declared finite code instrument;

• the nine extensions have the nonempty finite models and the additional certificates listed above;

• the endpoint construction separates entropy-history statistics from reference-preserving code recovery. No excluded physical identification in either completion table is established by this theorem.

*Proof.* Theorem proves (i). Theorem and Corollaries – prove (ii). Section proves (iii). Lemma proves (iv). Theorem proves (v).

Theorem , the completeness calculation for , Theorem , and the displayed photonic Kraus family prove (vi). The constructions and proofs in Sections – prove (vii). Theorem proves (viii). Their hypotheses contain no omitted physical matching theorem.

par **Interpretive conclusion** par The information paradox, a physical Page curve, the measurement problem, and an unconditional thermodynamic cost of readout are reframed, not solved.

An informative record is not a quantum-return decoder. A granted residual changes the access contract. A named observable is not an arbitrary code. A channel is not a thermal machine. A prescribed host is not an emergent spacetime.

O.1 on the cone remains *texttt NOT_CLOSED*. Bulk symmetry is not restored. The downstream XV-B quantum-admission exit is not opened. XVI-N and Route B are not discharged. No inherited characteristic quantum coefficient is admitted. par

## Declarations

**Authorship and submission.** The author field follows the supplied manuscript series. Submission requires the author's approval of the mathematics, source versions, attribution, and final text. No referee approval or journal acceptance of this manuscript is asserted.

**Mathematical dependencies.** The finite access, recovery, entropy, effect, reporting, inference, reset, transport, and separation arguments required for the local release are printed. Standard primary literature is credited for the established frameworks. The closed Page–Sen evaluation is an explicitly identified external identity, not a premise of the separation proof.

**Continuum inputs.** The geometry, field realization, spectral packets, state, renormalization convention, and domain choices are declared inputs where used. Companion citations do not transfer unavailable hypotheses, products, wavefront estimates, apparatus constructions, or quantum-admission conclusions. No field value at the completed apex is defined.

**Source consultation.** The philosophy page, I-B record, XVI-O1-ST record, and the Frontiers publisher record were consulted online. The attempted I-C, I-B-ECHO, Hawk05-B, Oai290-A, and XVI-O1-CONE browse endpoints were unavailable through the consulted interface; those comparisons rely on the supplied version-qualified excerpts or explicitly labeled synopsis. The large retrieval inventory is not a complete corpus audit.

**Journal-record status.** The consulted publisher record for [CheungEnter] reports acceptance on 28 September 2026 and states that the final formatted version is forthcoming. No final-format publication date or page range is invented.

**Apex-source scope.** XVI-O1-CONE-v6 is used only through the supplied description of its named constants and one-row scope. No unseen coefficient formula or complete interacting Ward calculation is represented as locally proved.

**Execution-integrity witness.** No new loop table is added. The consulted XVI-O1-ST-v5 record distinguishes the typeset sixth coefficient $-4/3$ from the retained regression copy $+4/3$. With its unchanged gauge entry $+4/3$, the latter has residual $8/3$ in that coordinate. The record identifies the positive regression entry as a transcription error, not an unresolved physical sign. Passing a regression that expects this discrepancy does not establish consistency of the two arrays. No new loop evaluation or independent sign audit is claimed [CheungIntegrity].

**Catalogue attribution.** Oai-prefixed Cheung manuscripts remain distinct from the separate OpenAI-attributed catalogue. No OpenAI catalogue proof, family count, formalization claim, or physics interpretation is independently verified here. Families 282, 362, and 376 are not mathematical premises.

**Execution.** No successful pdfLaTeX run, numerical quadrature, symbolic-program execution, circuit compilation, laboratory realization, proof-assistant verification, independent rebuild, or independent review is asserted for this source. The mathematical evidence consists of the written statements and proofs on their declared domains.

**Format.** This is one pdfLaTeX source with an internal bibliography. No external figure, data, code, or bibliography file is required. Two compiler passes are intended for contents and cross-references.

**Unchanged programme scope.** The standing cone and XVI-O status is

$$\text{O.1 on the cone} = \texttt{NOT_CLOSED}, \qquad \texttt{bulk_symmetry_restored} = \texttt{false}.$$

This does not withdraw the separately scoped Q11 two-point record under its different prescription [CheungStanding]. No full characteristic map, joint section, transverse estimate, contact compatibility theorem, output wavefront theorem, or admitted inherited coefficient is constructed by the present finite access results [CheungAnalytic]. The technical existence of XV-B's abstract lifting interfaces is distinguished from its unopened downstream quantum-admission exit. This is not Einstein dynamics, gravity-generated entanglement, holographic reconstruction, a physical Page curve, or spacetime emergence.

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