I-C-Ex2: Eight Further Extensions of the Shredder Cut Adaptive computation, finite optical cutoffs, redundant records, memory, sensing, and scheduled relativistic tasks on a prescribed conical host

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Abstract

Destructive measurement can support computation, sensing, and redundant classical witnessing without returning an arbitrary unknown quantum code. We give eight further finite extensions of the shredder-access framework, with complete instruments, declared residual ownership, and local proofs. The common sharp obstruction remains F_e ≤ k/d, 1/2‖𝒟𝒩− id ‖_{◇} ≥ 1 − k/d when the complete accessible output is classical together with quantum blocks of dimension at most k. Exact recovery additionally requires accessible dimension at least d times the rank of the constant complete complement. These statements are re-established locally rather than imported from companion manuscripts. The principal application-level findings are explicit separations. A three-carrier authorized share of a five-qubit perfect code has an exact two-live-qubit streaming decoder but no one-live-qubit decoder under the stated delivery rules. A finite optical cutoff retains the same access obstruction, while cutoff error is exactly the discarded probability only for block-diagonal preparations; coherent tails require a different allowance. Redundant pointer records can become perfectly readable without returning logical coherences. A finite reservoir exhibits complete distinguishability revival and exact recovery-error oscillation with zero reservoir energy change. A destructive multiparameter sensor attains its classical Fisher bound while its classical output remains unable to return an unrestricted unknown code. Scheduled summoning examples separate feasible routing, deadline-limited quantum access, and independent-output impossibility. The contribution is an explicit, error-controlled synthesis of access dimension, executable decoder resources, finite reporting, and statistical precision across these settings. Its potential impact is methodological: apparently favorable records, spectra, memory revivals, or sensing precision can be evaluated without being mistaken for whole-code recovery. The underlying Schmidt-number, error-correction, Gaussian, statistical, and relativistic principles are credited; no priority claim is made for them. The metric, field realizations, clocks, and any Haar hypothesis remain inputs. No fault-tolerance threshold, infinite-dimensional capacity theorem, continuum detector, physical soft hair, inflationary geometry, Page curve, island, Einstein dynamics, or spacetime emergence is constructed. O.1 on the cone remains texttt NOT_CLOSED.

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Introduction: Enter and the scope of this successor

Enter, spacetime Enter, and the shredder philosophy

In Cheung's Enter calculus, a runtime quantum name is an access right to a quantum tensor factor. Destructive computational-basis readout consumes the measured name and returns classical data. The operational characterization of that primitive is relative to its stated strategy class; it is not a record-free derivation of measurement or a continuum field-apparatus construction [CheungEnter].

The *Philosophy of Enter* distinguishes destructive readout from quantum recovery, classical reporting from quantum return, channel specification from thermal implementation, and analogy from physical identification [CheungPhil]. Spacetime Enter, in I-B, places finite names, calibrated clocks, location maps, and complete resource interfaces on a prescribed classical geometry [CheungIB]. The shredder philosophy of I-C makes the resulting access question precise: an informative public report can fail to preserve the coherence and untouched-reference correlations needed to return an arbitrary unknown code [CheungIC].

Consumption does not assert physical annihilation of every implementing degree of freedom. A residual may retain information, and an earlier coherent transfer may already have moved the code into another live register. Which register is granted to a decoder must be specified before a recovery claim is made.

Relation to the preceding nine extensions

I-C-Ex-v2 supplies nine explicit models and a common access framework [CheungEx]. Those models are not replaced here. The present successor adds Extensions X–XVII:

X. measurement-based computation and adaptive correction;

XI. truncated optical modes and Gaussian reporting;

XII. redundant environmental pointer records;

XIII. a declared finite horizon-memory candidate;

XIV. a fixed-mode late-access surrogate;

XV. finite non-Markovian memory and back-flow;

XVI. finite multiparameter sensing;

XVII. scheduled summoning and deadline access. Finite siblings XVIII–XIX address ensemble Holevo information and encoded partner ownership.

The foundational facts needed below are proved locally. The preceding nine models remain in their own manuscript; their full proofs are not silently made premises of this one.

Contribution and novelty boundary

The sharp dimension law is a Schmidt-number argument [TerhalHorodecki,ChruscinskiKossakowski]. Its classical endpoint is the entanglement-breaking mechanism [HSR]. Exact recovery and complement constancy belong to the established error-correction/no-hiding framework [KnillLaflamme,BraunsteinPati]. Measurement-based computation, Gaussian information, quantum Darwinism, non-Markovianity, and summoning have independent literatures [RaussendorfBriegel,Weedbrook,ZurekDarwinism, BreuerLainePiilo,Kent,HaydenMay].

The new contribution claimed here is the printed assembly: explicit finite instruments, complete ownership accounts, a self-contained perfect-code streaming witness, cutoff and reporting distinctions, exact finite revival diagnostics, and a simultaneous sensing/recovery certificate. Repeated specializations of the dimension law are applications, not independent discoveries of that law.

Standing host, field, and evidence contract

Prescribed geometry

Use $c = \hbar = k_B = 1$. The regular host is

$$M_\alpha = \mathbb{R}_t \times (0,\infty)_r \times S^1_\varphi \times \mathbb{R}_z,$$
$$g_\alpha = -\mathrm{d}t^2 + \mathrm{d}r^2 + \alpha^2 r^2 \mathrm{d}\varphi^2 + \mathrm{d}z^2, \qquad 0 < \alpha \le 1,$$

with angular period 2π. Its completion has a missing apex worldsheet, not a worldline. The metric and deficit are inputs.

Proposition. The spacetime is not globally hyperbolic, including at $\alpha = 1$ if the axis remains removed.

Proof. Fix $r_0 > 0$, φ_0, $z = 0$ and $T > r_0$. The causal diamond between $(-T, r_0, \varphi_0, 0)$ and $(T, r_0, \varphi_0, 0)$ contains $(0, r_n, \varphi_0, 0)$ for every $r_n \downarrow 0$. This sequence has no convergent subsequence in the regular host. The diamond is noncompact.

The scalar Friedrichs choice and any alternative operator domain must therefore be explicitly supplied where relevant; they are not inferred from readout [CheungIBG,KayStuder]. A field finite-propagation theorem is not automatically preserved by a finite positive-frequency mode truncation.

The length-distance propagation envelope must also be distinguished from a regular curve-generated causal relation at unattained endpoints. For instance, opposite points of a punctured flat plane have infimum distance attained only by a segment through the missing point. Finite communication claims below use actual regular routes, not that unattained segment.

Clock and Einstein comparator

For a clock basis $\{|\tau_j\rangle\}$ and surviving system A, destructive clock readout is

$$\mathscr{M}_C(X) = \sum_j \operatorname{proj} j_R \otimes (\langle\tau_j| \otimes I_A) X (|\tau_j\rangle \otimes I_A).$$

Its Kraus completeness and clock-basis expansion give

$$\operatorname{Tr}_R \mathscr{M}_C(X) = \operatorname{Tr}_C X,$$

also with every untouched reference. The clock names are absent from the returned quantum context. Clock calibration and any Page–Wootters dynamical interpretation require their separately stated hypotheses [CheungIB,CheungEn09,CheungXK].

The comparator

$$G_{\mu\nu}(g) + \Lambda g_{\mu\nu} = \kappa_{\mathrm{grav}} T_{\mu\nu}$$

is not an update rule contained in . A symmetric conserved traceless tensor need not source its background through this equation. A constant nonzero radiation tensor in a flat chart already supplies a counterexample when $\Lambda = 0$. This is the I-B prop:not-source separation [CheungIB].

Field layer: visible but not imported into finite proofs

toprule Source record Permitted comparison Not supplied by citation

midrule I-B-G Specified scalar inverses and propagation domains. A premise of the finite recovery law.

I-B-G-S Free Maxwell–Friedrichs source-test identity with compatible scalar ghost: $C_1 G_{(1)}^{\pm} = G_{(0)}^{\pm} C_1$. An interacting Ward identity or physical optical detector.

I-B-G-S-B Positive tensor-core domain selection in its declared class. A photon identification or recovery theorem.

I-B-VP; VP-0; VP-1 State-qualified one-point data, finite parts, and contact spaces. A quantum residual Hilbert space or acquired charge.

I-B-ECHO Specified window functionals and report-error comparisons. Acquisition of an unknown continuum amplitude.

Hawk05-ST A source-qualified finite Enter/Page assembly. A gravitational Page curve or endpoint.

bottomrule

These comparisons retain their source hypotheses [CheungIBG,CheungIBGS,CheungTensor,CheungVP, CheungVP0,CheungVP1,CheungEcho,CheungHawkST]. No full field proof is reconstructed from a theorem label.

Five objects frozen for every extension

Before a recovery assertion, declare:

• the complete message-bearing accessible algebra at the cut;

• its quantum blocks and their dimensions;

• the full append-only classical transcript, including failures;

• a separate geometric and thermal ledger;

• terminal ownership: consume, keep inaccessible, or grant. Previously transmitted quantum systems, available entanglement, controllers, and storage that can carry the message are included. Fresh input-independent ancillas are allowed, but they do not permit an omitted quantum bypass.

All new operational models below have finite input, finite quantum output, and finite exported classical alphabets. Continuous distributions are used only to define compressed effects or supplied reporting probabilities on those finite interfaces.

Common finite tools, proved locally

Distances, instruments, and the complete cut

For states and channels set

$$d_{\mathrm{tr}}(\rho,\sigma)=\frac{1}{2}\|\rho-\sigma\|_1, \qquad d_\diamond(\Phi,\Psi)=\frac{1}{2}\|\Phi-\Psi\|_\diamond.$$

The complete norm includes arbitrary untouched references [Watrous]. For $Q \cong Q\{'\} \cong \mathbb{C}^d$, define

$$|\Phi_d\rangle=\frac{1}{\sqrt{d}}\sum_{j=0}^{d-1}|j\rangle_E|j\rangle_Q, \qquad \Phi_d=|\Phi_d\rangle\langle\Phi_d|,$$

$$F_{\mathrm{e}}(T)=\mathrm{Tr}\,[\Phi_d(\mathrm{id}_E\otimes T)(\Phi_d)], \qquad e_{\mathrm{rec}}(\mathscr{N})=\inf_{\mathscr{D}} d_\diamond(\mathscr{D}\mathscr{N},\mathrm{id}_Q).$$

A target unitary can be absorbed into the decoder.

A finite complete instrument has CP branches $\mathscr{N}_r: \mathscr{B}(Q) \to \mathscr{B}(K_r)$ with trace-preserving sum. The accessible channel is

$$\mathscr{N}(X)=\mathrm{bigoplus}_r\mathscr{N}_r(X), \qquad k_r=\dim K_r, \qquad k=\max_r k_r.$$

Classical labels are a direct-sum index, not coherent quantum storage. Conditional residuals are normalized only on branches of positive probability [DaviesLewis].

Every finite adaptive tree has this form: compose its Kraus maps along complete histories and retain every exported label. For a classical-only output,

$$\mathscr{N}(X)=\sum_r \mathrm{Tr}\,(E_rX)\mathrm{proj}\,r, \qquad E_r\ge 0, \quad \sum_r E_r=I.$$

A deterministic decoder then prepares a state σ_r in each classical block. This is measure and prepare [HSR].

The sharp law and its finite attainment

Theorem. For and $1 \le k \le d$, every deterministic decoder satisfies

$$F_{\mathrm{e}}(\mathscr{D}\mathscr{N})\le\frac{k}{d}, \qquad d_\diamond(\mathscr{D}\mathscr{N},\mathrm{id}_Q)\ge 1-\frac{k}{d}.$$

More precisely, with $p_r = Tr\,\mathscr{N}_r(I/d)$,

$$F_{\mathrm{e}}(\mathscr{D}\mathscr{N})\le\frac{1}{d}\sum_r p_rk_r.$$

Both constants in are attained by a finite-outcome instrument for every d, k. For larger blocks one may replace k_r by $min\,(k_r, d)$.

Proof. A pure bipartite vector of Schmidt rank at most k has coefficient matrix A with $\|A\|_2 = 1$, $rank\,A \le k$. Its Bell overlap obeys

$$\frac{1}{d}|\mathrm{Tr}\,A|^2\le\frac{1}{d}\|A\|_1^2\le\frac{k}{d}.$$

Convexity gives the same bound for states of Schmidt number at most k; local CP maps cannot increase that

number, because each Kraus map multiplies a coefficient matrix on one side [TerhalHorodecki,ChruscinskiKossakowski].

The normalized Bell output in branch *r* lives on $E \otimes K_r$. Its decoded Bell overlap is at most $p_r k_r/d$. Summing proves the fidelity bound. Testing the difference from Φ_d with its own projector gives distance at least $1 - F_e$.

For attainment, let $P_U = UP_0U^{\dagger}$, with P_0 a rank-k projection, and define

$$T_{d,k}(X) = \frac{d}{k}\int P_U X P_U \, \mathrm{d}U.$$

The Haar average of P_U is kI/d, so this is a channel. Its Kraus trace formula gives $F_e(T_{d, k}) = k/d$. Unitary covariance implies the isotropic Choi state

$$J_{\mathrm{n}}(T_{d,k}) = \frac{k}{d}\Phi_d + \frac{1 - k/d}{d^2 - 1}(I - \Phi_d).$$

Equivalently, in a Weyl Bell basis,

$$T_{d,k} = \frac{k}{d}\mathrm{id} + \frac{1 - k/d}{d^2 - 1}\sum_{u \neq 0} \mathrm{Ad}_{W_u}.$$

The triangle inequality gives distance at most $1 - k/d$, matching the lower bound.

Finally, the maps $X \mapsto P_UXP_U$ form a compact set in a real vector space of dimension d^4. Its average is a convex combination of at most $d^4 + 1$ members: affine dependence removes excess terms, and compactness passes to limits. Write the combination as $\sum_j w_jP_jXP_j$, with $P_j = V_jV_j^{\dagger}$, $V_j: \mathbb{C}^k \to \mathbb{C}^d$. The encoder Kraus maps

$$A_j = \sqrt{dw_j/k}\, V_j^{\dagger}$$

are complete. Return *j* and a *k*-dimensional residual; decode block *j* by $Y \mapsto V_jYV_j^{\dagger}$. The composite is $T_{d, k}$.

This is finite exact attainment, not efficient synthesis. At $k = 1$, the sharp half-diamond lower bound is $1 - 1/d$; the unhalved norm has twice that bound. The distinct Haar-average pure-state fidelity is at most $2/(d + 1)$, by the standard identity $F_{av} = (1 + dF_e)/(d + 1)$ [NielsenAverage].

Proposition. Suppose a finite adaptive process, including all its controls, has final output . Classical programming changes its branch maps but not the declared bound *k*. Hence applies.

If an implemented channel is within δ of that complete comparison channel on common output spaces, then

$$d_{\diamond}(\widetilde{\mathscr{D}\mathscr{N}}, \mathrm{id}) \geq [1 - k/d - \delta]_+.$$

Complete stage allowances δ_j add to at most $\min(1, \sum_j \delta_j)$.

Proof. Compose the Kraus maps along each adaptive history. The final codomain fixes its block size. Postcomposition contracts diamond distance, so the reverse triangle inequality proves the recovery allowance. Replacing stages one at a time proves the telescoping bound.

Adaptivity can allocate or retain more quantum systems if the contract permits it. In that case the new systems must be counted; the proposition does not prohibit actual enlargement.

Complete complements and the rank correction

Theorem. For a finite isometry $V: Q \to A \otimes B$, let

$$\mathscr{M}(X) = \mathrm{Tr}_B\, VXV^{\dagger}, \qquad \mathscr{E}(X) = \mathrm{Tr}_A\, VXV^{\dagger}.$$

Exact recovery from *A* is equivalent to

$$\mathscr{E}(X) = \mathrm{Tr}\,(X)\sigma_B \quad \text{for every} X.$$

If $s = \mathrm{rank}\, \sigma_B$, then

$$\mathscr{M}(X) = J(X \otimes \sigma_S)J^{\dagger}, \qquad J : Q \otimes \mathbb{C}^s \to A \text{ isometric}, \qquad \dim A \geq ds.$$

If one complementary channel is constant, the other recovers. For $d \geq 2$, the two outputs cannot independently

recover the whole code.

Proof. Assume and diagonalize $\sigma_B = \sum_{a=1}^{s} p_a |b_a\rangle\langle b_a|$, $p_a > 0$. Write

$$V|i\rangle = \sum_a \sqrt{p_a}\,|u_{ia}\rangle_A |b_a\rangle_B.$$

The complementary matrix-unit equations imply $\langle u_{jb} \mid u_{ia}\rangle = \delta_{ij}\delta_{ab}$. Thus $J(|i\rangle|a\rangle) = |u_{ia}\rangle$ is an isometry, proving the normal form and rank bound. The channel

$$\mathscr{D}(Y) = \mathrm{Tr}_S\,(J^\dagger Y J) + \mathrm{Tr}\,[(I - JJ^\dagger)Y]\tau_Q$$

is a trace-preserving CP decoder on all of A.

Conversely, dilate an exact decoder by $L: A \to Q \otimes G$. Purity of its decoded basis outputs gives

$$(L \otimes I_B)V|i\rangle = |i\rangle|\xi_i\rangle_{GB}.$$

Exactness on off-diagonal matrix units gives $\langle \xi_j \mid \xi_i\rangle = 1$; all vectors coincide. Tracing QG gives . Interchange A, B for the final assertions.

This is the standard whole-code condition [KnillLaflamme,BraunsteinPati], locally printed to fix its complete output convention. It agrees with the scoped En07 and Hawk04 comparisons [CheungRecovery].

Corollary. If is exactly recoverable, each nonzero branch has input-independent probability p_r, and $\mathscr{N}_r/p_r$ is an exactly recoverable channel. If its complete constant complement has rank s_r, then

$$k_r \geq d s_r \geq d.$$

Proof. The positive Choi matrices of $\mathscr{D}_r\mathscr{N}_r$ sum to the rank-one Choi matrix of the identity. Each is therefore p_r times that matrix. Trace preservation of $\mathscr{D}_r$ gives $Tr\,\mathscr{N}_r(X) = p_r Tr\,X$. Apply Theorem to the normalized branch.

Remark. The quantity s is the rank of a constant *complete complementary state*. It is not the rank of an accessible marginal or of a classical probability vector. An identity channel on $\mathbb{C}^d$ has a rank-one complement even though its maximally mixed accessible output has rank d. Replacing s by that accessible rank would give the false requirement $d \geq d^2$.

The full-code criterion is not the criterion for arbitrary subsystem recovery. For example, Tr_S recovers a logical subsystem while its complement can depend on the discarded subsystem [CheungOai280].

Entropy, ownership, and thermal accounting

All entropies in this manuscript use natural logarithms. To compare with bit-valued source formulas, divide by $\ln 2$.

Proposition. For an append-only finite transcript R_n,

$$H(R_{n+1}) = H(R_n) + H(X_{n+1} \mid R_n) \geq H(R_n).$$

For $\omega_{RK} = \bigoplus_r p_r \rho_{K_r \mid r}$,

$$S(RK) = H(R) + \sum_r p_r S(\rho_{K_r \mid r}), \qquad H(R) \leq S(RK) \leq H(R) + \log k.$$

Neither the residual contribution nor the total hybrid entropy is generally monotone.

For any recorded prefix, hiding, consuming, or granting a residual preserves the old-record marginal when the continuation is trace preserving. These are different access specifications; ownership changes need not be different unrestricted dynamics.

Proof. The classical chain rule proves the first assertion. The block eigenvalues are $p_r\lambda_{r,j}$, giving . A channel can reset an independent mixed residual while preserving the record, so hybrid entropy can decrease. Tracing a trace-preserving local continuation leaves the old-record marginal unchanged.

These are the I-C B/E/F distinctions, now proved locally. The hybrid identity also appears in the supplied Hawk05-A record, whose title-page authorship is not established by the present extract [Hawk05ARecord].

Proposition. Let the complete non-reservoir system M initially be independent of a finite Gibbs reservoir $\tau_B = e^{-\beta H_B}/Z$, $\beta > 0$. For a unitary step with final state ω_{MB}, define

$$Q_B = \mathrm{Tr}\,[H_B(\omega_B - \tau_B)], \qquad \Delta S_M = S(\rho_M) - S(\omega_M).$$

Then

$$\beta Q_B = \Delta S_M + I(M:B)_\omega + D(\omega_B \| \tau_B).$$

Strict $\Delta S_M > 0$ is sufficient, not necessary, for $Q_B > 0$. An abstract instrument fixes no heat bill.

Proof. Unitary entropy preservation gives $S(\omega_B) - S(\tau_B) = \Delta S_M + I(M: B)_\{\omega\}$. The Gibbs logarithm gives $D(\omega_B \| \tau_B) = \beta Q_B - [S(\omega_B) - S(\tau_B)]$. Combine them [ReebWolf].

For nonnecessity, SWAP a memory qubit of excited probability $1 - p_\{\beta\}$ with a Gibbs qubit of probability $p_\{\beta\} = (1 + e^\{\beta E\})^\{-1\} < 1/2$, with equal Hamiltonians. The entropy decrease is zero while the bath gains $E(1 - 2p_\{\beta\}) > 0$.

A retained archive belongs in M whenever its correlations enter the reset account. A heat-meter label is classical access, not coherent access to the reservoir.

Named effects and fixed finite reports

For an effect F, write

$$\mathcal{M}_F(X) = \mathrm{Tr}\,[(I - F)X]\mathrm{proj}\,0 + \mathrm{Tr}\,(FX)\mathrm{proj}\,1.$$

Theorem. For effects F, G,

$$d_\diamond(\mathcal{M}_F, \mathcal{M}_G) = \|F - G\|_\infty.$$

For a channel $\mathcal{N}: Q \to A$, define

$$e_F(\mathcal{N}) = \inf_{0 \le G \le I_A} \|\mathcal{N}^*(G) - F\|_\infty.$$

Then

$$e_F(\mathcal{N})\mathrm{lee}_{\mathrm{rec}}\,(\mathcal{N}).$$

A small one-effect error does not conversely imply small whole-code recovery error.

Proof. Put $H = F - G$. For an amplified input X, the difference output has blocks $T, -T$, where $T = Tr_Q\,[(H \otimes I)X]$. Trace-norm duality gives $\|T\|_1 \le \|H\|_\{\infty\}\|X\|_1$. An eigenstate of H attains equality. For a quantum decoder choose $G = \mathcal{D}^\{*\}(F)$ and use contraction under $\mathcal{M}_F$. For failure of the converse, dephasing reconstructs every diagonal effect exactly but has recovery error $1 - 1/d$.

The direction in matters. An intrinsic one-effect lower bound can lower-bound code recovery; a code lower bound does not imply loss of that effect. This is the numerical comparison in Oai290-A [CheungOai290].

Lemma. Fix a finite program space P, a trace-norm continuous state preparation $w \mapsto \tau(w)$, and one complete channel

$$\mathcal{J}: \mathcal{B}(Q \otimes P) \to \mathrm{bigoplus}_r\, \mathcal{B}(K_r).$$

For $\mathcal{N}_w(X) = \mathcal{J}(X \otimes \tau(w))$,

$$d_\diamond(\mathcal{N}_w, \mathcal{N}_{w'})\mathrm{led}_{\mathrm{tr}}\,(\tau(w), \tau(w')).$$

Fixed-budget product uses have at most the sum of the single-use allowances. The output block dimensions do not increase in the limit.

Proof. Before $\mathcal{J}$, the amplified difference is $X \otimes [\tau(w) - \tau(w^\{'\})]$. Its trace norm factors. Apply complete contraction and divide by two. Tensor telescoping gives the repetition bound.

This lemma concerns a *supplied* program state. It is not a map acquiring an unknown continuum functional, and a nonlinear state-dependent programming prescription is not automatically a quantum channel [CheungIC,CheungEcho,CheungXJ].

Extension X: measurement-based computation and adaptive correction

A graph-state instrument with an unknown input

Let

$$R_z(\theta) = e^{-i\theta Z/2}, \qquad |s_\theta\rangle = \frac{|0\rangle + (-1)^s e^{i\theta}|1\rangle}{\sqrt{2}}.$$

Attach $|+\rangle$ to an unknown input qubit, apply controlled Z, and destructively measure the old input in this basis. The branch map to the surviving qubit is

$$K_s(\theta) = (\langle s_\theta| \otimes I)\, CZ\, (I \otimes |+\rangle) = \frac{1}{\sqrt{2}} X^s H \mathrm{diag}\,(1, e^{-i\theta}).$$

Thus the consumed input name is not returned, but its logical state has been transferred to the surviving qubit, up to a known unitary and byproduct.

For a chain of three vertices, measure the first at angle α, and the second at $\beta_s = (-1)^s\beta$. Up to irrelevant branch phases, the final branch maps are

$$K_{s,t} = \frac{1}{2} X^t Z^s (H R_z(-\beta))(H R_z(-\alpha)).$$

Applying Z^sX^t and the inverse target unitary returns the unknown input exactly. The two record bits are uniform and independent of that input, including every reference.

Proof. For input $a|0\rangle + b|1\rangle$, controlled Z produces $a|0\rangle|+\rangle + b|1\rangle|-\rangle$. Contraction with $\langle s_{\theta}|$ gives . For the second step use

$$(H R_z(-\theta)) X^s = Z^s H R_z(-(-1)^s\theta)$$

up to scalar phase. The adaptive angle cancels that sign, giving . Every branch has squared modulus $1/4$.

This is a finite graph-state computation, not the monitored brickwork model of the predecessor. The computational principle is established measurement-based computation [RaussendorfBriegel].

Theorem. For any finite graph-state or code-block adaptive instrument whose complete final output is , the bounds apply to its d-dimensional logical code.

In , granting the final qubit permits exact recovery; granting only the two classical bits gives a constant channel, with optimal recovery error $3/4$. The classical syndrome or byproduct log does not replace a missing coherent logical output.

Proof. The general statement is Theorem , applied after complete branch composition. The branch formulas provide an exact decoder when the qubit is granted. Without it the record is uniform. A replacement channel for a qubit has optimal recovery error $1 - 2^{-2} = 3/4$: the Bell test gives this lower bound, and preparing $I/2$ attains it through the uniform Pauli mixture.

Corollary. If a normalized syndrome branch returns the whole logical code exactly, its quantum output satisfies $k_r \geq d\, s_r$, with s_r its constant complete-complement rank. A single independently granted factor is assessed with its own complete complement, not with the rank of a selected syndrome marginal.

Proof. Apply Corollary and Theorem .

A finite perfect-code correction block

The streaming witness below uses a concrete five-qubit perfect code. Its algebra is derived here rather than imported from En02-B. The historical code is credited to the primary literature [Laflamme].

Let indices be cyclic modulo five and set

$$K_j = X_j Z_{j-1} Z_{j+1}.$$

The K_j commute and are independent. Let $\mathscr{S}$ be the group of even products of the K_j. It has 16 elements. Its common $+1$-eigenspace has dimension two, with projector

$$P = \frac{1}{16} \sum_{S \in \mathscr{S}} S.$$

Choose the logical axes on this code so that

$$\overline{X} = -K_2, \qquad \overline{Z} = Z_1Z_2Z_3Z_4Z_5.$$

They anticommute and preserve the code. Let *V_5: ℂ^2 → (ℂ^2)^{⊗5}* intertwine these axes with the ordinary logical Paulis.

Lemma. Every nonidentity Pauli of weight at most two has *PEP = 0*. Hence the code corrects arbitrary one-qubit errors and any specified two-qubit erasure.

On the authorized triple *123*, define

$$A = -Z_1X_2Z_3, \qquad B = -X_1Y_2X_3, \qquad C = -Y_1Z_2Y_3.$$

They satisfy *AB = iC*, and the triple channel is

$$\mathcal{N}_3(\rho) = \mathrm{Tr}_{45}\,(V_5\rho V_5^\dagger) = \frac{1}{8}(I + r_xA + r_yB + r_zC)$$

for *ρ = (I + r_xX + r_yY + r_zZ)/2*.

Proof. A nonidentity stabilizer has weight four. For an even subset of size two, this follows by inspecting adjacent and nonadjacent pairs on the five-cycle; both give weight four. For a subset of size four, its four *X*-positions already give weight at least four.

The Pauli centralizer of $\mathcal{S}$ is generated, up to phases, by all *K_j* and *Z^{⊗5}*. This follows from their six independent binary symplectic generators, the required centralizer dimension *10 − 4 = 6*. A nontrivial product of at least three *K_j* has at least three nonidentity positions. A single *K_j* has weight three, and a two-generator product has weight four. For *Z^{⊗5}* times zero, one, or two *K_j*, direct inspection gives weights five or three; with at least three *K_j*, the *X*-positions again give weight at least three. Thus no nonidentity centralizer Pauli has weight at most two. Any such low-weight Pauli therefore anticommutes with a stabilizer, giving *PEP = 0*.

Products of two one-qubit errors have weight at most two unless scalar. The corresponding correction condition follows, or equivalently the erased-pair complement is constant by expansion in its Pauli basis and Theorem .

The representative *A = −K_2* is supported on *123*. Multiplication of *Z^{⊗5}* by *K_1K_3 ∈* $\mathcal{S}$ gives *C*. Their Pauli product gives *B*. No two distinct representatives of the same logical coset can both be supported on *123*, since their product would be a nonidentity stabilizer of weight at most three. Tracing the stabilizer expansion over *45* consequently retains exactly *I, A, B, C*, each with coefficient *1/8*.

For the noise model

$$\mathcal{E}(\rho) = \sum_a p_aE_aV_5\rho V_5^\dagger E_a^\dagger,$$

let *E_a* range over the identity and the fifteen single-site Paulis. Their encoded ranges are orthogonal and exhaust the five-qubit space. Measuring the range projectors, consuming the syndrome encoding, and applying *E_a^{†}* returns the logical qubit. A complete corrected interface is

$$\rho\mathrm{longmapstobigoplus}_a\, p_a\rho.$$

Its classical syndrome is independent of the logical state; the residual logical block is two-dimensional. This is exact correction for the declared finite error family, not a noisy-gate threshold theorem.

A genuine decoder-peak obstruction

Definition. The supplier delivers carriers *1, 2, 3* of in order. Every delivered, undestroyed carrier counts toward laboratory peak occupancy. Undelivered carriers remain outside the laboratory. There is no quantum return to the supplier, no early quantum output, and no additional uncounted quantum storage. Classical storage and feed-forward are unrestricted. The final target is the logical qubit.

Lemma. The exact decoder for is unique as a channel on all three-carrier input operators. Its adjoint is

$$\mathcal{D}^*(X) = A, \qquad \mathcal{D}^*(Y) = B, \qquad \mathcal{D}^*(Z) = C, \qquad \mathcal{D}^*(I) = I.$$

Proof. The Pauli representation generated by *A, B, C* is unitarily equivalent to *M_2 ⊗ I_4*. Thus *𝒩_3(ρ) = J(ρ ⊗ I_4/4)J^{†}* for a unitary *J: ℂ^2 ⊗ ℂ^4 → ℂ^8*. Write *J_s|ψ⟩ = J(|ψ⟩|s⟩)*. If *{L_{ℓ}}* are Kraus maps of any exact decoder, the Kraus maps *L_{ℓ}J_s/2* implement the identity. Rank one of its Choi matrix gives *L_{ℓ}J_s = c_{ℓs}I*.

Trace preservation implies

$$\sum_{\ell} \overline{c_{\ell s}} c_{\ell t} = \delta_{st}.$$

Therefore $\mathscr{D}(J_{sXJ_t\{\dagger\}}) = \delta_{stX}$, which determines $\mathscr{D}$ on a matrix basis. Its adjoint is the displayed representation.

Proposition. Under Definition , exact recovery from the perfect-code triple has minimum peak occupancy two. This holds even with arbitrary classical memory.

Proof. If the peak were one, carrier *1* would have to cease being a live quantum input before carrier *2* arrived. Its only continuing influence would be a classical record. Hence the implemented channel would have the form

$$\Lambda(\rho_{123}) = \sum_{h} \Lambda_h(\mathrm{Tr}_1 [(E_h \otimes I_{23})\rho_{123}]).$$

Its Choi operator is separable across input-reference position *1* versus the other factors, and therefore has positive partial transpose on position *1*.

By Lemma , exact recovery would require $\Lambda = \mathscr{D}$. Ordered as output *0* and input-reference positions *123*, the normalized Choi matrix is

$$J_{\mathrm{n}}(\mathscr{D}) = \frac{1}{16}(I - X_0Z_1X_2Z_3 + Y_0X_1Y_2X_3 - Z_0Y_1Z_2Y_3).$$

Set

$$S_1 = -X_0Z_1X_2Z_3, \qquad S_3 = -Z_0Y_1Z_2Y_3.$$

They are independent commuting Paulis, with $S_1S_3 = Y_0X_1Y_2X_3$. Partial transpose on position *1* gives

$$J_{\mathrm{n}}(\mathscr{D})^{T_1} = \frac{1}{16}(I + S_1 + S_1S_3 - S_3).$$

On their simultaneous eigenspace $S_1 = -1$, $S_3 = +1$, the eigenvalue is $-1/8$. This contradicts positivity.

For the upper bound, retain the first two carriers and jointly measure the commuting pair

$$L_1 = Z_1X_2, \qquad L_2 = X_1Y_2.$$

Their projectors

$$F_{ab} = \frac{1}{4}(I + aL_1)(I + bL_2), \qquad a, b = \pm 1,$$

are rank-one rotated Bell projectors. They can be measured by a two-qubit Clifford basis change and destructive computational readout. For example, $I \otimes R_z(\pi/2)H$ maps the pair to ZZ, XX, after which the usual Bell measurement applies.

Consume both carriers, retain a, b classically, and receive carrier *3*. From , its conditional state is

$$\frac{1}{2}(I - ar_xZ - br_yX + abr_zY).$$

The three signed axes have the correct Pauli orientation, so a single-qubit Clifford correction returns ρ. Each outcome has probability $1/4$. The peak is two.

This reproduces the shape of En02-B's decoder-resource separation with the finite block and proof made explicit [CheungEn02]. It is not an unrestricted memory lower bound for all codes. The source En02-C timeline and its particular optimal one-live-qubit error concern a separately specified benchmark.

Five-object account and exclusions. The complete output is a syndrome/byproduct transcript plus the granted logical block. Exact branches satisfy the rank condition. Transcript entropy is monotone; residual entropy need not be. Gate permissions and energy costs remain separate inputs. Consuming the final logical block changes the endpoint to a classical-only task. No threshold, topological code, or volume-law transition is proved.

Extension XI: finite optical cutoffs and Gaussian reporting

Hard Fock support, not merely bounded mean energy

For m modes and total occupation cutoff L, define

$$\mathscr{H}_{m,L} = \operatorname{span}\{|n_1, \ldots, n_m\rangle : \sum_j n_j \le L\}.$$

Counting weak compositions gives

$$k_{m,L} = \dim \mathscr{H}_{m,L} = \frac{m+L}{m}.$$

A hard energy spectral cutoff for finitely many modes of strictly positive frequencies also has finite rank: $\sum_j \omega_j n_j \le E$ implies $\sum_j n_j \le E/\min_j \omega_j$.

Theorem. If the complete accessible output has classical labels and quantum blocks supported in $\mathscr{H}_{m, L}$, then

$$F_{\mathrm{e}} \le \frac{\min(k_{m,L}, d)}{d}, \qquad d_{\diamond}(\mathscr{D}\mathscr{N}, \mathrm{id}) \ge 1 - \frac{\min(k_{m,L}, d)}{d}.$$

Exact recovery on a nonzero branch additionally requires $k_{m, L} \ge d s_r$.

The model class is nonempty and sharp when $k_{m, L} \le d$: identify $\mathbb{C}^{k_{m, L}}$ with the occupation basis and use the finite encoder of Theorem .

Proof. Equation follows by adding a slack occupation $n_{m + 1} = L - \sum_j n_j$ and counting compositions of L into $m + 1$ nonnegative integers. Apply Theorem and Corollary . The stated basis identification gives the attaining instrument.

A mean-energy bound does not give this dimension. For a single oscillator, states with a small population ε_N at arbitrarily large N, with $N\varepsilon_N$ bounded, can obey a fixed mean-energy bound while using unbounded support. No infinite-dimensional capacity conclusion is made.

Flagged cutoff error: a necessary distinction

Let P_L be the total-number projection and introduce a flag orthogonal to the full Fock space. The complete cutoff channel is

$$\mathscr{C}_L(\rho) = P_L \rho P_L + \operatorname{Tr}[(I - P_L)\rho]|\mathrm{fail}\rangle\langle\mathrm{fail}|.$$

The success block has dimension $k_{m, L}$, the failure block dimension one.

Proposition. Set $p = \operatorname{Tr}[(I - P_L)\rho]$. After embedding the original and cutoff outputs in the common flagged space,

$$d_{\mathrm{tr}}(\rho, \mathscr{C}_L(\rho)) \le \sqrt{p} + \frac{p}{2}.$$

If $[\rho, P_L] = 0$, the distance is exactly p. For a finite encoded input channel, a uniform bound $p \le \varepsilon$ on every encoded state gives the corresponding complete-channel allowance $\min(1, \sqrt{\varepsilon} + \varepsilon/2)$.

For a one-mode thermal state

$$\rho_q = (1 - q) \sum_{n=0}^{\infty} q^n \operatorname{proj} n, \qquad 0 \le q < 1,$$

the exact state error is $q^{L + 1}$.

Proof. For a purification $|\Psi\rangle$, put $|\Psi_P\rangle = (P_L \otimes I)|\Psi\rangle$. Direct diagonalization of $\operatorname{proj}\Psi - |\Psi_P\rangle\langle\Psi_P|$ in their two-dimensional span gives trace norm $\sqrt{p(4 - 3p)} \le 2\sqrt{p}$. Partial trace contracts that norm. The added flag has trace p, giving the bound.

If the state is block diagonal, the difference is its positive tail minus an orthogonal flag of the same trace, so its half-trace norm is p. The purification argument works with every reference. Uniform encoded tail control therefore gives the complete channel statement. Finally, summing the thermal geometric tail gives $q^{L + 1}$.

For gauge-invariant finite-mode Gaussian states, passive normal-mode rotations preserve total number, so total-number cutoff is block diagonal. This is the setting of the state-qualified packet comparisons in En02-E and Em03-B [CheungGaussianPackets]. A fixed-state tail probability is not a uniform unknown-code allowance unless the stated uniform hypothesis is proved.

Quadrature and number effects on the finite code

Use the standard oscillator realization on $L^2(\mathbb{R})$. Let $V_L: \mathbb{C}^{L+1} \to L^2(\mathbb{R})$ embed the first $L+1$ real normalized Hermite functions ψ_n. For a position-threshold effect define

$$F_t = V_L^\dagger \, 1_{[t,\infty)}(\widehat{q}) \, V_L, \qquad (F_t)_{ab} = \int_t^\infty \psi_a(x)\psi_b(x)\,\mathrm{d}x.$$

This is a finite effect, not generally a projection. It is different from taking a spectral threshold of the finite matrix $V_L^\dagger \hat{q} V_L$.

Theorem. The complete binary record channel of F_t satisfies

$$d_\diamond(\mathscr{M}_{F_t}, \mathscr{M}_{F_u}) = \|F_t - F_u\|_\infty.$$

Moreover, for

$$C_L = \sup_{x \in \mathbb{R}} \sum_{n=0}^{L} |\psi_n(x)|^2 < \infty,$$

$$\|F_t - F_u\|_\infty \le C_L |t-u|.$$

The same effect-distance identity applies to a declared finite photon-number threshold.

These named-effect errors are not whole-code recovery errors.

Proof. The matrix-valued density in is a positive rank-one matrix with operator norm $\sum_{n=0}^{L} |\psi_n(x)|^2$. Its supremum is finite because it is a finite sum of polynomial times Gaussian functions. Integrate over the interval between the thresholds. Then apply Theorem . Number thresholds are finite effects by spectral calculus.

A finite complete residual instrument can be installed as

$$\mathscr{I}_1(X) = \sqrt{F_t} X \sqrt{F_t}, \qquad \mathscr{I}_0(X) = \sqrt{I-F_t} X \sqrt{I-F_t}.$$

Its residual has dimension $L+1$. This is a declared finite instrument, not a physical homodyne apparatus.

Lemma. Suppose $t_n \to t$ are finite-resolution threshold codes. Then the binary record channels in converge in diamond distance, with allowance $C_L|t_n - t|$. For the specifically chosen square-root residual instruments, their complete channels also converge, with allowance at most

$$\min\{1, \sqrt{2C_L|t_n - t|}\}.$$

Alternatively, if finite-resolution quadrature data program one fixed finite channel through a continuous joint program state, Lemma gives complete-channel convergence with unchanged residual dimension.

Proof. The record estimate is . For the residual instrument use the isometry with orthogonal record and environment flags

$$W_F v = \sum_{b=0}^{1} |b\rangle_R |b\rangle_E \sqrt{F_b}\, v, \quad F_1 = F, \quad F_0 = I - F.$$

For positive matrices, $\|\sqrt{A} - \sqrt{B}\|_\infty \le \sqrt{\|A-B\|_\infty}$. Hence $\|W_{F_{t_n}} - W_{F_t}\|_\infty$ is at most $\sqrt{2\|F_{t_n} - F_t\|_\infty}$. Expanding the difference of the two isometric channels gives half-diamond distance at most their isometry norm difference. Partial trace preserves this upper bound. The programming assertion is Lemma .

Remark. A finer discrete record, embedded as point masses on $\mathbb{R}$, can have TV distance one from an absolutely continuous homodyne law for every mesh size. Weak convergence is not TV convergence. The lemma uses fixed coarse binary outputs or one fixed finite programmed interface.

Likewise, equality or convergence of effects does not determine arbitrary residual instruments. The square-root rule above is explicitly chosen; otherwise a separate instrument estimate is required.

Gaussian moments are a restricted inference task

Proposition. Suppose a supplied calibration law is $X \sim N(\mu, s^2)$, with $s > 0$. For fixed $t_0 < t_1$, the finite three-bin law determines μ, s exactly as a probability model.

Writing $p_j = \Pr(X \le t_j)$, $z_j = \Phi^{-1}(p_j)$,

$$s = \frac{t_1 - t_0}{z_1 - z_0}, \qquad \mu = t_0 - s z_0.$$

Its realized finite records do not return an unrestricted unknown oscillator code.

Proof. The Gaussian cumulative law gives $z_j = (t_j - \mu)/s$; subtract and solve. The outcome is still a finite classical record channel and obeys with $k = 1$.

Even a full position law need not identify all logical coherences on $span\{|0\rangle, |1\rangle\}$: the orthogonal states

$$(|0\rangle + i|1\rangle)/\sqrt{2}, \qquad (|0\rangle - i|1\rangle)/\sqrt{2}$$

have the same position density because the Hermite functions are real. This is a concrete complementary-coherence ambiguity, not a claim about every Gaussian channel.

Gaussian optical theory provides context [Weedbrook]; Em03-D's local Weyl comparison and Em03-E/F's designed channels retain different apparatus and domain hypotheses [CheungPacketBridges]. The free Maxwell identity is not this finite effect model, and X-J-ST-2's report does not acquire the continuum amplitude [CheungIBGS,CheungXJ].

Five-object account and exclusions. The cut is finite bins plus the explicitly granted Fock block; exact branches require $k_{m, L} \geq ds_r$. Hybrid entropy obeys . Optical energies and packet-domain identifications are separate. Final occupation support is not a continuum capacity theorem. No homodyne apparatus, infinite-dimensional Gaussian capacity, or Hawking identification is constructed.

Extension XII: redundant environmental pointer records

A finite copying interaction with a logical degeneracy

Let

$$Q = P \otimes L, \qquad P \cong \mathbb{C}^2, \quad L \cong \mathbb{C}^{\ell}.$$

Choose the pointer projectors $P_z = proj\, z_P \otimes I_L$. Prepare N fragment qubits in $|0\rangle$ and apply controlled X from P to every fragment:

$$W|z, \psi\rangle_Q = |z, \psi\rangle_S |z\rangle_{F_1} \cdots |z\rangle_{F_N}.$$

This is an isometry. The pointer basis is supplied, not selected by the proof.

Destructively measuring every F_j gives the joint record $R_N = (z, \ldots, z)$. Its law is

$$p_z = \mathrm{Tr}\,(P_z \rho).$$

The recorded residual-system blocks are $P_z \rho P_z$.

Theorem. The joint fragment record channel obeys the classical recovery obstruction for every installed d-dimensional code. No phrase such as ``orthogonal to the pointer'' is needed: the obstruction concerns any unknown quantum code whose complete accessible output is classical.

For the full 2ℓ-dimensional code with the system residual granted, the output is dephasing on P tensored with identity on L, with the pointer flag retained. Its optimal whole-code recovery error is $1/2$.

For a code entirely inside one pointer eigenspace, the record is constant and its optimal record-only recovery error is $1 - \ell^{-2}$; the retained L-factor recovers exactly.

Proof. The record-only map is a POVM channel. For the larger output, the two branches have quantum support dimension ℓ, so Theorem gives $1 - \ell/(2\ell) = 1/2$. Discarding the flag and returning S realizes

$$\tfrac{1}{2}(\mathrm{id} + \mathrm{Ad}_{Z_P \otimes I_L}),$$

with distance at most $1/2$, proving optimality.

On a fixed pointer eigenspace, every record is deterministic, while L is unchanged. The replacement benchmark follows from the Bell test and the Weyl mixture, as in Theorem .

Corollary. For a single accessible quantum fragment A, exact recovery of an unknown d-dimensional code requires

$$\dim A \geq d \operatorname{rank} \sigma_B,$$

provided its *complete* complement is constant. For a flagged fragment, apply this branchwise. The rank of the fragment's own state is not a substitute.

In , every fragment marginal and even the joint fragment marginal is diagonal in the pointer basis. No fragment independently recovers a nontrivial quantum code.

Proof. Apply Theorem . Tracing S in removes off-diagonal pointer terms and gives a mixture of repeated basis labels.

Proposition. For $N \geq 1$,

$$H(R_N) = h(p_1),$$

not $Nh(p_1)$. Each additional perfect copy has zero conditional entropy given the first. With the system residual,

$$S(R_N S) = H(p) + \sum_z p_z S(\rho_{L \mid z}).$$

Thus classical redundancy need not produce additional joint Shannon entropy, and residual evolution can remain nonmonotone.

Proof. The joint law is supported on two repeated words. Apply the chain rule and .

This model isolates the record-redundancy motif studied in quantum Darwinism [ZurekDarwinism,ZwolakQuanZurek]. It does not derive an objective pointer ontology. En01-A's recorded architecture and Em03-F's classical witness/entanglement-breaking separation are comparisons, not premises [CheungRecords,CheungPacketBridges]. Psy01-A's analytic pruning framework is not a basis-selection theorem [CheungPruning].

Five-object account and exclusions. The exported algebra is the joint pointer record, optionally with declared quantum fragments or S. The appropriate rank is always complementary rank. The transcript is append-only even when its entropy stops growing. The copying gates have no automatic heat account. Granting or consuming a remaining fragment is a terminal choice. No environment-selected basis, biological mechanism, or gravitational radiation is identified.

Extension XIII: a declared finite horizon-memory candidate

A nonempty finite memory model

Let $d = qk$, with $q \geq 2$, and identify $Q = \mathbb{C}^q \otimes \mathbb{C}^k$. Fix a unitary U on Q. Define

$$A_r = (\langle r| \otimes I_k)U, \qquad \mathcal{N}_{\mathrm{mem}}(X) = \operatorname{bigoplus}_{r=0}^{q-1} A_r X A_r^\dagger.$$

The direct-sum label is classical; the returned quantum memory has dimension k. The operators obey $\sum_r A_r^\dagger A_r = I$.

The word ``memory candidate" is an access label. No identification with a physical soft-charge sector is made.

Theorem. The model has

$$e_{\mathrm{rec}}(\mathcal{N}_{\mathrm{mem}}) = 1 - \frac{1}{q} = 1 - \frac{k}{d}.$$

For arbitrary declared finite candidate spaces, the general bound remains .

Proof. The lower bound is Theorem . A decoder reinserts $|r\rangle$ and applies $U^\dagger$. The composite dephases the q-dimensional first factor in the U-basis. Writing that dephasing as the average of q phase conjugations gives distance at most $1 - 1/q$.

Corollary. If the memory alone, or a normalized memory branch, exactly recovers the full code, its accessible Hilbert dimension must be at least d times its constant complete-complement rank. In , $k < d$, so exact recovery is impossible.

Proof. Apply Theorem or Corollary .

A charge effect can survive exactly

Let

$$F = U^{\dagger}\Big(\sum_{r} f_r \mathrm{proj}\, r \otimes I_k\Big)U, \qquad 0 \le f_r \le 1.$$

Classical postprocessing of the label r with probability f_r realizes $\mathscr{M}_F$ exactly.

Proposition. For every bounded candidate charge $C = C^{\dagger}$ on a declared finite space, a threshold or normalized affine function of C is an effect. Its readout error is given by Theorem . In particular, the effect above is exact while $e_{rec}(\mathcal{N}_{mem}) = 1 - k/d > 0$.

An unbounded physical charge must first have a declared domain and a bounded effect or finite compression. A state-qualified c-number current is not itself a quantum residual algebra.

Proof. Finite spectral calculus gives the effect. The identity $\mathcal{N}_{mem}^{*}(\bigoplus_r f_r I_k) = F$ proves exact readout. The recovery error was computed above. The final distinctions follow from the different mathematical types: an expectation value is a number, not a decoder domain.

The conical current remains a named functional

In the supplied I-B-VP-0 normalization,

$$\mathscr{J} = \frac{\beta(1-\beta)(2\beta-1)}{12\pi^2\alpha^2 r^2}\,\mathrm{d}\varphi.$$

This is the coordinate one-form current in its stated field, state, charge, and orientation conventions [CheungVP0]. The orthonormal azimuthal component divides its coordinate coefficient by αr. The charged convention in I-B-VP may include the explicit charge e; the two expressions must not be compared after silently dropping that factor [CheungVP].

The formula is state-qualified one-point data. It is not soft hair, a BMS charge algebra, a reservoir Hilbert space, or an acquired observable of the present finite instrument. Supplying its value as a classical program is covered by Lemma ; acquisition is not.

Likewise, I-B-VP-1's finite parts and contact space, and the source-named apex coefficients in the predecessor, are not interacting apex quantum algebras [CheungVP1,CheungEx]. The soft-hair proposal literature concerns additional asymptotic and horizon structures not present in [HawkingPerryStrominger].

Proposition. After the prefix , the k-dimensional memory can be granted, retained inaccessible, or destructively read. Each option preserves the old classical label law. Only the granted option permits coherent use of that factor; none is selected by a current expectation.

Proof. Apply Proposition to the terminal trace-preserving operation or ownership restriction.

Five-object account and exclusions. The model has algebra $\bigoplus_r M_k$, a classical charge label, and the exact error above. Hybrid entropy obeys . The host and current are separate input ledgers. Terminal memory ownership is additional data. No BMS group, physical horizon memory, Page curve, or gravitational matching obligation is constructed. The local Ward comparisons in Oai282-A/B and the XVI-O counterterm record do not change that conclusion [CheungWard,CheungStanding].

Extension XIV: a fixed-mode late-access surrogate

A mode label and a finite power effect

Fix one packet label κ. Calling it ``comoving'' does not change the host. Choose a finite code $V: Q \to \mathscr{H}_{m,L}$, and a finite self-adjoint quadrature matrix Q_L. For $c > 0$, define the bounded power proxy

$$F_{\mathrm{pow}} = \frac{Q_L^2}{c + \|Q_L^2\|_{\infty}}.$$

It is an effect. A binary power-report instrument may be declared by the square-root branches of F_{pow} and $I - F_{pow}$, with the finite mode retained or consumed according to the output contract.

Theorem. For the fixed packet label and any complete late instrument with output R plus blocks of dimension at

most k, the early unknown code obeys . Exact normalized branches require $k_r \geq ds_r$.

If only the binary power record is exported, $k = 1$. If the unmeasured finite mode is granted, its actual matrix dimension is counted; the power record alone does not certify its recovery.

Proof. Composition with the encoding V is a finite channel. Its declared late output has the form . Apply the common access and rank theorems.

A complete qubit power-report example

For a two-level mode truncation let $n = proj\ 1$, and choose

$$F_1 = \frac{I + \eta Z}{2}, \qquad F_0 = I - F_1, \qquad 0 \leq \eta \leq 1.$$

The installed instrument is

$$\mathcal{I}_s(X) = \sqrt{F_s} X \sqrt{F_s}, \qquad s = 0, 1.$$

Its record estimates the occupation through $Pr\ (1) = [1 + \eta(1 - 2\rho_11)]/2$.

At $\eta = 0$, the record is a fair coin and the returned mode is unchanged. At $\eta = 1$, it is a complete occupation measurement; record plus residual is then measure and prepare. For every η, the record-only whole-code error is at least $1/2$. The code states $|+\rangle$ and $|-\rangle$ have identical occupation laws, showing the loss of a phase-sensitive task from that record.

Proposition. For any fixed finite sequence of complete power-report attempts, retaining old outcomes gives nondecreasing $H(R_n)$. If a residual is returned,

$$S(R_n K_n) = H(R_n) + \sum_r p_r S(\rho_{K_n \mid r}).$$

A selected mode entropy can fall, for example when a maximally mixed mode is completely occupation-measured and conditioned on its record. The classical spectrum log is not an arbitrary-state decoder.

Proof. Use Proposition . The example gives pure conditional eigenstates from the maximally mixed input, while retaining the nontrivial classical outcome entropy.

Proposition. Attaching a fixed classical label ``exit'' to a scheduled event changes neither the installed channel nor its access algebra. If different continuations are selected by that label, their channels must be declared independently. No expanding scale factor follows from the label or its record.

Proof. A fixed label is tensoring with a fixed classical state. It has no action on the metric or quantum input. A nontrivial controlled continuation requires a separately specified family of maps.

Em02-E's relational lapse belongs to its independently prescribed power-law background, not to this cone [CheungInflation]. VIII-A/B/C's conical matching and the QdS external-leg prescription supply neither inflation nor a Bunch–Davies state here [CheungConical,CheungStanding]. A scheduled ``horizon exit'' label is not a derived physical causal past.

Five-object account and exclusions. The early code, finite late output algebra, and residual dimension are explicit. The transcript obeys the classical chain rule and exact branches the complementary-rank condition. Geometry and thermal costs remain separate. The last mode can be consumed, hidden, or granted. No de Sitter state, super-horizon dynamics, physical classicalization mechanism, or wave-function of the universe is constructed.

Extension XV: finite non-Markovian memory and back-flow

A structured two-qubit reservoir

Let S, B be qubits and prescribe

$$H_{\text{int}} = g Z_S \otimes Z_B, \qquad g > 0, \qquad \rho_{SB}(0) = \rho_S \otimes I_B/2.$$

The unitary is

$$U_t = \cos(gt) I - i \sin(gt) Z_S \otimes Z_B.$$

The reduced channel is

$$\Lambda_t(\rho) = \begin{pmatrix} \rho_{00} & c(t)\rho_{01} \\ c(t)\rho_{10} & \rho_{11} \end{pmatrix}, \qquad c(t) = \cos(2gt).$$

A fixed purification of *I_B/2* is included in the complete complement whenever a pure complete dilation is required.

Theorem. For the two inputs *proj +*, *proj −*,

$$d_{\mathrm{tr}}(\Lambda_t(\mathrm{proj}\,+), \Lambda_t(\mathrm{proj}\,-)) = |c(t)|.$$

It decreases to zero and later returns to one. Consequently the reduced family is not CP-divisible across an interval containing a zero followed by a revival.

For input *proj +*,

$$S(\Lambda_t(\mathrm{proj}\,+)) = h\left(\frac{1 + |c(t)|}{2}\right),$$

which is nonmonotone. The optimal full-qubit recovery error is exactly

$$e_{\mathrm{rec}}(\Lambda_t) = \frac{1 - |c(t)|}{2}.$$

None of these revivals is recovery from a previously retained classical record alone.

Proof. Tracing *B* gives . The two output differences have eigenvalues *±c(t)*, giving the trace distance and entropy formula. A CPTP map cannot turn identical states at a zero of *c* into distinguishable states later, excluding CP divisibility.

Correct the sign of *c*, if necessary, by *Z*. The resulting dephasing channel is

$$\frac{1 + |c|}{2}\mathrm{id} + \frac{1 - |c|}{2}\mathrm{Ad}_Z.$$

Returning it gives distance at most *(1 − |c|)/2*. For any decoder with error *e*, trace-distance contraction and the two orthogonal target inputs give

$$1 \le 2e + |c|.$$

This proves optimality. The calculation concerns a live quantum system, not inversion of a classical-output channel.

The distinguishability diagnostic is related to the information-flow approach to non-Markovianity [BreuerLainePiilo]. It is an exact finite witness, not a general measure theorem for structured reservoirs.

An append-only monitored version

At predeclared times one may apply the noninformative complete system instrument

$$L_0 = L_1 = I_S/\sqrt{2},$$

implemented with a fresh finite meter and destructive meter readout. It appends an independent fair bit without changing *SB*. Thus after *n* such attempts,

$$H(R_n) = n\log 2,$$

while the residual-system entropy follows the nonmonotone formula above. The record is constant as a channel of the unknown input; its record-only recovery error is *3/4*. Record plus the live *S* has the error .

Informative measurements can instead be installed, but they change the joint process and need their own dynamics. One must not combine an unmonitored revival curve with a different destructive monitoring history.

Theorem. For any finite reservoir experiment, once the complete accessible cut is *R* plus blocks of dimension at most *k*, the whole-code law is . Reservoir memory outside that algebra does not increase *k*. An exactly recovering branch must meet *k_r ≥ ds_r*.

In the model , exact recovery from *S* occurs only when *|c(t)| = 1*; then its complete complement is constant. At other times the error is strictly positive.

Proof. The first assertions are the common theorems applied to the complete output. The last follows from , or from distinguishability loss and Theorem .

Zero reservoir energy does not forbid revival

Proposition. Choose the bare bath Hamiltonian $H_B = 0$, so its Gibbs state is $I_B/2$ at every finite $\beta > 0$. In the above model,

$$\omega_B(t) = I_B/2, \qquad Q_B(t) = 0.$$

For pure input $proj +$,

$$\Delta S_S = -S(\omega_S), \qquad I(S:B)_\omega = S(\omega_S), \qquad D(\omega_B \| \tau_B) = 0.$$

Thus is satisfied during both decoherence and revival, with zero bath-energy change.

Proof. In the bath Z-basis the dynamics is a mixture of the two system phase rotations with unchanged bath populations. Hence the bath marginal stays $I/2$. The joint entropy stays $\log 2$, while $S(B) = \log 2$, giving $I(S:B) = S(S)$. Substitution proves the equality.

The coupling preparation and switching ledger is not declared free. This proposition concerns the specified bath energy, not all work costs of an apparatus.

Em04-A's dissipative XY reduction has a separately assumed generator; Em05-A/B's signed fermionic channels are different models [CheungOpenSystems]. The complete-output and recycling comparisons in En07 do not identify those reservoirs [CheungRecovery].

Five-object account and exclusions. The live residual, record, and inaccessible purifier are explicit. Record entropy increases while residual entropy can fall. The complementary-rank test applies at each exact-recovery time. Thermal claims use their own Hamiltonians. Terminal bath access is not inferred from back-flow. No universal non-Markovian measure, XY-chain identification, gravitational endpoint, or channel-implied Landauer cost is proved.

Extension XVI: destructive multiparameter sensing

A differentiable finite statistical model

Fisher information is not defined by a finite list of isolated parameter values alone. Fix instead an open parameter region $\Theta \subset \mathbb{R}^p$ and a differentiable positive finite-alphabet law $P_{\theta}(x)$, with fixed support. Its score and Fisher matrix are

$$s_\theta(x) = \nabla_\theta \log P_\theta(x), \qquad I(\theta) = \mathbb{E}_\theta[s_\theta s_\theta^{\mathrm{T}}].$$

For n independent trials the score adds and the Fisher matrix is $nI(\theta)$.

Theorem. Assume the Fisher matrix is invertible and an estimator is locally unbiased with finite covariance. Then

$$\mathrm{Cov}_\theta(\hat{\theta}) \mathrm{succeq}\, I_{\mathrm{tot}}(\theta)^{-1}.$$

There is a finite destructive sensor attaining this bound exactly: for $\theta \in (a, 1-a)^p$, $0 < a < 1/2$, supply n fresh probes

$$\rho_\theta = \mathrm{bigotimes}_{j=1}^{p} [(1-\theta_j)\mathrm{proj}\, 0 + \theta_j \mathrm{proj}\, 1],$$

measure every qubit destructively, and use coordinate sample means. Then

$$I_{\mathrm{tot}}(\theta) = \mathrm{diag}\left(\frac{n}{\theta_j(1-\theta_j)}\right), \quad \mathrm{Cov}_\theta(\hat{\theta}) = \mathrm{diag}\left(\frac{\theta_j(1-\theta_j)}{n}\right).$$

Nevertheless the same measurement channel, regarded as acting on an arbitrary unknown pn-qubit code, is classical-only and has recovery error at least $1 - 2^{-pn}$.

Proof. Fixed support and normalization give $\mathbb{E}s_{\theta} = 0$. Differentiating local unbiasedness gives

$$\mathbb{E}[(\hat{\theta} - \theta) s_\theta^{\mathrm{T}}] = I_p$$

for the total score. Positivity of the covariance matrix of $\theta - \theta - I_{tot}^{-1} s_{\theta}$ gives the matrix bound.

For the product Bernoulli sensor, direct differentiation and independence give the displayed diagonal Fisher matrix. The sample means are unbiased, independent across coordinates, and have the displayed variances, attaining the bound. Its output on an arbitrary quantum input is still a classical record; apply Theorem .

Repeated preparations are additional inputs. This theorem does not supply a channel $\rho \mapsto \rho^{\otimes n}$ from one unknown specimen. Its efficiency is classical efficiency for the declared family, not a quantum Cramér–Rao optimality theorem for undeclared probes. General metrological distinctions have an independent literature [BraunsteinCaves].

Joint finite-confidence bounds

Theorem. Let $\Theta \subset \mathbb{R}^p$ be compact. For m fixed settings, let $q: \Theta \to [0,1]^m$ be continuous and satisfy

$$\|q(\theta) - q(\theta')\|_2 \geq b\|\theta - \theta'\|_2, \qquad b > 0.$$

Setting j has n_j independent Bernoulli attempts, fixed in advance, with actual mean $\tilde{q}_j$ satisfying

$$|\tilde{q}_j(\theta) - q_j(\theta)| \leq \eta_j.$$

Independence between settings is not required. Choose $\zeta_j > 0$, $\sum_j \zeta_j \leq \zeta$, and set

$$r_j = \sqrt{\frac{\log(2/\zeta_j)}{2n_j}}.$$

Let $\hat{\theta}$ minimize $\|q(\theta) - \bar{X}\|_2$ over Θ. Then with probability at least $1 - \zeta$,

$$\|\hat{\theta} - \theta\|_2 \leq \frac{2}{b}\left[\sum_{j=1}^{m} (r_j + \eta_j)^2\right]^{1/2}.$$

A deterministic tie rule can be fixed among minimizers.

For a rectangular coordinate model with separate monotone inverse-Lipschitz laws $q_j(\theta_j)$, clipping each empirical mean to its interval removes the factor two:

$$|\hat{\theta}_j - \theta_j| \leq (r_j + \eta_j)/b_j$$

simultaneously on the same confidence event.

Proof. For a Bernoulli variable the centered log moment-generating function is at most $u^2/8$: its second derivative is a tilted Bernoulli variance, at most $1/4$, and the value and first derivative vanish at zero. Exponential Markov bounds give

$$\Pr(|\bar{X}_j - \tilde{q}_j| > r_j) \leq \zeta_j.$$

The union bound yields the simultaneous event

$$\|\bar{X} - q(\theta)\|_2 \leq \left[\sum_j (r_j + \eta_j)^2\right]^{1/2}.$$

Compactness gives a minimizing estimator. Its residual norm is no larger than that of the true parameter, so

$$\|q(\hat{\theta}) - q(\theta)\|_2 \leq 2\|\bar{X} - q(\theta)\|_2.$$

Use the inverse-Lipschitz condition. For coordinate intervals, nearest-point clipping is nonexpansive and the one-dimensional inverse gives the last assertion.

The factor two is necessary for this general argument: projection onto a nonlinear likelihood image is not automatically nonexpansive.

A nonempty coupled model is

$$q_j(\theta) = \frac{1 + \tanh((A\theta + b_0)_j)}{2},$$

where A has full column rank. If $|(A\theta + b_0)_j| \leq Y$ on Θ, then

$$\|q(\theta) - q(\theta')\|_2 \geq \frac{1}{2}\operatorname{sech}^2(Y)\,\sigma_{\min}(A)\|\theta - \theta'\|_2.$$

This follows coordinatewise from the mean-value theorem.

Finite-resolution programs converging under Lemma add their complete allowance to the calibration budget without increasing the residual dimension. A bad-calibration event of probability at most β can be included by replacing confidence $1 - \zeta$ with $1 - \zeta - \beta$. No fixed-sample statement is reused after undeclared stopping.

Distinguishability, named effects, and nearby parameters

Proposition. For a differentiable family of effects F_{θ},

$$d_{\diamond}(\mathscr{M}_{F_\theta}, \mathscr{M}_{F_{\theta'}}) = \|F_\theta - F_{\theta'}\|_\infty.$$

For any fixed probe state, the resulting binary-law TV distance is at most this quantity. If $\|F_{\theta} - F_{\theta^{'}}\|_{\infty} \le L\|\theta - \theta^{'}\|$, that is a complete named-readout upper bound.

For a separate unknown-code task with output R plus quantum block dimension k, its recovery error still obeys $1 - k/d$. A large or efficiently estimable parameter signal does not reverse that lower bound.

Proof. Use Theorem and measurement contraction. The final assertion applies Theorem to its own declared input task.

The statistics must refer to one matched implemented channel, not different devices chosen after seeing favorable outcomes. This is consistent with the source-qualified complete-record validation contracts of Oai265-B and Oai280-C [CheungValidation].

A local echo-to-confidence specialization

The attached I-C-v2 gives a compact non-shadow scalar window model [CheungIC]. We reproduce the finite calibration consequence without importing its field kernel as a premise of the statistical theorems.

Fix a compact interval $J \subset (0, 1]$ avoiding $1 - \cos(\pi/\alpha) = 0$. Install a positive normalized weight v, independent of α, supported on $0 \le u \le u_{*}$, and define

$$s(\alpha) = \int q(\alpha, u)\, v(\mathrm{d}u),$$

$$q(\alpha, u) = -\frac{\sin(\pi/\alpha)}{\alpha[\cosh(u/\alpha) - \cos(\pi/\alpha)]}, \qquad q_0(\alpha) = -\frac{1}{\alpha}\cot\frac{\pi}{2\alpha}.$$

This is the normalized-window functional of the declared point-profile comparison, not a claim about every smeared detector.

Put

$$a = \min J, \quad d_* = \min_J [1 - \cos(\pi/\alpha)] > 0, \quad M_0 = \max_J |q_0|, \quad M_1 = \max_J |q_0^{'}|.$$

Define

$$L_J(u_*) = u_*^2 \cosh(u_*/a)\left[\frac{M_1}{2a^2 d_*} + \frac{M_0}{a^3 d_*} + \frac{\pi M_0}{2a^4 d_*^2}\right].$$

Corollary. If $L_J(u_{*}) < \pi/2$, then s is decreasing and

$$|s(\alpha) - s(\alpha^{'})| \ge m_J |\alpha - \alpha^{'}|, \qquad m_J = \pi/2 - L_J(u_*) > 0.$$

Let $B_J = \max_J |q_0|$, and install

$$p(\alpha) = \frac{1 + \tanh(\gamma s(\alpha))}{2}, \qquad \gamma > 0.$$

If the supplied window error is at most η_{win} and the device probability error at most η_{dev}, then n fresh attempts give, at confidence $1 - \zeta$,

$$|\widehat{\alpha} - \alpha| \le \frac{\sqrt{\log(2/\zeta)/(2n)} + \gamma\eta_{\mathrm{win}}/2 + \eta_{\mathrm{dev}}}{(\gamma m_J/2)\operatorname{sech}^2(\gamma B_J)}.$$

The finite instrument does not acquire the underlying continuum amplitude.

Proof. Write $D = 1 - \cos(\pi/\alpha)$ and $H_u = \cosh(u/\alpha) - 1$. Then $q = q_0 D/(D + H_u)$, so $|q| \le |q_0|$. With $x = \pi/(2\alpha)$,

$$q_0^{'}(\alpha) = -\frac{x - \sin x \cos x}{\alpha^2 \sin^2 x} \le -\pi/2.$$

Indeed, $x \ge \pi/2$, the numerator increases from $\pi/2$, and the denominator is at most one. Differentiating $q - q_0 = -q_0 H_u/(D + H_u)$, with

$$H_u \le \frac{u^2}{2a^2}\cosh(u_*/a), \quad |\partial_\alpha H_u| \le \frac{u^2}{a^3}\cosh(u_*/a), \quad |D^{'}| \le \pi/a^2,$$

gives the bound . Averaging over the α-independent weight gives $s^{'} \le -m_J$.

The derivative of the installed probability has absolute value at least $(\gamma m_J/2)\mathrm{sech}^2(\gamma B_J)$. The tanh map has Lipschitz constant $\gamma/2$. Apply the one-dimensional case of Theorem . The non-acquisition assertion is the distinction between this supplied functional and a constructed field-to-register map.

I-B-ECHO's readout, finite-injectivity, and core convergence results can supply calibration data only inside their declared window classes [CheungEcho]. Point-profile constants are not transferred to spatially smeared experiments by citation. En01-A/B/C and En10-A supply documentary testing and inference comparisons, not experimental exclusions here [CheungSensing]. XVII-A concerns one bounded sine effect, not a holographic code decoder [CheungXVII]. Oai276-A and Oai119-A retain their frozen Holevo and Hellinger tasks [CheungInformation].

Five-object account and exclusions. The sensor has finite complete outcome words, a declared quantum residual if any, and a joint confidence budget. Exact branches obey complement rank; appended attempts obey the entropy chain rule. Sampling resources and thermal/geometric interpretations are separate. Consuming the last probe is not returning its state. No undeclared-probe quantum Cramér–Rao theorem, gravitational spectroscopy, or continuum acquisition is proved.

Extension XVII: summoning, ancestry, and deadlines

Scheduled locality on the installed host

Let a finite network contain every quantum port, record, controller, and resource wire. At each tick j, its complete channel factors over a declared partition $\mathscr{B}_j$. For terminal ports Y, define

$$N_{T,T}(Y) = Y, \qquad N_{j,T}(Y) = \bigcup \{B \in \mathscr{B}_{j+1} : B \cap N_{j+1,T}(Y) \neq \varnothing\}.$$

Lemma. The terminal reduction factors through the initial reduction on $N_\{0, T\}(Y)$, including untouched references. Excluded complete gates can be replaced without changing that output when wiring and included gates are fixed.

If every dependency edge has a certified regular future-causal route in , ancestry is contained in the prescribed metric causal relation. Equality is not implied.

Proof. Pull an output observable backward. On an excluded gate it is identity, which the unital adjoint preserves. A meeting gate enlarges support only to its input predecessors. Induction proves the support containment. Appending any fixed state on the initial complement gives a reduced channel independent of that appended state; the observable argument shows agreement even for correlated inputs and reference extensions. Concatenating certified routes proves metric containment.

Ancestry limits which slice factors can affect an output by its deadline. It does not prohibit other factors from remaining physically live elsewhere. These are the I-B/En05/En06/VII-J-ST locality distinctions [CheungIB,CheungTransport].

A cut theorem for call-dependent tasks

Let a classical call j be independent of the unknown code. The legal schedules and call distributions are fixed before decoding. For each permitted call, identify the complete interface that can influence its return port.

Theorem. Suppose, for call j, the complete message-bearing interface available before its deadline is

$$\mathscr{N}^{(j)} : Q \to \mathrm{bigoplus}_r \mathscr{B}(K_{j,r}), \qquad \dim K_{j,r} \le k_j,$$

with no omitted quantum bypass. Every deterministic deadline decoder satisfies

$$F_{\mathrm{e}} \le \frac{\min(k_j, d)}{d}, \qquad d_{\diamond} \ge 1 - \frac{\min(k_j, d)}{d}.$$

Exact branches require *k_{j, r} ≥ ds_{j, r}*. Complete implementation allowances subtract as in Proposition .

The call promise alone does not furnish the quantum factor.

Proof. Everything after the identified cut is postprocessing of that complete interface. Apply the common access, rank, and robustness results for each call.

Proposition. At an intermediate slice, let the only quantum factors in the full backward region of return *j* have dimensions *q_1, …, q_m*, with all relevant classical controls included. Their joint quantum dimension is at most *k_j = ∏_a q_a*.

If the slice encoding into that complete ancestral interface is the only route by which the unknown code can affect the return, Theorem applies with that *k_j*. Factors outside the backward region do not improve the unconditioned deadline output.

Proof. Lemma gives factorization through the slice reduction. The joint quantum Hilbert dimension is the product. Apply Theorem .

The hypothesis concerns the complete ancestral interface: entanglement across omitted quantum resources or prior transmissions must not be excluded by bookkeeping.

A feasible two-return task

Fix *r_0 > 0*, *φ_0*. The *t, z* plane at these coordinates has metric *−dt^2 + dz^2*. Supply a code at *s = (0, 0)*. A call *j ∈ { +, −}* is delivered at a common event *c = (2a, 0)*, *a > 0*. Return events are

$$r_{\pm} = (2a + T, \pm a), \qquad T > a.$$

Store the payload at *c*, then send it along the chosen regular route using equal-dimensional SWAP buffers. Unchosen buffers remain blanks.

The logical channel at the requested return is exactly the identity, including every reference. The call record is append-only and independent of the unknown input; the granted payload has dimension *d*. Its complement is constant. This is a nonempty finite summoning-type task with a declared causal route, not a classification of all summoning tasks.

An incompatible independent-call task

Now place calls at

$$c_{\pm} = (2a, \pm a),$$

and returns at

$$r_{\pm} = (2a + \tau, \pm a), \qquad 0 < \tau < 2a.$$

The input at *s = (0, 0)* can reach both call regions. However, *c_{+}* cannot influence *r_{−}*, nor *c_{−}* influence *r_{+}*, under the installed causal permissions.

Proposition. Assume at most one call is promised and the local operations are defined also on the unpromised two-call input. If opposite calls cannot influence the other return, no protocol can guarantee exact quantum return for both possible single-call cases on a nontrivial unknown code.

Proof. Consider the counterfactual two-call run. No signalling from the opposite call makes its *r_{+}* marginal equal to that of the +-only run, and similarly for *r_{−}*. If both promised single-call runs were exact, both independent return marginals in this one complete run would recover the whole code.

Purify the complete run and fold every other output into the complement of *r_{+}*. Theorem makes that complete complement constant. Its *r_{−}* reduction is then constant and cannot also return a nontrivial code. Contradiction.

This is the familiar no-cloning/no-signalling mechanism of no-summoning, proved for the displayed finite task. The general summoning literature has broader task classifications and hypotheses [Kent,HaydenMay]. No such classification is claimed here.

Proposition. At either deadline, a live factor can be granted, destructively read, or left inaccessible. The old call

and arrival records do not select among these choices. Consuming the last coherent factor gives a different task from granting it to a decoder.

Proof. This is the terminal trace-preservation and ownership statement of Proposition .

Proposition. The same finite channel and access algebra can be assigned to different regular laboratory placements and different timed route permissions. Its Kraus operators therefore do not determine the background metric or its causal cones.

A finite acyclic schedule alone also does not determine a unique geometric causal relation.

Proof. Place a one-site identity channel at any two distinct regular locations; its algebraic map is unchanged. More generally, any finite schedule can be given slower edge durations while preserving its logical channel, whenever the longer-duration regular routes are supplied. The numerical channel data are identical while the timed embedding data differ. Thus they cannot determine those data.

En08-A's scheduler/Lorentz-cone separation and the prescribed speed constructions in En05-B/En06-B are documentary comparisons, not a derivation of light cones here [CheungTransport]. A tensor-free dependency algebra can be used instead of a spatial tensor slice, through the finite algebraic criterion proved in Appendix [CheungEn05C].

Five-object account and exclusions. Call-dependent output algebras, complete residual dimensions, records, routes, and terminal ownership are explicit. The rank theorem concerns each matched complete complement. Hybrid entropy is the record/residual identity, not a causal metric. Thermal costs require their own ledger. No physical light cone inferred from records, horizon geometry, universal summoning classification, or gravitational communication channel is constructed.

Two finite siblings

XVIII: ensemble Holevo information versus whole-code recovery

For a declared ensemble $\{p_x, \rho_x\}$, define

$$\chi = S\Big(\sum_x p_x\rho_x\Big) - \sum_x p_x S(\rho_x).$$

Proposition. The identity channel and computational dephasing on $\mathbb{C}^d$ have identical outputs on the uniform basis ensemble $\{1/d, \text{proj } j\}$, with $\chi = \log d$. Their whole-code recovery errors are respectively 0 and $1 - 1/d$.

Proof. Both channels return the same pure basis states, with average I/d, so the displayed Holevo quantity is $\log d$. The identity recovers exactly. Dephasing has a classical-only measure-and-reprepare factorization and the sharp error from Theorem .

This computes a finite ensemble quantity; it does not claim a regularized capacity theorem. Oai276-A's frozen one-use channel certificate and Oai290-A's typed decision remain different source records [CheungInformation,CheungOai290].

XIX: encoded partners are not extra commuting copies

Let $J: Q \otimes S \to A$ be an isometry. On the encoded range, the algebra

$$\pi(X) = J(X \otimes I_S)J^\dagger$$

is a logical matrix algebra. Its identity is the range projection, not necessarily the identity on the entire physical space.

Proposition. Suppose two represented algebras preserve the encoded range and satisfy

$$\pi_1(X)J = J(X \otimes I), \qquad \pi_2(Y)J = J(Y \otimes I)$$

for every $X, Y \in \mathcal{B}(Q)$. If $d \geq 2$, the two algebras cannot commute elementwise.

Proof. Their commutator on the encoded range is

$$[\pi_1(X), \pi_2(Y)]J = J([X, Y] \otimes I).$$

Choose noncommuting logical matrices. Since J is injective, the right side is nonzero.

A partner algebra represented inside one accessible factor is therefore not automatically an additional independent tensor copy. This is the finite ownership distinction compared with Hawk03-A [CheungPartner]. The physical firewall problem has additional horizon and state assumptions [AMPS]; none is supplied by this proposition.

The preceding entropy-history separation also survives. For any isometry $J_0: Q \to R_1 \cdots R_N$, unitary U, and inaccessible $L \cong Q$, compare

$$V_{\text{good}}|\psi\rangle = UJ_0|\psi\rangle|0\rangle_L, \qquad V_{\text{bad}}|\psi\rangle = UJ_0|0\rangle|\psi\rangle_L.$$

For input $|0\rangle$, every accessible prefix agrees. For Haar U, every fixed pure input has the same prefix entropy distributions in the two ensembles, by invariance of the uniform unit-vector distribution. At the endpoint the recovery errors are 0 and $1 - d^{-2}$, because one channel is isometric and the other replacement. This is a local proof of the mechanism, not a gravitational Page curve [CheungHawk05B,CheungEx].

Completion ledgers and release

The preceding nine models are not displaced

The following is a carry-forward scope ledger, not a new release of their unprinted proofs [CheungEx].

begingroup toprule Earlier Retained in I-C-Ex-v2 Not inferred there or here

midrule endfirsthead toprule Earlier Retained in I-C-Ex-v2 Not inferred there or here

midrule endhead bottomrule endfoot I Weak, diffusive, jump, and finite photonic instruments. A continuum optical detector or collapse ontology.

II Finite monitored brickwork and complete record boundary. A thermodynamic measurement-induced transition.

III Regulated accelerated scalar detector. A pointlike continuum detector or universal acceleration identifiability.

IV Late-observer packet surrogate. An expanding geometry, de Sitter state, or cosmological horizon.

V Two-packet exchange and flagged finite star model. A laboratory or gravitational horizon.

VI Environment instrument and decoherence comparisons. Pointer ontology, compulsory heating, or annihilation.

VII Two-zone sensing and a specified SWAP reset. Universal heat per readout or a complete work-extraction cycle.

VIII SWAP route and algebraic dependency-cut example. Derived physical causality or universal summoning.

IX Terminal ownership and entropy-history separation. Evaporation, remnant selection, or a physical Page curve.

endgroup

The eight new models

begingroup toprule Extension Constructed here Not inferred

midrule endfirsthead toprule Extension Constructed here Not inferred

midrule endhead bottomrule endfoot X Graph-state transfer, adaptive byproduct correction, perfect-code triple, unique decoder, and exact two-live-qubit streaming optimum. A fault-tolerance threshold, unrestricted decoder optimum, topological code, or volume-law transition.

XI Hard Fock cut, exact block-diagonal cutoff error, coherent-tail allowance, finite quadrature effects, common-interface resolution convergence, and Gaussian moment reporting. Mean-energy-to-dimension equivalence, full Gaussian capacity, a physical homodyne apparatus, or continuum acquisition.

XII Redundant pointer-fragment instrument, exact record laws, and logical-degeneracy/recovery separation. An environmentally selected basis, pointer ontology, biological pruning, or gravitational radiation.

XIII Finite candidate memory with sharp recovery error, exact surviving charge effects, and terminal ownership options. Physical soft hair, BMS algebra, an interacting apex algebra, or a source law.

XIV Fixed-mode power-effect and late-access instruments. Inflation, a de Sitter or Bunch–Davies state, Em02-E's lapse, or a wave-function of the universe.

XV Finite dephasing revival, exact recovery error, append-only monitored version, and zero bath-energy account. A universal non-Markovian measure, an XY-chain identification, or channel-implied heat cost.

XVI Efficient finite Bernoulli Fisher model, joint calibrated confidence regions, effect gaps, and a conditional echo-report radius. Undeclared-probe quantum optimality, gravitational spectroscopy, or acquisition of a continuum window.

XVII Feasible two-return route, independent-call obstruction, deadline capacity bound, and ownership/non-cone separations. A general summoning classification, a metric inferred from records, a horizon, or a gravitational transport channel.

endgroup

Debugging corrections and limits

• **Half-diamond normalization is fixed.** The bound $1 - k/d$ is for half the diamond norm.

• **A residual is not classical.** Its complete branch dimension must be included.

• **Complement rank is not marginal rank.** The factor s in $\dim A \geq ds$ refers to a constant complete complement. It does not price every arbitrary subsystem by the rank of its own state.

• **Adaptive syntax is not free quantum memory.** Classical choices do not enlarge an already fixed codomain. Actually allocated or retained quantum factors do enlarge it and must be counted.

• **Peak occupancy is model-dependent.** The perfect-code result excludes early quantum output, return to the supplier, and hidden storage. It is not an unrestricted fault-tolerance theorem.

• **A hard cutoff is not a mean-energy promise.** A finite-dimensional access theorem requires the former or another explicit finite code.

• **Cutoff error depends on coherence.** The discarded probability is the exact flagged error in the block-diagonal case, not for arbitrary coherent tails.

• **Continuous records require a common interface.** Fine discrete laws do not generally converge to continuous laws in TV. Here only fixed coarse finite outputs or fixed continuous programs are compared.

• **Effects do not determine residual instruments.** The chosen square-root rule is part of the finite optical model.

• **The effect/code comparison has a direction.** Intrinsic optimal effect error lower-bounds code error; small error for one effect does not imply code recovery.

• **Fisher information needs differentiability.** A finite list of hypotheses supports a testing task, not automatically a differential Fisher calculation.

• **Multiparameter projection retains its factor two.** A nonlinear likelihood image is not a convex interval.

• **Back-flow is not classical-record inversion.** A live reservoir/system process can revive while its classical record alone remains nonrecovering.

• **Heat has independent hypotheses.** Strict complete entropy decrease is sufficient, not necessary, for positive bath heat. Zero bath-energy revival is allowed by the printed model.

• **Ancestry does not determine liveness or geometry.** It bounds potential influence on specified terminal ports. Physical causal containment needs separately certified routes.

• **The cone remains ultrastatic.** Mode and exit labels do not make it inflationary.

• **Topic identifiers are not theorem transfers.** Oai-prefixed Cheung sheets are distinct from the separately OpenAI-attributed catalogue. No catalogue family is a premise.

Gravitational and analytic obligations remain outside

The five source-named gravitational obligations of Hawk05-B remain open in this manuscript. Their complete original list is not reconstructed from an unavailable title-page or summary. At minimum, a physical identification would need a named gravitational encoding, same-model dynamics and state, outgoing/inaccessible assignment, endpoint and conservation account, and a receiver/observable correspondence. This is a local matching checklist, not a claim that the source's five clauses have been discharged [CheungHawk05B,CheungHawkST].

The standing inherited analytic ledger is version-qualified: toprule Source gate Status not changed here

midrule XIV-H H2 Historical uniform internal-amplitude stop remains unresolved within XIV-H's release.

XIV-I I2/I3 The narrower internal amplitude is constructed at I2; the required full characteristic joint map remains unresolved at I3.

XIV-J J2 The supported-sector off-locus reference is not the full six-branch characteristic map; J2 remains unresolved.

Later analytic chain No joint section/remainder, uniform transverse estimates, contact compatibility, full output wavefront theorem, or inherited quantum admission is supplied.

XV-B Its abstract technical interfaces are not the opened downstream quantum-admission exit.

bottomrule

A finite normal-jet formula, a reconstructed internal kernel, and a finite instrument are different mathematical objects. No distributional graph product is constructed by a record operation [CheungAnalytic].

Mathematical release

Theorem. For the printed finite models and contracts:

• the sharp access law, exact complementary-channel criterion, complementary-rank bound, hybrid entropy identity, effect-distance formula, and fixed-program allowance hold by the local proofs in Section ;

• Extensions X–XVII each contain a nonempty declared finite instrument, its complete accessible algebra, its quantum residual assignment, an append-only record, a separate physical ledger, and terminal ownership options;

• the perfect-code authorized triple has exact streaming peak two under Definition ;

• the finite optical model has the printed cutoff, threshold, and common-interface convergence allowances;

• the redundant-record, finite-memory, and late-mode models have the stated code/observable separations;

• the revival model has exact error $(1 - |\cos 2gt|)/2$, with the zero-bath-energy realization of Proposition ;

• the sensor attains its stated classical Fisher bound and has the simultaneous finite-confidence certificate ;

• the displayed summoning tasks have the constructive and obstructed outcomes proved locally;

• siblings XVIII–XIX give the stated ensemble-information and encoded-algebra separations. The cone, field realizations, clock calibration, and any Haar hypothesis remain inputs. No physical Page curve, island, continuum detector, gravitational memory, Einstein dynamics, spacetime emergence, or inherited characteristic quantum coefficient is constructed.

Proof. The foundational assertions are Theorems , , , Corollary , Proposition , and Lemma .

The complete maps in , , , the threshold square-root instrument, the fixed-mode instrument, the revival unitary with finite meters, the Bernoulli sensor, and the controlled SWAP route establish nonemptiness. Their sections state the five-object contracts.

Proposition proves (iii). Theorem , Proposition , Theorem , and Lemma prove (iv). The corresponding extension theorems prove (v). Theorems – and Proposition prove (vi). Theorems and prove (vii). Theorem , the explicit route, and Proposition prove (viii). Propositions and prove (ix). No excluded physical correspondence appears among their constructed channels, Hamiltonians, algebras, or proofs.

par **Final interpretation** par An informative record is not an arbitrary quantum-return decoder. An available quantum code is not necessarily executable under a restricted decoder-memory model. A named charge, power, or quadrature effect is not a full code. A memory revival is not inversion of a classical log. An efficient sensor is not thereby a coherent receiver. A scheduled network is not a derived spacetime.

O.1 on the cone remains *texttt NOT_CLOSED*. *texttt bulk_symmetry_restored = texttt false*. The downstream XV-B admission exit is not opened. XVI-N and Route B are not discharged. No inherited characteristic quantum coefficient is admitted. par

appendix

Finite algebraic dependency without a spatial tensor cut

Proposition. Let $\mathscr{A}_{in} \subseteq M_n(\mathbb{C})$ be a unital finite-dimensional C^{*}-subalgebra and $T: \mathscr{A}_{out} \to M_n(\mathbb{C})$ be *textup UCP*. The following are equivalent:

• equal state restrictions to $\mathscr{A}_{in}$ imply equal output expectations through T;

• $T(\mathscr{A}_{out}) \subseteq \mathscr{A}_{in}$;

• T factors through a unique *textup UCP* map $\mathscr{A}_{out} \to \mathscr{A}_{in}$ and the inclusion. No complementary spatial tensor factor is required.

Proof. Containment gives the factor and equality of state expectations. Complete positivity is inherited because the subalgebra has the same matrix-positive cones.

Conversely, take $h = T(b)$, $b = b^{\dagger}$. If $h \notin \mathscr{A}_{in}$, its Hilbert–Schmidt component H perpendicular to $(\mathscr{A}_{in})_{sa}$ is nonzero. Since $I \in \mathscr{A}_{in}$, $\mathrm{Tr}\, H = 0$. For sufficiently small $u > 0$, the states $I/n \pm uH$ have equal restrictions to $\mathscr{A}_{in}$ but values on h differing by $2u\|H\|_2^2 > 0$. This contradicts (i). Linear decomposition gives containment.

This is the finite specialization of the tensor-free comparison recorded in En05-C [CheungEn05C]. It need not make the factor map multiplicative and constructs no continuum local net.

Declarations

Authorship and approval. The author field follows the supplied I-C manuscript series. Submission requires the author's approval of the final mathematics, source versions, attribution, and text. No independent referee approval or journal acceptance of I-C-Ex2 is asserted.

Proof dependency. Every finite mathematical assertion included in the release has its proof or explicit construction in this file. Companion citations provide provenance and comparisons, not unprinted physical premises. Standard quantum-channel representation results and elementary finite spectral calculus are background mathematics.

Source access. The attached I-C-v2 and I-C-Ex-v2 sources were available in full. Other companion material is used only at the supplied excerpt or synopsis level. The public philosophy and I-B pages were consulted. The attempted I-C, En02-B, En02-C, and I-B-VP-0 browse endpoints did not yield usable pages through the consulted interface; their contents were not represented as independently retrieved. The large supplied retrieval inventory is not a corpus-wide audit.

Journal metadata. The consulted Frontiers record for [CheungEnter] reports acceptance on 28 September 2026. No final formatted page range or later publication status is invented.

Hawk05-A attribution. The supplied Hawk05-A extract is cited by identifier. Its title-page authorship is not inferred from its placement among Cheung-related records. No local theorem depends on that attribution.

Continuum and apparatus scope. Finite compressed quadrature effects and supplied Gaussian probabilities are not a homodyne apparatus. Finite packet labels are not a continuum field realization. One-point currents and finite parts are not acquired by citation. No local coupling, regulator-removal theorem, or physical gravitational dictionary is supplied.

Execution and numerical scope. No successful pdfLaTeX execution, symbolic-program run, numerical simulation, gate compilation, laboratory realization, proof-assistant check, independent rebuild, or independent technical review is asserted. All displayed constants used in proofs are analytical. No loop table or independent sign audit is added.

Catalogue scope. Oai-prefixed Cheung sheets remain distinct from the separate OpenAI-attributed catalogue. No catalogue proof, family, formalization percentage, or external physics claim is verified or used as a premise. Families 282, 362, and 376 remain documentary.

Unchanged programme status. The historical H2 verdict is not retroactively changed, XIV-I's narrower I2 construction is not withdrawn, and the full I3/J2 characteristic target remains unresolved. The technical existence of XV-B's lifting interfaces is distinguished from its unopened downstream quantum-admission exit. XVI-N, Route B, and the five source-named Hawk05-B gravitational obligations are not discharged. The separately scoped Q11 two-point record is not converted into blanket cone closure. O.1 remains *texttt NOT_CLOSED*, and bulk symmetry is not restored.

Format. This is one pdfLaTeX source with an internal bibliography. No external figures, data, scripts, bibliography database, or shell escape are required. Two passes are intended for the contents and cross-references.

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N. Cheung, *The Philosophy of Enter: Destructive Readout, Quantum Recovery, and the Refusal of Identification*, version 2, EntPhil-v2.tex, Enter Calculus Companion. Documentary orientation. https://enter.grok.me/enter.

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N. Cheung, *I-C-Ex-v2: Sharp Access Bounds, Residual Capacity, and Nine Extensions of Destructive Enter*, 18 October 2026, supplied full source I-C-Ex-v2.tex. Predecessor framework and nine finite models. Its results needed here are re-established locally.

N. Cheung, *I-B-G*, version 2, supplied excerpts, Enter Calculus Companion. Scalar Green operators, prop:nonGH, thm:green, and thm:speed. Domain and support conventions remain source-qualified.

N. Cheung, *I-B-G-S*, version 2, supplied excerpts, Enter Calculus Companion. Especially thm:spin1: free Maxwell–Friedrichs source-test intertwining with compatible scalar ghost. Not an interacting detector theorem.

N. Cheung, *I-B-G-S-B*, version 1, supplied excerpts, Enter Calculus Companion. thm:selection, cor:exclude: positive tensor-core Friedrichs selection and its exceptional branch exclusion. Not a recovery theorem.

N. Cheung, *I-B-VP*, version 3, supplied excerpts, Enter Calculus Companion. thm:friedrichs, thm:current: state-qualified one-point quantities and current conventions.

N. Cheung, *I-B-VP-0*, version 1, supplied excerpts, Enter Calculus Companion. thm:current: charged Friedrichs vacuum current. Coordinate one-form normalization is retained.

N. Cheung, *I-B-VP-1*, version 3, supplied excerpts, Enter Calculus Companion. thm:U, thm:classification: radial finite parts and stated contact-space classification. Not an interacting apex quantum algebra.

N. Cheung, *I-B-ECHO*, version 2, supplied excerpts, Enter Calculus Companion. thm:readout, thm:TV, thm:finiteinjectivity, thm:core, and cor:nonreconstruction. Specified window reports do not acquire continuum amplitudes.

N. Cheung, *Hawk05-ST*, version 1, supplied excerpts, Enter Calculus Companion. thm:main: a finite Enter/Page assembly on the prescribed cone. Not a gravitational Page curve or endpoint.

N. Cheung, *En09-A: Realization of Relational Time by Destructive Enter on a Clock Register*, version 1, supplied excerpts, Enter Calculus Companion. thm:main; conditioning is distinct from dynamical interpretation.

N. Cheung, *X-K-ST*, version 1, supplied excerpts, Enter Calculus Companion. Finite complete clock composition, locality, and resource-fitting instruction certificates.

N. Cheung, *X-J-ST-2*, version 1, supplied excerpts, Enter Calculus Companion. prop:noacquisition, thm:TV, and thm:clock. Supplied reports are not continuum acquisition.

N. Cheung, *Hawk04-A*, version 1; *Hawk04-B*, version 2; *Hawk04-C*, version 1; *En07-A* and *En07-B*, version 1, supplied excerpts, Enter Calculus Companion. Complete restoration, exact whole-code recovery, recycling, and conditional endpoint interfaces. Model-specific channels are not identified.

Hawk05-A, version 1, supplied manuscript excerpts, Enter Calculus Companion. thm:global, thm:exact, cor:nohiding, and thm:recordentropy. Title-page authorship is not established by the supplied extract and is not inferred here.

N. Cheung, *Oai280-C*, version 1, supplied excerpts, Enter Calculus Companion. Full-code, subsystem, and

finite-algebra targets; thm:complement is a full-code criterion. Not verification of an external catalogue family.

N. Cheung, *Oai290-A*, version 1, supplied excerpts, Enter Calculus Companion. thm:effect: referenced binary-effect reconstruction; typed code/observable/model separation.

N. Cheung, *En02-A*, *En02-B*, and *En02-C*, version 1, supplied excerpts, Enter Calculus Companion. Cut monotones and sub-capacity rigidity; En02-B thm:B5 streaming peak; En02-C thm:C2, thm:C3 availability/execution separation. No fault-tolerance threshold is imported.

N. Cheung, *En02-E*, version 2, and *Em03-B*, version 2, supplied excerpts, Enter Calculus Companion. Finite-mode Gaussian restriction, flagged cutoffs, and normal geometric packet marginals. Not a full continuum Gaussian-channel capacity theorem.

N. Cheung, *Em03-D*, *Em03-E*, and *Em03-F*, version 1, supplied excerpts, Enter Calculus Companion. Local Weyl comparisons, designed finite channels, and classical-witness/entanglement-breaking distinctions. Their apparatus and state identifications remain separate.

N. Cheung, *En01-A* and *En03-A*, version 1, supplied excerpts, Enter Calculus Companion. Recorded architectures, observation boundaries, controls, guards, and retained storage. Not a pointer-ontology theorem.

N. Cheung, *Psy01-A*, version 1, supplied excerpts and description, Enter Calculus Companion. Analytic pruning and information-obstruction comparisons. Not an environmental basis-selection theorem.

N. Cheung, *Oai282-A* and *Oai282-B*, version 1, supplied excerpts, Enter Calculus Companion. Finite local Ward and coverage comparisons. Not BMS charges, Einstein dynamics, or verification of Families 282, 362, or 376.

N. Cheung, *Em02-E*, version 1, supplied excerpts, Enter Calculus Companion. thm:lapse: a classical relational lapse on its declared power-law background. Not the present conical host.

N. Cheung, *VIII-A*, *VIII-B*, and *VIII-C*, version 3, supplied excerpts, Enter Calculus Companion. Conical matching and finite block-channel comparisons. No inflationary or Einstein-dynamical transfer is made.

N. Cheung, *Em04-A*, version 2; *Em05-A* and *Em05-B*, version 1, supplied excerpts, Enter Calculus Companion. A specified dissipative XY reduction and distinct signed fermionic extraction channels. None supplies the generator of the revival model here.

N. Cheung, *Oai265-B*, version 2, and *Oai280-C*, version 1, supplied excerpts, Enter Calculus Companion. Fixed implemented-channel, complete-record, calibration, and finite-confidence validation contracts. Not an area law or an external family verification.

N. Cheung, *En01-A*, version 1; *En01-B*, version 2; *En01-C*, version 1; and *En10-A*, version 1, supplied excerpts, Enter Calculus Companion. Testing, contrasts, preparation-aware bounds, and declared source-weight inference. Not experimental exclusions or gravitational source identification.

N. Cheung, *XVII-A*, version 1, supplied excerpts, Enter Calculus Companion. Exact reconstruction of one named bounded sine effect from one declared region. Not arbitrary bulk-code recovery.

N. Cheung, *Oai276-A* and *Oai119-A*, version 1, supplied excerpts, Enter Calculus Companion. One frozen one-use Holevo certificate and one finite Hellinger comparison. Not general metrology theorems.

N. Cheung, *En05-A*, *En05-B*, *En06-A*, *En06-B*, *En08-A*, *En08-B*, *VII-J*, and *VII-J-ST*, version-qualified supplied excerpts, Enter Calculus Companion. Scheduled ancestry, prescribed-speed transport, causal embedding, and scheduler/geometry separations. Regular-part laboratories do not silently import apex-register permissions from other models.

N. Cheung, *En05-C*, version 1, supplied excerpts, Enter Calculus Companion. thm:comparison: tensor-free snapshot/channel/adjoint equivalence. Its finite specialization is proved here.

N. Cheung, *Hawk03-A*, version 1, supplied excerpts, Enter Calculus Companion. State-independent encoded partner algebras, commuting-copy obstructions, and separately conditional firewall benchmarks.

N. Cheung, *Hawk05-B*, version 1, supplied excerpts, Enter Calculus Companion. thm:retain: entropy-history/recovery separation. The five source-named gravitational obligations remain outside the finite constructions here.

N. Cheung, *XIV-H*, *XIV-I*, *XIV-J*, version 1, and *XV-B*, version 2, supplied excerpts, Enter Calculus Companion. Historical H2 stop, constructed I2 internal amplitude, unresolved full I3/J2 map, and abstract reconstruction-lifting interfaces. No inherited coefficient is admitted here.

N. Cheung, *XVI-N*, revision r12; *XVI-O*, revision o70-sync; *XVI-P*, supplied p23 records; and *XVI-Q*, release Q11, supplied scope records, Enter Calculus Companion. Prepared-source, loop, and separately scoped two-point records are not blanket cone closure. QdS is an external-leg scheme, not a de Sitter state.

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