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I-C-CERT-v6: Exact Dual Certificates for the Shredder Recovery Obstruction

The spectral law, the certified shredder floor, all composites at the floor, an exact non-classical subclass, the gap (d − 1)/d^2 between optimal accounts, the best rank-one dual, when it is exact, and why it generically is not

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Abstract

This is version 6 of the standalone certificate sheet I-C-CERT. It contains version 5 (I-C-CERT-v5, which contains versions 4, 3, 2 and 1; all stay untouched) and adds the items of request t268u (no plan was attached). It supersedes version 5 by addition; the earlier corrections stay in place (Sections,,), and the errors of earlier versions found in version 6 are recorded in Section. The parent sheet I-C (latest version I-C-v2) is cited as a source record only and is not revised. For a classical-output channel followed by a decoder on a d-dimensional code, the half-diamond error to the identity is certified from both sides by explicit witnesses. From version 1: every record-ignoring decoder has error exactly $1 - 1/S$, $S = \sum_i \lambda_i^{-1}$; the optimum over all decoders is $1 - 1/d$, certified by the dual operator $(1 - 1/d)\Omega$; the gap $(d - 1)/d^2$ is the exact value of reading the record, in the sense of best record-ignoring against best unrestricted account (Proposition); and the errors of the full class fill $[1 - 1/d, 1]$, so the spectral law fails there (plan correction). New in version 2. (i) On the basis-measurement interface, every classical decoder with stochastic matrix M has error exactly $1 - 1/\sum_x M_{xx}^{-1}$; general decoders are pinned between $1 - 1/\sum_x \langle x|\sigma_x|x\rangle^{-1}$ and $1 - 1/\sum_x \langle x|\sigma_x^{-1}|x\rangle$. So the error is fixed by the decoder's action on the record, not by its output spectrum; every relabelling other than the identity has error 1, and along the path of version 1 the error is exactly $1 - (1 - s)/d$. (ii) Complete dephasing attains the shredder floor and is a thermal operation with a zero-Hamiltonian bath. (iii) A thermal replacement on the record has error $1 - 1/(Z_{\beta}Z_{-\beta})$, strictly above the shredder floor, certified by the same dual sandwich. (iv) Under every reorganization of the interface the retained value stays at most $1/d$, and the gap $(d - 1)/d^2$ is exactly the gain from reading the record when the starting account is the best record-ignoring one (version-2 wording corrected in version 3; the version-3 abstract repeated it uncorrected, version-3 error V3E.1). The plan's claim that the two extremes lie on a continuous path checked by C3 is confirmed, and now proved exactly. New in version 3. (i) For every channel, the error equals $1 - F_e$ if and only if the Choi operator has $|\Omega\rangle$ as eigenvector, if and only if $J \geq F_e\Omega$; covariance is sufficient, not necessary. (ii) A measure-and-prepare composite is at the shredder floor $1 - 1/d$ exactly when every nonzero effect has rank one and the decoder prepares the normalized effects. (iii) On the basis-measurement interface, outputs with $\sigma_x|x\rangle = q|x\rangle$ give the exact error $1 - q/d$; for $d \geq 3$ this includes non-classical, non-covariant decoders. (iv) Correction of version 2: $(d - 1)/d^2$ is the difference between the best record-ignoring and the best unrestricted retained values; the increase from an arbitrary record-ignoring account is $1/d - 1/S_{\sigma}$, up to $1/d$. (v) The Hawk07 and PG replacement certificates share the value and the primal witness of the spectral law, but their dual operator is not the rank-one $(1 - t)\Omega$ (plan correction; qualified in version 4, since PG-OB-1-CERT-v2 also has the rank-one witness and the block dual corresponds to it under a sign change). New in version 4. (i) A witness in the Hawk07/PG convention ($W \geq 0$, $W \geq J(\mathscr{I}) - \Omega$) is turned into one of this sheet by $W \mapsto W - J(\mathscr{I}) + \Omega$, with the same partial trace; the block dual of the Hawk07/PG replacement certificate becomes exactly the rank-one witness $(1 - 1/S)\Omega$, so ``the same dual certificate" holds up to this convention change, not as operators. (ii) The best dual of the form $(1 - t)\Omega$ has $t_{*} = 1/\langle\Omega|J^{+}|\Omega\rangle$; it is exact for replacement channels (without Bell saturation) and on the Bell-saturated class. (iii) On the basis-measurement interface, the version-2 upper bound is exactly this best rank-one value, and the exact law $1 - 1/\sum_x q_x^{-1}$ extends to every decoder with $\sigma_x|x\rangle = q_x|x\rangle$. (iv) Readings of PG-OB-1-CERT-v2, Hawk07-A/B/C-CERT-v3, En08-ST-DSS-CERT-v3 and Big01-A; prop:shredsharp records $(d - 1)/d^2$ as a difference of two floors, not a gain from every start (plan correction). Two version-3 errors are recorded. New in version 5. (i) For every channel, the best rank-one dual $(1 - t_{*})\Omega$ is exact if and only if $\ker (J - t_{*}\Omega)$ contains $\mathrm{vec}(R)$ for a state R; for

full-rank J, if and only if the matrix of J^{−1}|Ω⟩ is Hermitian positive semidefinite. Exactness is invariant under unitary conjugation. (ii) On the basis-measurement interface the lower bound 1 − 1/S_q is exact only under the eigenvector condition σ_x|x⟩ = q_x|x⟩, so dephasing the decoder's outputs never increases the error and strictly lowers it otherwise. (iii) The upper bound 1 − 1/S_r is exact if and only if G_xy = ⟨y|σ_x^{−1}|x⟩ is Hermitian positive semidefinite; an explicit non-classical decoder outside the eigenvector condition has exact error 11/14. Outside both conditions the error lies strictly between the two bounds; this is a partial result, and the exact value there stays open. (iv) Readings of PG-OB-1-CERT-v3, Hawk07-A/B/C-CERT-v3 and En08-ST-DSS-CERT-v4. Two version-4 errors are recorded (Section). New in version 6. (i) The fidelity primal L_F, a convex program over states, satisfies 1 − F_e ≤ L_F ≤ ε ≤ 1 − t_{*}, and the best rank-one dual is exact if and only if L_F = 1 − t_{*}. (ii) Non-exactness is generic: the channels whose matrix M_{$\mathscr{T}$} is not Hermitian form an open dense set of full measure, on which ε < 1 − t_{*} strictly. A depolarized unitary pAd_U +(1 − p)$\mathscr{R}$_{I/d} is exact if and only if Tr U = 0 or U is a phase times a Hermitian unitary. (iii) On the basis-measurement interface the exactness criterion extends to singular outputs as a linear matrix inequality; for d = 2 with one pure output it is αdet σ_1 ≥ |β|^2, with error α/(α + det σ_1) (for example 12/19). The fidelity primal is never below 1 − 1/S_q. (iv) Readings of the newly delivered OB CERT v4, Hawk07 CERT v4 and DSS CERT v5 sheets; one imprecision of version 5 and one omission of version 4 are recorded (Section). The exact error of non-exact channels stays open. All proofs are local. Big01-A, I-C-Th, Hawk07-A/B/C-CERT, PG-OB-1-CERT and En08-ST-DSS-CERT-v2 are source records, not premises. No quantum coefficient is admitted. Host C is not created; Host A and Host B stay separate. O.1 stays NOT_CLOSED. XVI-N is not discharged (N-Ins.4 unresolved), and there is no Route-B successor. XV-B is not opened. H2, I3 and T3 stay unresolved; J2 stays Unresolved. U6-sel is not moved. Res02-A Q1–Q4 and A1–A2 stay open. The last reading bars (quantized observable, no external clock) stay open. No Einstein dynamics, derived metric, spacetime emergence, external-clock removal or Page curve is claimed. Gate 0 stays NOT_SUPPLIED, and thm:J is not substituted. Continuum acquisition stays a negative certificate. No compilation is asserted. Citations identify source records only; no proof transfer. Authorship requires confirmation. Nothing is pushed or registered.

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Parent, sources and scope

Previous version. I-C-CERT-v1 (sha256 4f932078b3c653f94093b76ee99a5f78990027ff26d1da1b1914a6a12b4c3980) is retained in Sections – and in the plan audit of plan t256u, unchanged except for the cross-reference paragraph added at the end of Section ; the file I-C-CERT-v1 itself is not edited.

Version 2. I-C-CERT-v2 (sha256 a850d04730a82dd0658262fc616abc03a851ecddd83625c10458af416658512c)

is retained in Sections – and in its plan audit; Section was new in version 3. The file I-C-CERT-v2 itself is not edited.

Version 3. I-C-CERT-v3 (sha256 143d3d1ebe12d081e6a5e3f0b48e82a4461fee2442cfe367686a07c9024f8f09) is retained in the sections before Section and in its plan audits; Section and the plan-t266u audit were new in version 4. The file I-C-CERT-v3 itself is not edited.

Version 4. I-C-CERT-v4 (sha256 9917ea20e07a3483e08fbd1a7de01eb2084eb3f7f5be23f2474a7c6283496487) is retained in the sections before Section and in its plan audits; Section and Section were new in version 5. The file I-C-CERT-v4 itself is not edited.

Version 5. I-C-CERT-v5 (sha256 56c30a1a434ce3cec589929093d7764a97330fbc27748afdf8e6f5bc82b984bf) is retained in the sections before Section and in its plan audits; Section and Section are new. The file I-C-CERT-v5 itself is not edited.

Parent. The parent sheet is I-C; its latest version, I-C-v2 (sha256 6684c95253a6b61d5690ad3374c6a38779c11d569fcc5d4c45a6bea639c5d187), was read by read-only retrieval and is a source record [ICparent]. The parent's recovery bound is I-C-v2 thm:A; its interface is I-C-v2 eq:qc, and its measure-and-prepare lemma is I-C-v2 lem:EB. Those facts that this sheet needs are re-proved in Section . The parent stays untouched; this sheet is not a version of it.

Other source records. Big01-A version 5 (sha256 e1160613ea8e02b37fb45954fe677d89f5fa7cca90062d42b68973b03a2bff1d): Big01-A thm:Bexact, Big01-A lem:shredder, Big01-A prop:shredsharp, Big01-A thm:Bthermal, Big01-A thm:Bdephase, Big01-A def:reorg, Big01-A thm:tick. I-C-Th-v1 (sha256 3d8d2a76deea9469326a2fc864368c9503eb93acea792787eb45adc4610e3f60): its witness Hamiltonian. The companion sheets I-C-Ex-CERT-v1 and I-C-Ex2-CERT-v1, for the cross-reference of Section . New in version 3: Hawk07-A-CERT-v2 (sha256 9f4a813e5ed721ac074c05dfd6316a591bac28ccb1be7e23c4972cc32cab0cca), Hawk07-B-CERT-v2 (47d9b5e886faf80023963a32953a899bba46df79f15ac5dbac33a51e41b5ba5b), Hawk07-C-CERT-v2 (c54297c3f2e64ff35bf5261c259e9172140927502e5868f13a4e9f92a5e2ce25), PG-OB-1-CERT-v2 (015f230247fc86885e278ecf2b796b14652f7742c57e200fde8e670fbaf09a40) and En08-ST-DSS-CERT-v2 (40d0dfb5adda8e3279bdb814d8d7912c0e94be51965927a057a5c5f63ac0ebc2); their v1 files (Hawk07, PG) and v3 file (En08) were also checked and contain the cited labels. None is a premise [BigA]. New in version 4: Hawk07-A-CERT-v3 (sha256 1e91c9fb7bb97c16d9b4b0d48e821d511badea6e4ace77fc6643a61c76eee776), Hawk07-B-CERT-v3 (2639d64e7ec3340017a91ed62e487b857317f2cef51966a2086fd016ffcf4176), Hawk07-C-CERT-v3 (4352ee5e6a754a153631ab755a44e16ea9273fb1b6b9950e8fc29711b983a416), the newest delivered Hawk07 CERT versions; the cited statements are unchanged from v2 (checked). Hawk07 CERT v3 corrects the version-2 Definition def:v2-H and the name ``conformal warp in the sheets that contain them; neither is cited here. En08-ST-DSS-CERT-v3 (sha256 3882f4e56296b5fa8aecc5cb359bbba8e0b72dac7d2ccc6f2d040c631e7271ff), the version named by the plan; En08-ST-DSS-CERT-v4 (98c42ee7397263bc694ebed7d66c682b090df437a3230f294b3245f4bd01bee9) is newer and has the cited statements unchanged. PG-OB-1-CERT-v2 as above (only v2 is delivered; its v3, in preparation elsewhere, was not read). None is a premise. New in version 5: PG-OB-1-CERT-v3 (sha256 8c0a7e2fdbb7e4558bad17bc35df10a66feea6ba4959af524921393a61eafbcc), newest delivered, superseding the PG-OB-1-CERT-v2 cited above; Hawk07-A/B/C-CERT-v3 as above; En08-ST-DSS-CERT-v4 (sha256 98c42ee7397263bc694ebed7d66c682b090df437a3230f294b3245f4bd01bee9). Newer versions of Hawk07 CERT (version 4), of the OB CERT sheets (version 4) and of DSS CERT (version 5) are being built concurrently and were not read. None is a premise. New in version 6, each verified against its delivery manifest: PG-OB-1-CERT-v4 (sha256 3dc921457bd781606e9e673231a8c832be104435950913521fdfb2ed737fc1c1); Hawk07-A-CERT-v4 (sha256 a88a2430a9e3672a9cbdb7c661dc021154dd3ac4237a1040b4eab097194fceb5); En08-ST-DSS-CERT-v5 (sha256 9be4fe4fdd9cf1164a75f0218035f0ee3a4c340ed7f15787551de3997fe5af64); Red06-G-DSS-CERT-v5 (sha256 e4a1b02552c138b8f9f009f2e71e391825ad0014f717b43f96883c445605213a); V-DST-DSS-CERT-v5 (sha256 4133961f566c059c22054b5922423d1ff94cb0d1c888ca353a7c143bba1541be);

VC-OB-1-CERT-v4 (sha256 f9802da66896159503132b736164016c1d06d2a6530bbbc06cb4ad3af0256387); DS-OB-1-CERT-v4 (sha256 0b5b86f87f91503ba9e96cf221b2ef96a4b4e96e3f27489336eb2b89848463c6); Red06-G-DSS-CERT-v4 and V-DST-DSS-CERT-v4 are read for their error lists. None is a premise. Second wave read in version 6: DS-, VC- and PG-OB-1-CERT-v5, Hawk07-A/F/H-CERT-v5 and En08-ST/Red06-G/V-DST-DSS-CERT-v6, verified against their manifests and cited by sha256 in Section as source records only.

Standing ledger (verbatim). No quantum coefficient is admitted. Host C is not created; Host A and Host B stay separate. O.1 stays NOT_CLOSED. XVI-N is not discharged (N-Ins.4 unresolved), and there is no Route-B successor. XV-B is not opened. H2, I3 and T3 stay unresolved; J2 stays Unresolved. U6-sel is not moved. Res02-A Q1–Q4 and A1–A2 stay open. The last reading bars (quantized observable, no external clock) stay open. No Einstein dynamics, derived metric, spacetime emergence, external-clock removal or Page curve is claimed. Gate 0 stays NOT_SUPPLIED, and thm:J is not substituted. Continuum acquisition stays a negative certificate. No compilation is asserted. Citations identify source records only; no proof transfer. Authorship requires confirmation. Nothing is pushed or registered.

Plan refusal lines. Host stays prescribed, labels stay named. No finite Hilbert space, temperature, or Einstein dynamics derived. None imports Big01-A or any other sheet as a premise. The certificates are documentary separations: they do not discharge O.1, Gate 0, XV-B, J2, U6-sel or any standing gate, and they do not create Host C. No quantum coefficient is admitted. No derived metric, spacetime emergence or Page curve is claimed. This sheet revises no other sheet.

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The primal–dual sandwich

Throughout this sheet $Q \cong Q^{\{'\}} \cong E \cong \mathbb{C}^d$ with $d \geq 2$, and a fixed intended identification of $Q^{\{'\}}$ with Q. Put

$$|\Omega\rangle = \sum_{j=0}^{d-1} |j\rangle_E |j\rangle_Q = \sqrt{d}\,|\Phi_d\rangle, \qquad \Omega = |\Omega\rangle\langle\Omega|, \qquad J(\mathscr{T}) = (\mathrm{id}_E \otimes \mathscr{T})(\Omega),$$

so $J(\mathscr{T})$ is the unnormalized Choi operator of a channel $\mathscr{T}: Q \to Q^{\{'\}}$, $J(id) = \Omega$, $Tr_Q\, J(\mathscr{T}) = I_E$, and $F_e(\mathscr{T}) = \langle\Omega|J(\mathscr{T})|\Omega\rangle/d^2$. The error of $\mathscr{T}$is

$$\varepsilon(\mathscr{T}) = d_\diamond(\mathscr{T}, \mathrm{id}_Q) = \tfrac{1}{2}\|\mathscr{T} - \mathrm{id}_Q\|_\diamond.$$

Lemma. Let $\mathscr{T}: Q \to Q^{\{'\}}$ be a channel.

• *(Primal witness.)* For every unit vector $\psi \in E \otimes Q$, $\varepsilon(\mathscr{T}) \geq 1 - \langle\psi|(id_E \otimes \mathscr{T})(proj\ \psi)|\psi\rangle$.

• *(Dual witness.)* For every operator Z on $E \otimes Q$ with

$$Z \geq 0, \qquad Z \geq \Omega - J(\mathscr{T}),$$

one has $\varepsilon(\mathscr{T}) \leq \|Tr_Q\, Z\|_{\{\infty\}}$.

• *(Exact dual certificate.)* If a unit vector ψ and an operator Z as in (a) and (b) give the same value t, that is $1 - \langle\psi|(id_E \otimes \mathscr{T})(proj\ \psi)|\psi\rangle = t = \|Tr_Q\, Z\|_{\{\infty\}}$, then $\varepsilon(\mathscr{T}) = t$ exactly. The pair (ψ, Z) is then called an *exact dual certificate* for $\mathscr{T}$.

Proof. (a) Put $\rho = proj\ \psi$ and $\rho^{\{'\}} = (id_E \otimes \mathscr{T})(\rho)$. Testing $\rho - \rho^{\{'\}}$ with the projector ρ gives $1/2\|\rho - \rho^{\{'\}}\|_1 \geq Tr\,[\rho(\rho - \rho^{\{'\}})] = 1 - \langle\psi|\rho^{\{'\}}|\psi\rangle$. The half-diamond distance is at least this trace distance.

(b) The half-diamond norm of the difference of two channels on a d-dimensional input is attained on pure inputs with a reference of dimension d [Watrous]; we use this standard fact only. Every unit vector of $E \otimes Q$ has the form $|\psi\rangle = (A \otimes I_Q)|\Omega\rangle$ with $Tr\, A^{\{\dagger\}}A = 1$. Since $A \otimes I_Q$ acts on the untouched reference,

$$\mathrm{proj}\,\psi - (\mathrm{id}_E \otimes \mathscr{T})(\mathrm{proj}\,\psi) = (A \otimes I)(\Omega - J(\mathscr{T}))(A \otimes I)^\dagger =: \Delta_\psi,$$

which is Hermitian and traceless. Hence $1/2\|\Delta_{\{\psi\}}\|_1 = max_{\{0 \leq P \leq I\}}\, Tr\,(P\Delta_{\{\psi\}})$. For every such P, using $Z - (\Omega - J(\mathscr{T})) \geq 0$, then $P \leq I$ and $Z \geq 0$,

$$\mathrm{Tr}\,(P\Delta_\psi) \leq \mathrm{Tr}\,(P(A \otimes I)Z(A \otimes I)^\dagger) \leq \mathrm{Tr}\,((A^\dagger A \otimes I)Z) = \mathrm{Tr}\,(A^\dagger A\,\mathrm{Tr}_Q\, Z) \leq \|\mathrm{Tr}_Q\, Z\|_\infty.$$

Taking the maximum over ψ proves (b). (c) is (a) and (b) together.

par *Does not imply*. The lemma is the standard primal–dual pair for the half-diamond distance to the identity; it is proved here, not imported from any cited sheet, and it does not imply any physical recovery protocol. par

Corollary. Let $0 \le t \le 1$.

• If $J(\mathscr{T}) \ge t\,\Omega$, then $Z = (1-t)\Omega$ is a dual witness and $\varepsilon(\mathscr{T}) \le 1 - t$.

• For every channel, $\varepsilon(\mathscr{T}) \ge 1 - F_e(\mathscr{T})$, with the Bell vector Φ_d as primal witness.

• If $J(\mathscr{T}) \ge F_e(\mathscr{T})\,\Omega$, then $(\Phi_d, (1 - F_e(\mathscr{T}))\Omega)$ is an exact dual certificate and $\varepsilon(\mathscr{T}) = 1 - F_e(\mathscr{T})$.

Proof. (a) $Z \ge 0$, and $Z - (\Omega - J(\mathscr{T})) = J(\mathscr{T}) - t\Omega \ge 0$. Moreover $\mathrm{Tr}_Q\,\Omega = I_E$, so $\mathrm{Tr}_Q\,Z = (1-t)I_E$. (b) is Lemma (a) with $\psi = \Phi_d$, since $\langle\Phi_d|(\mathrm{id}\otimes\mathscr{T})(\Phi_d)|\Phi_d\rangle = F_e(\mathscr{T})$. (c) combines (a) with $t = F_e(\mathscr{T})$ and (b).

par *Does not imply*. The corollary gives explicit constants only. It does not imply that every channel satisfies the Bell criterion of part (c). par

Cross-reference. The same lemma and corollary, with the same local proofs, are in the companion sheets I-C-Ex-CERT and I-C-Ex2-CERT (I-C-Ex-CERT-v1 lem:cert-sandwich, I-C-Ex-CERT-v1 cor:cert-bell; I-C-Ex2-CERT-v1 lem:cert-sandwich, I-C-Ex2-CERT-v1 cor:cert-bell; unchanged in their versions 2). Each sheet proves them independently; nothing here depends on the companions.

Setting: the classical-record interface

Definition. Let $Q \cong Q' \cong \mathbb{C}^d$, $d \ge 2$, with a fixed identification. A finite classical-output channel is $\mathscr{C}(\rho) = \sum_x \mathrm{Tr}\,(E_x\rho)\,\mathrm{proj}\,x$ with $E_x \ge 0$, $\sum_x E_x = I$ (the interface I-C-v2 eq:qc). A decoder is a channel $\mathscr{D}: R \to Q'$ on the record. A decoder *ignores the record* if $\mathscr{D}(Y) = \mathrm{Tr}\,(Y)\sigma'$ for a fixed state σ'; then $\mathscr{D}\mathscr{C} = \mathscr{R}_{\sigma'}$, $\mathscr{R}_{\sigma'}(\rho) = \mathrm{Tr}\,(\rho)\sigma'$.

par *Does not imply*. The definition fixes the finite interface of the parent. It does not model a horizon, a black hole or any physical shredder. par

Lemma. For every $\mathscr{C}$ and $\mathscr{D}$ as in Definition , $\mathscr{D}\mathscr{C}(\rho) = \sum_x \mathrm{Tr}\,(E_x\rho)\sigma_x$ with $\sigma_x = \mathscr{D}(\mathrm{proj}\,x)$. Consequently $F_e(\mathscr{D}\mathscr{C}) \le 1/d$ and $\varepsilon(\mathscr{D}\mathscr{C}) \ge 1 - 1/d$, where F_e and ε are as in Section .

Proof. The formula is linearity of $\mathscr{D}$. On Φ_d the classical channel gives $\sum_x \frac{1}{d}E_x^{T} \otimes \mathrm{proj}\,x$, so the composite gives $\sum_x \frac{1}{d}E_x^{T} \otimes \sigma_x$, a separable state. For states a, b, $\mathrm{Tr}\,[\Phi_d(a\otimes b)] = \frac{1}{d}\mathrm{Tr}\,(a^{T}b) \le 1/d$, and convexity gives $F_e \le 1/d$. The Bell vector is a primal witness: $\varepsilon \ge 1 - F_e$ (Lemma (a)).

par *Does not imply*. This re-proves the lower bound of I-C-v2 thm:A for local use; it does not import that proof and does not imply sharpness, which is Theorem . par

Record-ignoring decoders: the exact spectral law

A decoder $\mathscr{D}: R \to Q'$ *ignores the record* if $\mathscr{D}(Y) = \mathrm{Tr}\,(Y)\,\sigma'$ for a fixed state σ'. For every $\mathscr{C}$, the composite is then the replacement channel $\mathscr{R}_{\sigma'}$ of Definition .

Theorem. Let $\lambda_1, \ldots, \lambda_d$ be the eigenvalues of σ', with eigenvectors $e_1, \ldots, e_d$, and put

$$S = \sum_{i=1}^{d} \lambda_i^{-1} \in [d^2, \infty], \qquad 1/0 = \infty, \quad 1/\infty = 0.$$

Then

$$\varepsilon(\mathscr{R}_{\sigma'}) = 1 - \frac{1}{S}.$$

An exact dual certificate in the sense of Lemma (c) is:

• if $\sigma' > 0$: the Schmidt vector $\psi_p = \sum_i \sqrt{(p_i)}\,|f_i\rangle_E|e_i\rangle_Q$, for any orthonormal basis (f_i) of E, with $p_i = \lambda_i^{-1}/S$, and the dual operator $Z = (1 - 1/S)\,\Omega$;

• if $\lambda_k = 0$ for some k: the product vector $|f_k\rangle|e_k\rangle$ and $Z = \Omega$. Moreover $S \ge d^2$, with equality if and only if $\sigma' = I/d$. Hence the minimum error over record-ignoring decoders is exactly $1 - 1/d^2$, attained only by $\sigma' = I/d$.

Proof. The Choi operator is $J(\mathscr{R}_{\sigma^{'}}) = \sum_{i,j} |i\rangle\langle j| \otimes Tr\,(|i\rangle\langle j|)\sigma^{'} = I_E \otimes \sigma^{'}$.

Primal. For $\sigma^{'} > 0$, the output on ψ_p is $\rho_E \otimes \sigma^{'}$ with $\rho_E = \sum_i p_i |f_i\rangle\langle f_i|$. Its overlap with ψ_p is

$$\sum_{i,j} \sqrt{p_i p_j}\, \langle f_i|\rho_E|f_j\rangle\, \langle e_i|\sigma^{'}|e_j\rangle = \sum_i p_i^2 \lambda_i = \frac{1}{S^2}\sum_i \lambda_i^{-1} = \frac{1}{S}.$$

If $\lambda_k = 0$, the output on $|f_k\rangle|e_k\rangle$ is $|f_k\rangle\langle f_k| \otimes \sigma^{'}$, whose overlap with the input is $\langle e_k|\sigma^{'}|e_k\rangle = 0$.

Dual. If $\lambda_k = 0$, $Z = \Omega$ satisfies $Z \geq 0$ and $Z - (\Omega - J) = J \geq 0$, with $Tr_Q\, Z = I_E$. If $\sigma^{'} > 0$, then for every vector z the weighted Cauchy–Schwarz inequality gives

$$|\langle \Omega, z\rangle|^2 \leq \langle \Omega, (I \otimes \sigma^{'\wedge -1})\Omega\rangle\, \langle z, (I \otimes \sigma^{'})z\rangle = S\,\langle z, (I \otimes \sigma^{'})z\rangle,$$

because $\langle \Omega|(I \otimes B)|\Omega\rangle = Tr\, B$. Thus $J(\mathscr{R}_{\sigma^{'}}) = I \otimes \sigma^{'} \geq \Omega/S$, and Corollary (a) with $t = 1/S$ shows that $Z = (1 - 1/S)\Omega$ is a dual witness with $\|Tr_Q\, Z\|_\infty = 1 - 1/S$. Lemma (c) gives equality in both cases.

Finally $d = \sum_i \lambda_i^{1/2}\lambda_i^{-1/2} \leq (\sum_i \lambda_i)^{1/2}(\sum_i \lambda_i^{-1})^{1/2} = S^{1/2}$, with equality if and only if all λ_i are equal, that is $\sigma^{'} = I/d$.

par *Does not imply*. Theorem is a finite statement about replacement composites. The same exact law appears in Big01-A (Big01-A thm:Bexact), for decoders acting after a replacement channel that Big01-A attributes to placement beyond a horizon; there the Schmidt vector ψ_p is the same witness. That source record is not used as a premise: the proof above is self-contained. The theorem does not imply that a record-ignoring decoder models any horizon, and it derives no horizon, temperature, finite Hilbert space of a host, or Einstein dynamics. par

The full classical-record class

Theorem. For every finite classical-output channel $\mathscr{C}$ and every decoder $\mathscr{D}$, $\varepsilon(\mathscr{D}\mathscr{C}) \geq 1 - 1/d$ (Lemma). The pinching $\Delta(\rho) = \sum_j \mathrm{proj}\, j \rho \mathrm{proj}\, j$, obtained by measuring a basis and repreparing the outcome, has the exact dual certificate

$$(\Phi_d,\ Z_\Delta = (1 - \tfrac{1}{d})\Omega), \qquad J(\Delta) - \tfrac{1}{d}\,\Omega \geq 0.$$

Hence

$$\min_{\mathscr{C},\,\mathscr{D}} \varepsilon(\mathscr{D}\mathscr{C}) = 1 - \frac{1}{d},$$

and the minimum is certified from below by the Bell witness and from above by the explicit dual operator Z_{Δ}, without the triangle inequality over phase conjugations used in the parent (I-C-v2 thm:A).

Proof. $J(\Delta) = \sum_{i,j} |i\rangle\langle j| \otimes \Delta(|i\rangle\langle j|) = \sum_j \mathrm{proj}\, j \otimes \mathrm{proj}\, j$. On $V = \mathrm{span}\{|jj\rangle\}$, which contains $|\Omega\rangle$, $J(\Delta)$ is the identity and $\Omega = 11^{T}$ in the basis $(|jj\rangle)_j$, so $J(\Delta) - \Omega/d = I_V - 11^{T}/d$ is an orthogonal projector; both operators vanish on $V^{\perp}$. So $J(\Delta) \geq F_e(\Delta)\Omega$ with $F_e(\Delta) = 1/d$, and Corollary (c) applies. The lower bound for every composite is Lemma .

par *Does not imply*. The theorem certifies the optimum of the class. It does not imply that every decoded classical-record channel has error $1 - 1/d$; Proposition shows the opposite. par

Proposition. Let $X|j\rangle = |j + 1 \bmod d\rangle$.

• Δ and $\mathrm{Ad}_X \circ \Delta$ are decoded classical-record channels: the second decoder relabels the record before repreparing. Both send I/d to I/d, so their output spectra agree and give $S = d^2$, $1 - 1/S = 1 - 1/d^2$. Yet

$$\varepsilon(\Delta) = 1 - \frac{1}{d}, \qquad \varepsilon(\mathrm{Ad}_X \circ \Delta) = 1.$$

Neither equals $1 - 1/S$.

• For every $e \in [1 - 1/d,\ 1]$ there are $\mathscr{C}$ and $\mathscr{D}$ with $\varepsilon(\mathscr{D}\mathscr{C}) = e$. Every decoded classical-record channel has error in $[1 - 1/d,\ 1]$.

• Consequently no function of the output spectrum $\mathrm{spec}\,(\mathscr{D}\mathscr{C})(I/d)$ equals the error on the full classical-record class.

Proof. (a) The first error is Theorem . For the second, the input $|0\rangle_E|0\rangle_Q$ is sent to $\mathrm{proj}\, 0 \otimes \mathrm{proj}\, 1$, which is

orthogonal to it, so Lemma (a) gives error at least *1*; the dual witness $Z = \Omega$ gives at most *1*. (b) Fix the basis measurement $\mathscr{C}$. For $0 \le s \le 1$ the map $\mathscr{D}_s = (1 - s)\mathscr{D}_0 + s\,\mathrm{Ad}_X\,\mathscr{D}_0$, with $\mathscr{D}_0(\mathrm{proj}\,x) = \mathrm{proj}\,x$, is a decoder, and $\mathscr{D}_s\mathscr{C} = (1 - s)\Delta + s\,\mathrm{Ad}_X\,\Delta$. By the triangle inequality, $|\varepsilon(\mathscr{T}) - \varepsilon(\mathscr{T}')| \le d_\diamond(\mathscr{T}, \mathscr{T}')$, and $d_\diamond(\mathscr{D}_s\mathscr{C}, \mathscr{D}_{s'}\mathscr{C}) = |s - s'|\, d_\diamond(\Delta, \mathrm{Ad}_X\,\Delta) \le |s - s'|$. So the error is continuous in s; the intermediate value theorem and (a) give every $e \in [1 - 1/d, 1]$. The interval bound is Lemma and $\varepsilon \le 1$. (c) follows from (a).

par *Does not imply*. The proposition does not imply that the exact spectral law of Theorem fails; it fails only outside the record-ignoring subclass. par

Corollary. Inside the interface of Definition , with both values exactly certified,

$$\min_{\mathscr{C},\ \mathscr{D}\text{record-ignoring}} \varepsilon(\mathscr{D}\mathscr{C}) - \min_{\mathscr{C},\ \mathscr{D}} \varepsilon(\mathscr{D}\mathscr{C}) = (1 - \frac{1}{d^2}) - (1 - \frac{1}{d}) = \frac{d-1}{d^2} > 0.$$

The left minimum is certified by Theorem , and it does not depend on $\mathscr{C}$. The right minimum is certified by Theorem and is attained by the basis measurement with the identity relabelling.

Proof. Combine the two theorems.

par *Does not imply*. The gap is the same number that Big01-A records between its horizon floor $1 - 1/d^2$ and the shredder floor $1 - 1/d$ (Big01-A lem:shredder, Big01-A prop:shredsharp). Here it is obtained for the record-ignoring subclass of the I-C interface, with no horizon, causal structure, or host geometry. It does not imply that a horizon exists, that a horizon acts as a record-ignoring decoder, or that the host is anything other than prescribed. The host stays prescribed and the labels stay named. par

Version 2: classification, dephasing, thermal separation and the capacity reading

This section is new in version 2 (plan t258u). Everything in it is proved locally. Big01-A and I-C-Th are cited as source records only.

The error depends on the decoder's action on the record

Fix the basis measurement $\mathscr{C}_0(\rho) = \sum_x \langle x|\rho|x\rangle \mathrm{proj}\,x$ of Definition . A decoder is then given by its output states $\sigma_x = \mathscr{D}(\mathrm{proj}\,x)$, and

$$\mathscr{T}_\sigma(\rho) := \mathscr{D}\mathscr{C}_0(\rho) = \sum_x \langle x|\rho|x\rangle\, \sigma_x, \qquad q_x := \langle x|\sigma_x|x\rangle.$$

Call q_x the *record faithfulness* of the decoder at x. The decoder is *classical* if every σ_x is diagonal in the basis $(|y\rangle)$, that is $\sigma_x = \sum_y M_{yx}\mathrm{proj}\,y$ for a column-stochastic matrix M; then $q_x = M_{xx}$. Relabellings are the permutation matrices $M = P_\pi$.

Theorem. Put $S_q = \sum_x q_x^{-1} \in [d, \infty]$ (with $1/0 = \infty$, $1/\infty = 0$).

• *(Lower bound.)* $\varepsilon(\mathscr{T}_\sigma) \ge 1 - 1/S_q$ for every decoder.

• *(Upper bound.)* If every $\sigma_x > 0$, then $\varepsilon(\mathscr{T}_\sigma) \le 1 - 1/S_r$ with $S_r = \sum_x \langle x|\sigma_x^{-1}|x\rangle \ge S_q$.

• *(Exact law for classical decoders.)* If the decoder is classical, then $\varepsilon(\mathscr{T}_\sigma) = 1 - 1/\sum_x M_{xx}^{-1}$. When every $M_{xx} > 0$, an exact dual certificate is $(\psi_p, (1 - 1/S_q)\Omega)$ with $\psi_p = \sum_x \sqrt{(p_x)}|x\rangle_E|x\rangle_Q$, $p_x = q_x^{-1}/S_q$; if some $M_{kk} = 0$, it is $(|k\rangle|k\rangle, \Omega)$.

• *(Relabellings.)* $\varepsilon(\mathrm{Ad}_{P_\pi} \circ \Delta) = 1 - 1/d$ if π is the identity and $= 1$ otherwise; all relabellings have the same output $(\mathrm{Ad}_{P_\pi} \circ \Delta)(I/d) = I/d$.

• *(The path of Proposition .)* For $M_s = (1 - s)I + sP_X$, $0 \le s \le 1$, $\varepsilon(\mathscr{T}_\sigma) = 1 - (1 - s)/d$ exactly, with all outputs at I/d equal to I/d. So on classical decoders the error is a function of the diagonal (M_{xx}), the decoder's action on the record, and not of the output spectrum; for general decoders it is pinned between two functions of (σ_x).

Proof. The Choi operator is $J = \sum_x \mathrm{proj}\,x \otimes \sigma_x$. (a) If some $q_k = 0$, the input $|k\rangle|k\rangle$ goes to $\mathrm{proj}\,k \otimes \sigma_k$, whose overlap with the input is $q_k = 0$; Lemma (a) gives $\varepsilon \ge 1$. Otherwise the output on ψ_p is $\sum_x p_x \mathrm{proj}\,x \otimes \sigma_x$, with overlap $\sum_x p_x^2 q_x = S_q^{-2}\sum_x q_x^{-1} = 1/S_q$; apply Lemma (a). (b) Write a vector as $z = \sum_x |x\rangle \otimes z_x$. Then $\langle\Omega, z\rangle = \sum_x \langle x, z_x\rangle$ and $\langle z, Jz\rangle = \sum_x \langle z_x, \sigma_x z_x\rangle$. By Cauchy–Schwarz, $|\langle x, z_x\rangle| =$

$|\langle\sigma_x^{-1/2}x, \sigma_x^{1/2}z_x\rangle| \le \langle x|\sigma_x^{-1}|x\rangle^{1/2}\langle z_x, \sigma_{xz}x\rangle^{1/2}$, and once more over x, $|\langle\Omega, z\rangle|^2 \le S_r\langle z, Jz\rangle$. So $J \ge \Omega/S_r$ and Corollary (a) applies. Also $\langle x|\sigma_x^{-1}|x\rangle \ge \langle x|\sigma_x|x\rangle^{-1}$ by Cauchy–Schwarz, so $S_r \ge S_q$. (c) If some $M_{kk} = 0$, (a) gives $\varepsilon = 1$ and $Z = \Omega$ is a dual witness ($Z \ge \Omega - J$ since $J \ge 0$, and $\mathrm{Tr}_Q\,\Omega = I$). If all $M_{xx} > 0$: for a classical decoder $\langle z_x, \sigma_{xz}x\rangle = \sum_y M_{yx}|\langle y, z_x\rangle|^2 \ge M_{xx}|\langle x, z_x\rangle|^2$, so the argument of (b) gives $J \ge \Omega/S_q$ without invertibility; with (a), Lemma (c) applies. (d) P_{π} has $M_{xx} = 1$ at fixed points and 0 elsewhere; apply (c). Each relabelling sends I/d to I/d. (e) $M_{xx} = 1 - s$ for all x (for $d \ge 2$, X has no fixed point), so $S_q = d/(1 - s)$; apply (c). The output at I/d is I/d because M_s is doubly stochastic.

par *Does not imply*. The theorem classifies decoders on one fixed interface, the basis measurement. It does not imply an exact formula for non-classical decoders (only the two-sided bound (a),(b)) or for other measurements $\mathscr{C}$, and it does not model a horizon or any physical shredder. Check C9 corroborates (a)–(e) by semidefinite programming, including the path of check C3, whose six values $2/3, \ldots, 1$ are those of (e). par

The shredder floor as a named object, and complete dephasing

Definition. For the interface of Definition on $\mathbb{C}^d$, the *shredder floor* is $f_{sh}(d) = \min_{\mathscr{C}, \mathscr{D}} \varepsilon(\mathscr{D}\mathscr{C})$ and the *record-ignoring floor* is $f_{ri}(d) = \min_{\mathscr{C}, \mathscr{D}\text{record-ignoring}} \varepsilon(\mathscr{D}\mathscr{C})$.

par *Does not imply*. The names are interface quantities. ``Record-ignoring floor'' is used instead of Big01-A's ``horizon floor'': no horizon is modelled here. par

Theorem.

• $f_{sh}(d) = 1 - 1/d$ and $f_{ri}(d) = 1 - 1/d^2$, each with an exact dual certificate.

• Complete dephasing Δ attains $f_{sh}(d)$, with the exact dual certificate $(\Phi_d, (1 - 1/d)\Omega)$ of Theorem .

• Let a Hamiltonian H on $\mathbb{C}^d$ diagonal in the basis $(|j\rangle)$ and $\beta > 0$ be supplied, and let the bath be $\mathbb{C}^d$ with zero Hamiltonian, so its Gibbs state is I/d at every β. With $Z|j\rangle = \omega^j|j\rangle$, $\omega = e^{2\pi i/d}$, the unitary $U = \sum_a Z^a \otimes \mathrm{proj}\, a$ commutes with $H \otimes I$, and $\mathrm{Tr}_B\,[U(X \otimes I/d)U^{\dagger}] = \Delta(X)$. So Δ is a thermal operation in the sense of energy-conserving unitary plus Gibbs bath, and it attains the shredder floor.

Proof. (a) Theorems and . (b) is Theorem . (c) U is block diagonal in the bath basis with diagonal blocks Z^a, which commute with the diagonal H. Tracing out the bath gives $1/d\sum_a Z^aXZ^{-a}$, whose (j, k) entry is $X_{jk}1/d\sum_a \omega^{a(j-k)} = \delta_{jk}X_{jk}$.

par *Does not imply*. The thermal-operation model is a finite ancillary model with a supplied Hamiltonian and a supplied β; β plays no role in the output. It does not imply a reservoir or temperature of any host. The same statement is recorded in Big01-A (Big01-A thm:Bdephase(a), Big01-A prop:shredsharp); that source record is not used. Check C11 corroborates (c). par

A thermal replacement on the record stays strictly above the shredder floor

Theorem. Let a Hamiltonian H on $Q^{'} \cong \mathbb{C}^d$ with eigenvalues E_i, and $\beta > 0$, be supplied; put $\gamma_{\beta} = e^{-\beta H}/Z_{\beta}$ and $Z_{\pm\beta} = \sum_i e^{\mp\beta E_i}$. Let the decoder ignore the record and prepare γ_{β} (a thermal operation: swap with a bath copy of $(Q^{'}, H)$ in the state γ_{β}).

• $\varepsilon(\mathscr{R}_{\gamma_{\beta}}) = 1 - 1/(Z_{\beta}Z_{-\beta})$, with the exact dual certificate of Theorem : Schmidt weights $p_i \propto e^{\beta E_i}$ and $Z = (1 - 1/(Z_{\beta}Z_{-\beta}))\Omega$.

• $\varepsilon(\mathscr{R}_{\gamma_{\beta}}) \ge f_{ri}(d) > f_{sh}(d)$; the first inequality is an equality iff H is a multiple of I, and the excess over the shredder floor is at least $(d - 1)/d^2$.

• For the qubit Hamiltonian $H = \varepsilon\,\mathrm{proj}\, 1$ of the I-C-Th witness family (I-C-Th-v1 thm:underdetermination; source record), $\varepsilon(\mathscr{R}_{\gamma_{\beta}}) = 1 - 1/(2 + 2\cosh\beta\varepsilon)$.

Proof. The swap of system and bath commutes with $H \otimes I + I \otimes H$ and outputs the bath state, so the composite is $\mathscr{R}_{\gamma_{\beta}}$. (a) The eigenvalues of γ_{β} are $e^{-\beta E_i}/Z_{\beta} > 0$, so $S = \sum_i Z_{\beta}e^{\beta E_i} = Z_{\beta}Z_{-\beta}$; apply Theorem . (b) $S \ge d^2$ with equality iff $\gamma_{\beta} = I/d$ (Theorem), iff H is scalar at finite β; and $1 - 1/d^2 - (1 - 1/d) = (d - 1)/d^2 > 0$ for $d \ge 2$. (c) $Z_{\beta}Z_{-\beta} = (1 + e^{-\beta\varepsilon})(1 + e^{\beta\varepsilon}) = 2 + 2\cosh\beta\varepsilon$.

par *Does not imply*. The separation is a statement about finite supplied Hamiltonians and supplied β. It does not imply a temperature of any host, a thermal noise model of a horizon, or a reservoir; no temperature is derived. Big01-A records the same formula (Big01-A thm:Bthermal); there the thermal replacement is compared with its

horizon floor, here with the shredder floor. Check C10 corroborates (a) by semidefinite programming. par

Capacity reading of the gap

Definition. For fixed d, an *account* is a pair $a = (\mathscr{C}, \mathscr{D})$ as in Definition ; its *retained value* is $v(a) = 1 - \varepsilon(\mathscr{D}\mathscr{C})$. A *reorganization* is a map on accounts that keeps d and acts by any combination of: (i) a unitary pre-encoding $\mathscr{C} \mapsto \mathscr{C} \circ Ad_V$; (ii) a classical post-processing or relabelling of the record, absorbed into the decoder; (iii) a change of decoder; (iv) a classical coin mixing two accounts. A reorganization is *record-ignoring* if it maps record-ignoring accounts to record-ignoring accounts.

par *Does not imply*. This is the local pattern of Big01-A's reorganization monoid (Big01-A def:reorg), restricted to the finite interface; no host, causal structure or account space over a host is introduced. par

Theorem.

• The set of composites $\mathscr{D}\mathscr{C}$ of accounts is closed under every reorganization, and the set of composites of record-ignoring accounts (the replacement channels) under every record-ignoring reorganization.

• $\sup_a v(a) = 1/d$ and $\sup_{a \text{ record-ignoring}} v(a) = 1/d^2$; both suprema are attained and certified on both sides.

• Hence for every account a and every reorganization ρ, $v(\rho(a)) \le 1/d$; for every record-ignoring account and every record-ignoring reorganization, $v(\rho(a)) \le 1/d^2$. The largest gain in retained value obtainable by passing from a record-ignoring account to any account is exactly $1/d - 1/d^2 = (d-1)/d^2$, attained by reading the record with the identity relabelling.

Proof. (a) $\mathscr{C} \circ Ad_V$ is again a finite classical-output channel, with POVM $(V^\dagger E_x V)$; a classical post-processing of the record followed by a decoder is a decoder; a coin mixture of two accounts is the account with the union of the records, tagged by the coin outcome, and the blockwise decoder. Record-ignoring decoders stay record-ignoring under (i), (ii) and changes among record-ignoring decoders, and a coin mixture of two replacement composites $\mathscr{R}_{\sigma_1}, \mathscr{R}_{\sigma_2}$ is the replacement composite $\mathscr{R}_{(1-\lambda)\sigma_1 + \lambda\sigma_2}$, realized by a record-ignoring decoder. (b) is Theorem (a). (c) follows from (a) and (b); the gain is attained by passing from the decoder with output I/d to the basis measurement with the identity relabelling (Theorem).

par *Does not imply*. The reading is a statement about a finite interface: the gap $(d-1)/d^2$ is not a capacity of any host, horizon or physical channel, and no host is present. Reorganizations cannot exceed the shredder floor; that is the only sense in which they ``do not close" the gap. Big01-A's tick ceiling (Big01-A thm:tick, Big01-A thm:tickexact) is a pattern of the same type for a declared tick rule; it is not used. par

Version 3: Bell saturation, all optimal measure-and-prepare composites, and the corrected gap

This section is new in version 3 (plan t262u). Everything in it is proved locally from Sections –. Hawk07-A/B/C-CERT, PG-OB-1-CERT, En08-ST-DSS-CERT-v2 and Big01-A are cited as source records only; I-C-CERT-v2 is the previous version of this sheet.

The Bell-saturation criterion

Theorem. Let $\mathscr{T}$ be a channel on $\mathbb{C}^d$ and $F = F_e(\mathscr{T})$. The following are equivalent:

• $\varepsilon(\mathscr{T}) = 1 - F$;

• $J(\mathscr{T})|\Omega\rangle = dF|\Omega\rangle$;

• $J(\mathscr{T}) \ge F\,\Omega$. When they hold, $(\Phi_d, (1-F)\Omega)$ is an exact dual certificate. In particular unitary covariance is sufficient, but not necessary, for $\varepsilon = 1 - F_e$.

Proof. Put $P = |\Phi_d\rangle\langle\Phi_d| = \Omega/d$, $B = J(\mathscr{T})/d$ (a state) and $H = P - B$, the output difference on the Bell input; $Tr\, H = 0$ and $Tr\,(PH) = 1 - F$. Write $H = H_{+} - H_{-}$ with $H_{\pm} \ge 0$, $H_{+}H_{-} = 0$, so $Tr\, H_{+} = 1/2\|H\|_1$.

(i)⇒(ii). By the definition of the diamond norm, $1/2\|H\|_1 \le \varepsilon(\mathscr{T}) = 1 - F$. Hence

$$\operatorname{Tr} H_{+} = \tfrac{1}{2}\|H\|_1 \le 1 - F = \operatorname{Tr}(PH) = \operatorname{Tr}(PH_{+}) - \operatorname{Tr}(PH_{-}) \le \operatorname{Tr}(PH_{+}) \le \operatorname{Tr} H_{+},$$

so all are equalities: $Tr\,[(I-P)H_{+}] = 0$ and $Tr\,(PH_{-}) = 0$. Since $(I-P)H_{+}(I-P) \ge 0$ has trace zero,

$H_{+}^{1/2}(I - P) = 0$, so $H_{+} = PH_{+}P = (Tr\ H_{+})P = (1 - F)P$; likewise $H_{-}^{1/2}P = 0$, so $H_{-}|\Phi_d\rangle = 0$. Therefore $H|\Phi_d\rangle = (1 - F)|\Phi_d\rangle$, i.e. $B|\Phi_d\rangle = F|\Phi_d\rangle$, which is equivalent to (ii) because $J = dB$ and $|\Omega\rangle = \sqrt{(d)}|\Phi_d\rangle$.

(ii)⇒(iii). $J = J(\mathscr{T}) \geq 0$ is Hermitian with eigenvector $|\Omega\rangle$ (norm $\sqrt{(d)}$) and eigenvalue dF, so $J = F\Omega + J^{'}$ with $J^{'} = (I - P)J(I - P) \geq 0$.

(iii)⇒(i). Corollary (a) with $t = F$ gives $\varepsilon \leq 1 - F$, and Corollary (b) gives $\varepsilon \geq 1 - F$. The same two statements give the exact certificate. A unitarily covariant channel satisfies $(U \otimes U)J(U \otimes U)^{\dagger} = J$ for every unitary U, and $|\Omega\rangle$ spans the joint fixed vectors of these operators; since J commutes with all of them, it maps $|\Omega\rangle$ to a multiple of $|\Omega\rangle$, which is (ii). Non-covariant channels satisfying (ii) are given below.

par *Does not imply*. The criterion characterizes when the scalar $1 - F_e$ is the error; it does not give a scalar error law for channels that fail (ii), and it does not use the companion sheets, where the same criterion is proved independently. Check C14 corroborates the equivalence by semidefinite programming. par

All optimal measure-and-prepare composites

Theorem. Let $\mathscr{T}(X) = \sum_x Tr\ (E_x X)\sigma_x$ be a composite of Definition (Lemma), with POVM (E_x) and states σ_x. Then $\varepsilon(\mathscr{T}) = 1 - 1/d$ if and only if for every x with $E_x \neq 0$ there are a unit vector a_x and $w_x > 0$ with

$$\sigma_x = |a_x\rangle\langle a_x|, \qquad E_x = w_x|a_x\rangle\langle a_x|$$

(and then $\sum_{x:\ E_x \neq 0} w_x|a_x\rangle\langle a_x| = I$). So a fixed measurement admits a decoder at the shredder floor exactly when all its nonzero effects have rank one, and an attaining decoder must prepare the normalized effect. On the basis-measurement interface the unique attaining assignment is $\sigma_x = proj\ x$, i.e. the identity relabelling of Theorem (d).

Proof. By Lemma , $J(\mathscr{T}) = \sum_x E_x^{T} \otimes \sigma_x$ and $F_e(\mathscr{T}) = d^{-2}\sum_x Tr\ (E_x\sigma_x) \leq d^{-2}\sum_x Tr\ E_x = 1/d$. If $\varepsilon(\mathscr{T}) = 1 - 1/d$, then $1 - 1/d \geq 1 - F_e \geq 1 - 1/d$ (Corollary (b)), so $Tr\ [E_x(I - \sigma_x)] = 0$ for every x. As $E_x \geq 0$ and $I - \sigma_x \geq 0$, the range of E_x lies in the eigenspace of σ_x for eigenvalue 1. For a state that eigenspace is nonzero only if $\sigma_x = |a_x\rangle\langle a_x|$ is pure, and then it is $\mathbb{C}a_x$; hence $E_x = w_x|a_x\rangle\langle a_x|$ with $w_x = Tr\ E_x > 0$. Conversely, under the condition, $\mathscr{T}(X) = \sum_x w_x\langle a_x rvert X\ |a_x\rangle|a_x\rangle\langle a_x|$ and $J(\mathscr{T}) = \sum_x w_x|\bar{a}_x\rangle\langle\bar{a}_x| \otimes |a_x\rangle\langle a_x|$, where $\bar{a}_x$ has the complex-conjugate coordinates. Since $|\bar{a}_x\rangle|a_x\rangle = (I \otimes |a_x\rangle\langle a_x|)|\Omega\rangle$ and $\langle\bar{a}_x \otimes a_x|\Omega\rangle = \sum_i a_{x,\,i}\bar{a}_{x,\,i} = 1$,

$$J(\mathscr{T})|\Omega\rangle = (I \otimes \sum_x w_x|a_x\rangle\langle a_x|)|\Omega\rangle = |\Omega\rangle = d \cdot \tfrac{1}{d}|\Omega\rangle,$$

and $F_e = 1/d$. Theorem gives $\varepsilon = 1 - 1/d$ with the exact certificate $(\Phi_d, (1 - 1/d)\Omega)$. For $E_x = proj\ x$ the condition forces $a_x = |x\rangle$ up to phase.

par *Does not imply*. The theorem classifies the attainers of the shredder floor only. It does not give the error of a non-attaining composite, and a rank-one measurement does not by itself fix the error: the decoder must prepare the normalized effects. Check C16 corroborates both directions numerically. par

Corollary. On the basis-measurement interface, $\varepsilon(\mathscr{T}_{\sigma}) = 1 - F_e(\mathscr{T}_{\sigma})$ holds if and only if there is one $q \in [0, 1]$ with $\sigma_x|x\rangle = q|x\rangle$ for every x. Then $F_e(\mathscr{T}_{\sigma}) = q/d$ and $\varepsilon(\mathscr{T}_{\sigma}) = 1 - q/d$, which equals the lower bound $1 - 1/S_q$ of Theorem (a). For $d \geq 3$ and $0 < q < 1$ the outputs

$$\sigma_x = q\,\mathrm{proj}\,x + (1 - q)|v_x\rangle\langle v_x|, \qquad |v_x\rangle = (d - 1)^{-1/2} \sum_{y \neq x} |y\rangle,$$

are not diagonal, so the decoder is not classical, and $\mathscr{T}_{\sigma}$ is not unitarily covariant.

Proof. $J(\mathscr{T}_{\sigma}) = \sum_x proj\ x \otimes \sigma_x$, so $J|\Omega\rangle = \sum_x |x\rangle \otimes \sigma_x|x\rangle$, and (ii) of Theorem reads $\sigma_x|x\rangle = dF|x\rangle$ for all x ; put $q = dF$. Then $q_x = q$ for all x and $1 - 1/S_q = 1 - q/d$. The displayed σ_x satisfy $\sigma_x|x\rangle = q|x\rangle$ because $v_x \perp x$, and they have nonzero off-diagonal entries $(1 - q)/(d - 1)$ between distinct $y, y^{'} \neq x$ when $d \geq 3$. Non-covariance: $\mathscr{T}_{\sigma}$ annihilates $|x\rangle\langle y|$ for $x \neq y$. Let U act as $|x\rangle \mapsto (|x\rangle + |y\rangle)/\sqrt{(2)}$, $|y\rangle \mapsto (|x\rangle - |y\rangle)/\sqrt{(2)}$ and fix the other basis vectors; then $U(proj\ x - proj\ y)U^{\dagger} = |x\rangle\langle y| + |y\rangle\langle x|$. A covariant channel would give $\mathscr{T}_{\sigma}(proj\ x - proj\ y) = U^{\dagger}\mathscr{T}_{\sigma}(|x\rangle\langle y| + |y\rangle\langle x|)U = 0$, i.e. $\sigma_x = \sigma_y$, which fails.

par *Does not imply*. This is a new exact subclass, not a closed formula for every non-classical decoder; outside the

eigenvector condition only the two-sided bounds of Theorem are proved. Check C17 corroborates the value and the non-covariance. par

The gap, with the correct quantifiers

Proposition. Put $g_d = 1/d - 1/d^2 = (d-1)/d^2$.

• On the basis-measurement interface, the best record-ignoring retained value is $1/d^2$, the best retained value is $1/d$, and their difference is g_d.

• For a record-ignoring decoder with output σ, put $S_{\sigma} = \sum_i \lambda_i(\sigma)^{-1}$ ($1/S_{\sigma} = 0$ if σ is singular). The largest increase of retained value obtainable by changing its decoder on the basis-measurement interface is $1/d - 1/S_{\sigma}$; it equals g_d only for $\sigma = I/d$.

• The supremum of these increases over all record-ignoring starting decoders is $1/d$, attained exactly when σ is singular.

• For every finite classical-output channel $\mathscr{C}$, put $v_{*}(\mathscr{C}) = 1 - \inf_{\mathscr{D}} \varepsilon(\mathscr{D}\mathscr{C})$. Then $0 \le v_{*}(\mathscr{C}) - 1/d^2 \le g_d$; the upper endpoint is attained by the basis measurement (more generally, by every measurement with rank-one nonzero effects), and the lower endpoint by a constant record.

Proof. (a) is Theorem (b). (b) The starting value is $1/S_{\sigma}$ (Theorem); every decoder gives at most $1/d$ (Lemma), and the identity relabelling gives $1/d$ (Theorem (d)). $1/d - 1/S_{\sigma} = g_d$ iff $S_{\sigma} = d^2$ iff $\sigma = I/d$ (Theorem). (c) $1/S_{\sigma} \ge 0$ with equality iff σ is singular. (d) Preparing I/d gives $v_{*} \ge 1/d^2$; Lemma gives $v_{*} \le 1/d$; Theorem gives $v_{*} = 1/d$ for rank-one effects. A constant record $\mathscr{C}(\rho) = \mathrm{Tr}\,(\rho)|x_0\rangle\langle x_0|$ makes every composite a replacement channel, so $v_{*} = 1/d^2$ by Theorem .

par *Does not imply.* The proposition is about a finite interface. It does not imply a capacity of a host or a horizon, and it does not monotonically order individual accounts: changing a decoder can lower the retained value. Check C18 computes v_{*} for five measurements by one semidefinite program over the decoder outputs. par

Correction to version 2 (v2 error). The last sentence of Theorem (c), ``the largest gain in retained value obtainable by passing from a record-ignoring account to any account is exactly $(d-1)/d^2$'', and item (iv) of the version-2 abstract are too strong if the starting account is arbitrary: by Proposition (b),(c) the increase from a record-ignoring account with output σ is $1/d - 1/S_{\sigma}$, which is $(d-1)/d^2$ only for $\sigma = I/d$ and approaches $1/d$ for singular σ. The correct statements are: $(d-1)/d^2$ is the difference between the *best* record-ignoring and the *best* unrestricted retained values, equivalently the increase from the *best* record-ignoring account. Parts (a), (b) and the first sentence of (c) of Theorem are unaffected. The version-2 file is not edited.

Documentary comparisons

Hawk07 and PG replacement certificates. Hawk07-A-CERT-v2 thm:s2-exact (identically Hawk07-B-CERT-v2 thm:s2-exact, Hawk07-C-CERT-v2 thm:s2-exact, PG-OB-1-CERT-v2 thm:s2-exact) states $d_{\diamond}(\mathscr{R}_{\sigma}, \mathrm{id}) = 1 - 1/\Xi(\sigma)$ in the half-diamond normalization, the value of Theorem . Its primal witness is the same Schmidt vector. Its dual witness is different: in an eigenbasis of σ, an operator $Y = \sum_{i \ne j} \lambda_i |ij\rangle\langle ij| \oplus (D_{\lambda} - \Xi^{-1} 1 1^{T})$ with $Y \ge \sigma \otimes I - \Omega$, written output-first and for the Choi operator of $\mathscr{R}_{\sigma} - \mathrm{id}$, i.e. with the opposite ordering and sign to Lemma (b). It is not of the rank-one form $(1-t)\Omega$. Both duals have partial trace $(1 - 1/\Xi)I$. The rank-one dual $(1 - 1/S)\Omega$ of Theorem is the one of this sheet. Neither is a Page curve.

Big01-A. Big01-A prop:shredsharp displays the same number $(1 - d^{-2}) - (1 - d^{-1}) = (d-1)/d^2$. Here it is an optimal-account gap inside a finite classical-record interface (Proposition (a)); the numerical equality does not identify the interface with a horizon or make a retained value a capacity of a host.

En08. En08-ST-DSS-CERT-v2 thm:EC-reorg keeps a lattice ceiling under reorganizations that preserve a declared class (lattice-node centres) and shows that it fails for an off-lattice centre. The reorganization ceiling $1/d$ of Theorem likewise holds for operations that preserve the declared interface. The lattice statement controls hull data; this one controls channel errors. Neither derives a finite Hilbert space of a prescribed host.

Typed cross-reference. The replacement sandwich is proved locally. The Hawk07 and PG replacement certificates share its value and its primal witness, not its dual operator. Big01-A prop:shredsharp shares its optimal-gap number, not an identified physical channel.

Version 4: sign conventions, the best rank-one dual, and the plan-t266u readings

This section is new in version 4 (plan t266u). Everything in it is proved locally from Sections –. Hawk07-A/B/C-CERT (v3 cited; v2 also read), PG-OB-1-CERT-v2, En08-ST-DSS-CERT-v3 and Big01-A are cited as source records only; I-C-CERT-v3 is the previous version of this sheet.

Sign conventions and the best rank-one dual

Throughout this subsection $\mathscr{T}$is a channel on $\mathbb{C}^d$, $J = J(\mathscr{T})$ is its Choi operator in the order $E \otimes Q$ of Section , and $J_{\Delta} := J - \Omega$ is the Choi operator of $\mathscr{T} - id$. The swap $F|e\rangle|q\rangle = |q\rangle|e\rangle$ fixes Ω, and $F(\cdot)F$ turns an operator written with the output factor first into one written in the order $E \otimes Q$, carrying the partial trace over the output factor to Tr_Q.

Lemma. For an operator W on $E \otimes Q$ put $Z = W - J_{\Delta} = W + \Omega - J$. Then

$$(W \geq 0 \text{ and } W \geq J_{\Delta}) \Leftrightarrow (Z \geq 0 \text{ and } Z \geq \Omega - J), \qquad \mathrm{Tr}_Q Z = \mathrm{Tr}_Q W.$$

Hence every $W \geq 0$ with $W \geq J_{\Delta}$ gives $\varepsilon(\mathscr{T}) \leq \|\mathrm{Tr}_Q W\|_{\infty}$, and the two witness sets, $\{W \geq 0, W \geq J_{\Delta}\}$ and $\{Z \geq 0, Z \geq \Omega - J\}$ of Lemma (b), have the same infimum of $\|\mathrm{Tr}_Q \cdot\|_{\infty}$; $W \mapsto W - J_{\Delta}$ is a bijection between them that preserves Tr_Q.

Proof. $W \geq J_{\Delta}$ is $Z \geq 0$, and $W \geq 0$ is $Z \geq -J_{\Delta} = \Omega - J$. Since $\mathrm{Tr}_Q J = I_E = \mathrm{Tr}_Q \Omega$, $\mathrm{Tr}_Q J_{\Delta} = 0$, so $\mathrm{Tr}_Q Z = \mathrm{Tr}_Q W$. The bound is Lemma (b) applied to Z. The inverse map is $Z \mapsto Z + J_{\Delta}$.

par *Does not imply*. The lemma is bookkeeping between two sign conventions for one semidefinite bound. It does not say that witnesses written in the two conventions are equal operators, it does not use any companion sheet, and it implies no recovery protocol. Check C21 corroborates the bijection and the equality of the two optima. par

Theorem. Put $t_*(\mathscr{T}) = \sup\{t \geq 0 : J \geq t\,\Omega\}$, and let J^+ be the Moore–Penrose inverse of J.

• $t_*(\mathscr{T}) = 1/\langle\Omega|J^+|\Omega\rangle$ if $|\Omega\rangle$ lies in the range of J, and $t_*(\mathscr{T}) = 0$ otherwise; the supremum is attained.

• $t_*(\mathscr{T}) \leq F_e(\mathscr{T}) \leq 1$ and $\varepsilon(\mathscr{T}) \leq 1 - t_*(\mathscr{T})$: among dual witnesses of the form $(1 - t)\Omega$, the best value is $1 - t_*$.

• $t_*(\mathscr{T}) = F_e(\mathscr{T})$ if and only if the equivalent conditions of Theorem hold; then $\varepsilon(\mathscr{T}) = 1 - t_*$ with the Bell primal witness.

• *(Replacement channels.)* Let $\mathscr{R}_{\sigma}(X) = \mathrm{Tr}\,(X)\sigma$, with eigenvalues λ_i of σ and $S_{\sigma} = \sum_i \lambda_i^{-1}$ ($1/S_{\sigma} = 0$ if σ is singular). Then $t_*(\mathscr{R}_{\sigma}) = 1/S_{\sigma}$, and $(\psi_{\sigma}, (1 - 1/S_{\sigma})\Omega)$ is an exact dual certificate, with $\psi_{\sigma} = \sum_i \sqrt{(p_i)}|i\rangle|i\rangle$ in an eigenbasis of σ (taken as the basis defining Ω), $p_i = \lambda_i^{-1}/S_{\sigma}$ if $\sigma > 0$, and $\psi_{\sigma} = |k\rangle|k\rangle$ with $\lambda_k = 0$ otherwise. So $\varepsilon(\mathscr{R}_{\sigma}) = 1 - 1/S_{\sigma}$, while $t_*(\mathscr{R}_{\sigma}) < F_e(\mathscr{R}_{\sigma}) = 1/d^2$ unless $\sigma = I/d$: the best rank-one dual can be exact without Bell saturation.

Proof. (a) We use: *for $J \geq 0$ and a vector v, $J \geq |v\rangle\langle v|$ iff $v \in \mathrm{ran}\,J$ and $\langle v|J^+|v\rangle \leq 1$.* If $J \geq |v\rangle\langle v|$ and $Jx = 0$, then $0 \leq |\langle x, v\rangle|^2 \leq \langle x|J|x\rangle = 0$, so $v \perp \ker J$, i.e. $v \in \mathrm{ran}\,J$; with $x = J^+v$, $\langle v|J^+|v\rangle^2 = |\langle x, v\rangle|^2 \leq \langle x|J|x\rangle = \langle v|J^+JJ^+|v\rangle = \langle v|J^+|v\rangle$, so $\langle v|J^+|v\rangle \leq 1$. Conversely, if $v = JJ^+v$ then for every x, by Cauchy–Schwarz for the positive form $\langle\cdot|J|\cdot\rangle$, $|\langle x, v\rangle|^2 = |\langle x|J|J^+v\rangle|^2 \leq \langle x|J|x\rangle\,\langle v|J^+JJ^+|v\rangle = \langle x|J|x\rangle\,\langle v|J^+|v\rangle \leq \langle x|J|x\rangle$. Apply this to $v = \sqrt{(t)}\,|\Omega\rangle$: if $|\Omega\rangle \notin \mathrm{ran}\,J$ only $t = 0$ is allowed; otherwise $\langle\Omega|J^+|\Omega\rangle > 0$ (J^+ is positive definite on $\mathrm{ran}\,J$) and the allowed t form the closed interval $[0, 1/\langle\Omega|J^+|\Omega\rangle]$. (b) $J \geq t_*\Omega$ gives $d^2F_e = \langle\Omega|J|\Omega\rangle \geq t_*d^2$. Corollary (a) with $t = t_*$ gives $\varepsilon \leq 1 - t_*$, and $(1 - t)\Omega$ is a witness exactly when $J \geq t\Omega$. (c) If $t_* = F_e$, then $J \geq F_e\,\Omega$, which is (iii) of Theorem . Conversely (iii) gives $t_* \geq F_e$, hence $t_* = F_e$ by (b); Theorem gives $\varepsilon = 1 - F_e$ with the Bell primal. (d) $J(\mathscr{R}_{\sigma}) = \sum_{i,j} |i\rangle\langle j| \otimes \mathrm{Tr}\,(|i\rangle\langle j|)\sigma = I \otimes \sigma$. Its range is $\mathbb{C}^d \otimes \mathrm{ran}\,\sigma$, which contains $|\Omega\rangle = \sum_j |j\rangle|j\rangle$ iff $\mathrm{ran}\,\sigma = \mathbb{C}^d$. If $\sigma > 0$, $J^+ = I \otimes \sigma^{-1}$ and $\langle\Omega|J^+|\Omega\rangle = \mathrm{Tr}\,\sigma^{-1} = S_{\sigma}$; otherwise $t_* = 0 = 1/S_{\sigma}$. So $Z = (1 - 1/S_{\sigma})\Omega$ is a witness with $\mathrm{Tr}_Q Z = (1 - 1/S_{\sigma})I$. Primal: if $\sigma > 0$, $(id \otimes \mathscr{R}_{\sigma})(|\psi_{\sigma}\rangle\langle\psi_{\sigma}|) = \mathrm{diag}\,(p) \otimes \sigma$, whose overlap with ψ_{σ} is $\sum_i p_i^2\lambda_i = S_{\sigma}^{-2}\sum_i \lambda_i^{-1} = 1/S_{\sigma}$; if $\lambda_k = 0$, the overlap of $\mathrm{proj}\,k \otimes \sigma$ with $|k\rangle|k\rangle$ is $\lambda_k = 0$. Lemma (c) gives exactness. Finally $F_e(\mathscr{R}_{\sigma}) = \langle\Omega|I \otimes \sigma|\Omega\rangle/d^2 = 1/d^2$ and $d^2 = (\sum_i \lambda_i^{1/2}\lambda_i^{-1/2})^2 \leq \sum_i \lambda_i \sum_i \lambda_i^{-1} = S_{\sigma}$, with equality iff all λ_i are equal.

par *Does not imply*. The theorem identifies the best certificate within one family of duals. It does not say that the best rank-one dual is exact in general (for generic channels $1 - t_*$ is strictly above the error; check C23), and it

gives no closed error formula outside (c) and (d). Check C23 corroborates (a), (b) and (d). par

Corollary. Let σ have eigenvalues λ_i in the basis $|i\rangle$ that defines Ω, $S = S_{\sigma}$, $V = span\{|i\rangle|i\rangle\}$, and define, in the order $E \otimes Q$,

$$Y_\sigma = \sum_{j \neq i} \lambda_i \, \mathrm{proj}\, j \otimes \mathrm{proj}\, i \; + \sum_{i,i'} (\lambda_i \delta_{ii'} - S^{-1}) |ii\rangle\langle i' i'|.$$

Then $Y_{\sigma} - J_{\Delta}(\mathscr{R}_{\sigma}) = (1 - 1/S)\,\Omega$. Consequently $Y_{\sigma} \geq 0$, $Y_{\sigma} \geq J_{\Delta}(\mathscr{R}_{\sigma})$, $Tr_Q\, Y_{\sigma} = (1 - 1/S)I$, and under the bijection of Lemma the witness Y_{σ} corresponds exactly to the rank-one witness $(1 - 1/S)\Omega$ of Theorem (d). If $\sigma > 0$ and $d \geq 2$, Y_{σ} has rank at least $d(d-1) \geq 2$, so the two witnesses are different operators.

Proof. $J_{\Delta}(\mathscr{R}_{\sigma}) = I \otimes \sigma - \Omega$ with $I \otimes \sigma = \sum_{j,i} \lambda_i \mathrm{proj}\, j \otimes \mathrm{proj}\, i$. All three of Y_{σ}, $I \otimes \sigma$ and Ω are reduced by V and $V^{\perp} = span\{|j\rangle|i\rangle : j \neq i\}$. On $V^{\perp}$, $Y_{\sigma} = I \otimes \sigma$ and $\Omega = 0$, so $Y_{\sigma} - J_{\Delta} = 0$. On V, in the basis $|ii\rangle$, $Y_{\sigma} = D_{\lambda} - S^{-1}11^T$, $I \otimes \sigma = D_{\lambda}$ and $\Omega = 11^T$, so $Y_{\sigma} - J_{\Delta} = (1 - S^{-1})11^T = (1 - 1/S)\Omega$. Now $Z = (1 - 1/S)\Omega$ satisfies $Z \geq 0$ and $Z \geq \Omega - I \otimes \sigma$ (the latter is $I \otimes \sigma \geq \Omega/S$, Theorem (a),(d)); Lemma gives the three properties of $Y_{\sigma} = Z + J_{\Delta}$. On $V^{\perp}$, Y_{σ} is diagonal with the $d(d-1)$ entries $\lambda_i > 0$.

par *Does not imply*. Written output-first, Y_{σ} is the operator displayed in Hawk07-A-CERT-v3 thm:s2-exact(b) and PG-OB-1-CERT-v2 thm:s2-exact(b) (source records; the formula is restated and verified here, not imported), and $(1 - 1/S)\Omega$ is the witness of PG-OB-1-CERT-v2 thm:v2-Z. The corollary makes precise in which sense these are ``the same dual certificate'': same value, same primal, corresponding duals under a convention change; not the same operator. No Page curve, interior or physical recovery protocol is implied. Check C22 corroborates the identity, including singular σ. par

The basis-measurement bound is the best rank-one dual

Corollary. In the notation of Theorem , let σ_x^{+} be the Moore–Penrose inverse of σ_x.

• If $|x\rangle \in ran\, \sigma_x$ for every x, then $t_{*}(\mathscr{T}_{\sigma}) = 1/\sum_x \langle x|\sigma_x^{+}|x\rangle$ and $\varepsilon(\mathscr{T}_{\sigma}) \leq 1 - 1/\sum_x \langle x|\sigma_x^{+}|x\rangle$. If $|x\rangle \notin ran\, \sigma_x$ for some x, then $t_{*}(\mathscr{T}_{\sigma}) = 0$ and no witness $(1-t)\Omega$ certifies an error below 1.

• If every $\sigma_x > 0$, $t_{*}(\mathscr{T}_{\sigma}) = 1/S_r$: the upper bound of Theorem (b) is exactly the best rank-one dual value, and (a) extends it to singular outputs with $|x\rangle \in ran\, \sigma_x$.

• $t_{*}(\mathscr{T}_{\sigma}) \leq 1/S_q$, with equality if and only if $\sigma_x|x\rangle = q_x|x\rangle$ for every x (all $q_x > 0$), or some $q_x = 0$. In the first case $\varepsilon(\mathscr{T}_{\sigma}) = 1 - 1/S_q$ exactly, with the certificate $(\psi_p, (1 - 1/S_q)\Omega)$ of Theorem (c).

Proof. $J = \sum_x \mathrm{proj}\, x \otimes \sigma_x$ is block diagonal, so $J^{+} = \sum_x \mathrm{proj}\, x \otimes \sigma_x^{+}$ and $ran\, J = \bigoplus_x |x\rangle \otimes ran\, \sigma_x$; $|\Omega\rangle = \sum_x |x\rangle|x\rangle$ lies in it iff $|x\rangle \in ran\, \sigma_x$ for all x, and then $\langle\Omega|J^{+}|\Omega\rangle = \sum_x \langle x|\sigma_x^{+}|x\rangle$. Theorem (a),(b) give (a), and (b) is the case $\sigma_x^{+} = \sigma_x^{-1}$. (c) If some $q_x = 0$, then $|x\rangle \notin ran\, \sigma_x$ (else $\langle x|\sigma_x|x\rangle > 0$), so $t_{*} = 0 = 1/S_q$. Otherwise, for $|x\rangle \in ran\, \sigma_x$, Cauchy–Schwarz gives $1 = |\langle x, x\rangle|^2 \leq \langle x|\sigma_x|x\rangle\langle x|\sigma_x^{+}|x\rangle$, i.e. $\langle x|\sigma_x^{+}|x\rangle \geq q_x^{-1}$, with equality iff $\sigma_x^{1/2}|x\rangle \parallel (\sigma_x^{+})^{1/2}|x\rangle$, i.e. iff $|x\rangle$ is an eigenvector of σ_x, and then its eigenvalue is q_x. Summing, $1/t_{*} \geq S_q$, with equality iff every $|x\rangle$ is an eigenvector. If some $|x\rangle \notin ran\, \sigma_x$ while all $q_x > 0$, then $t_{*} = 0 < 1/S_q$. In the eigenvector case, $t_{*} = 1/S_q$ and Theorem (a) give $\varepsilon = 1 - 1/S_q$; the primal ψ_p is that of the proof of Theorem (a).

par *Does not imply*. (c) recovers Theorem (c) for classical decoders and Corollary (equal q_x), and extends the exact law to decoders with $\sigma_x|x\rangle = q_x|x\rangle$ and unequal q_x. It is not a closed formula for decoders outside the eigenvector condition, where in the instances of check C23 the error lies strictly below the best rank-one value. Check C23 corroborates (a). par

Readings required by plan t266u

PG-OB-1-CERT-v2. PG-OB-1-CERT-v2 thm:s2-exact (statement identical to Hawk07-A-CERT-v3 thm:s2-exact) has the value $1 - 1/\Im$ and the Schmidt primal of Theorem (d) and of Theorem ; its dual is the block operator Y_{σ} of Corollary , in the other sign convention. The same sheet's PG-OB-1-CERT-v2 thm:v2-Z gives the witness $Z_{\sigma} = (1 - 1/\Im)\Omega$ in the convention of this sheet (verified in the file; label not named by the plan). By Corollary , Y_{σ} and Z_{σ} correspond under Lemma . No Page curve.

Hawk07 recovery certificates. The plan's ``thm:s2-exact of A/B/C gives the same dual certificate" is qualified: same value and same primal; the dual operators differ in form (block versus rank one) and sign convention, and they correspond exactly under the convention translation (Corollary). The statements of thm:s2-exact and prop:s2-ceiling are unchanged between Hawk07-A/B/C-CERT v2 and v3 (checked); v3 is cited. Hawk07-A-CERT-v3 prop:s2-ceiling is an entropy bound $S(\rho_R) \le \min(\log d_R, \log d_{R^c} + S(\rho))$, not ``the same hard ceiling" as the shredder floor $1 - 1/d$: both are fixed-resource bounds, of different quantities.

En08-ST-DSS-CERT-v3. En08-ST-DSS-CERT-v3 lem:EC-sandwich is a primal/dual bracket for the hull deficit τ_j of a lattice ball, exact when both witnesses are optimal (as for the witnesses of En08-ST-DSS-CERT-v3 thm:EC-dual(b)). It is a linear-inequality certificate for a convex hull; Lemma is a semidefinite certificate for a half-diamond distance. Both carry written witnesses; nothing is transferred, and there is no Page curve or interior. (En08-ST-DSS-CERT-v4 is delivered; its lem:EC-sandwich, thm:EC-dual and thm:EC-reorg are unchanged from v3.)

Big01-A shredder gap. Big01-A prop:shredsharp displays $(1 - d^{-2}) - (1 - d^{-1}) = (d - 1)/d^2$ as the difference of two exact *floors*: the minimal error over entanglement-breaking channels, $1 - 1/d$, and the horizon floor $1 - 1/d^2$ (Big01-A thm:Bexact). It does not record a gain from reading a record from every start. On this sheet's interface the two floors are the minimal error of record-ignoring composites, $1 - 1/d^2$ (Theorem), and of all composites, $1 - 1/d$ (Theorem); so the number agrees with the best-versus-best reading of Proposition (a), and the correction of Proposition (b),(c) stays. Neither sheet derives a physical separation between a shredder and a horizon.

Open item. The exact error of non-classical decoders outside the eigenvector condition of Corollary (c) stays open: each finite instance is a semidefinite program, and no closed formula is proved. No physical identification of the interface is made.

Typed cross-reference. Lemma is a source record for the dual-certificate pattern only. The Hawk07 and PG replacement certificates share its value and primal; their block dual corresponds to its rank-one dual under a sign-convention change; En08 shares only the existence of written witnesses; Big01-A shares the number $(d - 1)/d^2$ as a difference of floors.

Version 5: exactness of the best rank-one dual, and the companion readings

This section is new in version 5 (request t267u; no plan was attached). Everything in it is proved locally from the earlier sections, including Section ; computer-assisted checks are labelled as such. PG-OB-1-CERT-v3, Hawk07-A/B/C-CERT-v3 and En08-ST-DSS-CERT-v4 are cited by sha256 as source records only; I-C-CERT-v4 is the previous version of this sheet. Other streams are concurrently building Hawk07 CERT version 4, the OB CERT sheets version 4 and DSS CERT version 5; their files were not read, and any statement here about the companions refers only to the delivered versions cited by sha256.

When the best rank-one dual is exact

For an operator M on $E = \mathbb{C}^d$ write $\mathrm{vec}(M) = (M \otimes I_Q)|\Omega\rangle = \sum_{i,j} M_{ij}|i\rangle|j\rangle$, in the basis that defines Ω. Then vec is a linear bijection, $\langle \mathrm{vec}(M), \mathrm{vec}(N)\rangle = \mathrm{Tr}(M^\dagger N)$, $\langle \Omega|\mathrm{vec}(M)\rangle = \mathrm{Tr}\, M$, and $(X \otimes Y)\mathrm{vec}(M) = \mathrm{vec}(XMY^T)$.

Theorem. Let $\mathscr{T}$ be a channel on $\mathbb{C}^d$, $J = J(\mathscr{T})$, and $t_* = t_*(\mathscr{T})$ as in Theorem .

- $\varepsilon(\mathscr{T}) = 1 - t_*$ if and only if there is a state R on E with $(J - t_*\Omega)\mathrm{vec}(R) = 0$.
- For such R, $\psi_R = \mathrm{vec}(R^{1/2})$ is a unit vector and $(\psi_R, (1 - t_*)\Omega)$ is an exact dual certificate.
- If $J > 0$, then $\ker(J - t_*\Omega) = \mathbb{C}\, J^{-1}|\Omega\rangle$. Define $M_{\mathscr{T}}$ by $\mathrm{vec}(M_{\mathscr{T}}) = J^{-1}|\Omega\rangle$. Then $\mathrm{Tr}\, M_{\mathscr{T}} = 1/t_*$, and $\varepsilon(\mathscr{T}) = 1 - t_*$ if and only if $M_{\mathscr{T}}$ is Hermitian and positive semidefinite; then $R = t_* M_{\mathscr{T}}$.

Proof. Put $a = 1 - t_*$ and $K_0 = J - t_*\Omega \ge 0$ (Theorem (a)). If $a = 0$, then $J \ge \Omega$ and $\mathrm{Tr}\, J = d = \mathrm{Tr}\, \Omega$ give $J = \Omega$, $\mathscr{T} = \mathrm{id}$, $\varepsilon = 0$, and $R = I/d$ satisfies (a); assume $a > 0$. For $|\psi\rangle = (A \otimes I)|\Omega\rangle$ with $\mathrm{Tr}\, A^\dagger A = 1$ the proof of Lemma (b) gives $\Delta_\psi = (A \otimes I)(\Omega - J)(A \otimes I)^\dagger$, and since $(A \otimes I)\Omega(A \otimes I)^\dagger = \mathrm{proj}\,\psi$,

$$\Delta_\psi = a\,\mathrm{proj}\,\psi - K_\psi, \qquad K_\psi = (A \otimes I)K_0(A \otimes I)^\dagger \ge 0.$$

If. Let R be a state with $K_{0vec(R)} = 0$ and $A = R^{1/2}$. Then $\|\psi_R\|^2 = \mathrm{Tr}\, R = 1$ and $(A \otimes I)^\dagger|\psi_R\rangle = vec(R)$, so $\langle\psi_R|K_{\psi_R}|\psi_R\rangle = \langle vec(R)|K_0|vec(R)\rangle = 0$. The primal value of Lemma (a) is $1 - \langle\psi_R|(id \otimes \mathcal{T})(|\psi_R\rangle\langle\psi_R|)|\psi_R\rangle = \langle\psi_R|\Delta_{\psi_R}|\psi_R\rangle = a$, and $Z = a\Omega$ is a dual witness with $\|\mathrm{Tr}_Q Z\|_\infty = a$ (Theorem (b)). Lemma (c) gives $\varepsilon = a$ and (b). *Only if.* By the standard fact used in the proof of Lemma (b), and compactness of the unit sphere, there is $\psi = (A \otimes I)|\Omega\rangle$ with $\frac{1}{2}\|\Delta_\psi\|_1 = \varepsilon = a$. Since Δ_ψ is Hermitian and traceless, $a = \max_{0 \le P \le I} \mathrm{Tr}\,(P\Delta_\psi)$; let P attain it. As $\mathrm{Tr}\,(P\Delta_\psi) = a\langle\psi|P|\psi\rangle - \mathrm{Tr}\,(PK_\psi) \le a$, equality forces $\langle\psi|P|\psi\rangle = 1$ and $\mathrm{Tr}\,(PK_\psi) = 0$. The first gives $(I - P)^{1/2}\psi = 0$, so $P\psi = \psi$; the second gives $K_\psi^{1/2}P^{1/2} = 0$, so $K_\psi\psi = K_\psi P\psi = 0$. Hence $0 = \langle\psi|K_\psi|\psi\rangle = \langle vec(R)|K_0|vec(R)\rangle$ with $R = A^\dagger A$, a state, and $K_0 \ge 0$ gives $K_0 vec(R) = 0$. (c) If $K_0 v = 0$ then $Jv = t_*\langle\Omega, v\rangle|\Omega\rangle$, so $v \in \mathbb{C}J^{-1}|\Omega\rangle$ (if $\langle\Omega, v\rangle = 0$ then $Jv = 0$ and $v = 0$); conversely $K_0 J^{-1}|\Omega\rangle = |\Omega\rangle - t_*\langle\Omega|J^{-1}|\Omega\rangle|\Omega\rangle = 0$ by Theorem (a). Also $\mathrm{Tr}\, M_{\mathcal{T}} = \langle\Omega|J^{-1}|\Omega\rangle = 1/t_*$. A state R with $vec(R) \in \mathbb{C}J^{-1}|\Omega\rangle$ is $R = sM_{\mathcal{T}}$ with $\mathrm{Tr}\, R = 1$, i.e. $s = t_*$; it is a state iff $M_{\mathcal{T}} \ge 0$. Apply (a).

par *Does not imply.* The criterion characterizes when one particular dual family is optimal; it is not a closed error formula for channels that fail it, whose error lies strictly below $1 - t_*$ and is in each instance the value of a finite semidefinite program. Generic non-exactness is not proved: check C26 (computer-assisted) finds $M_{\mathcal{T}}$ not Hermitian positive semidefinite, and the error strictly below $1 - t_*$, for random channels, and confirms the criterion for replacement and isotropic channels. par

Corollary.

• *(Bell saturation.)* If $J|\Omega\rangle = dF_e|\Omega\rangle$, then $t_* = F_e$ and $R = I/d$ satisfies Theorem (a).

• *(Replacement.)* For $\sigma > 0$, $J(\mathcal{R}_\sigma) = I \otimes \sigma$, $M_{\mathcal{R}_\sigma} = (\sigma^{-1})^T \ge 0$ and $R = (\sigma^{-1})^T/S_\sigma$.

• *(Unitary conjugation.)* For a unitary U put $\mathcal{T}^U = Ad_U \circ \mathcal{T} \circ Ad_{U^\dagger}$. Then $\varepsilon(\mathcal{T}^U) = \varepsilon(\mathcal{T})$, $t_*(\mathcal{T}^U) = t_*(\mathcal{T})$, and $\mathcal{T}^U$ satisfies Theorem (a) with $\bar{U}RU^T$ iff $\mathcal{T}$ does with R.

Proof. (a) Theorem and Theorem (c) give $t_* = F_e$; then $(J - F_e\Omega)|\Omega\rangle = dF_e|\Omega\rangle - F_e d|\Omega\rangle = 0$ and $vec(I/d) = |\Omega\rangle/d$. (b) $J^{-1}|\Omega\rangle = (I \otimes \sigma^{-1})vec(I) = vec((\sigma^{-1})^T)$, and $\mathrm{Tr}\,\sigma^{-1} = S_\sigma$. (c) $\mathcal{T}^U - id = Ad_U \circ (\mathcal{T} - id) \circ Ad_{U^\dagger}$, and composing with unitary channels on either side does not change the diamond norm (submultiplicativity in both directions). With $W = \bar{U} \otimes U$, $(id \otimes Ad_{U^\dagger})(\Omega) = (\bar{U} \otimes I)\Omega(\bar{U} \otimes I)^\dagger$, hence $J(\mathcal{T}^U) = WJ(\mathcal{T})W^\dagger$, and $W|\Omega\rangle = vec(\bar{U}U^T) = |\Omega\rangle$. So $J(\mathcal{T}^U) - t\Omega = W(J - t\Omega)W^\dagger$ for every t, which gives the same t_*, and $Wvec(R) = vec(\bar{U}RU^T)$, a state iff R is.

par *Does not imply.* The corollary re-derives the two exact cases of Theorem (c),(d) from the criterion and shows that exactness is a property of unitary orbits. It does not say that these are the only exact channels; Theorem and the examples below show that they are not. Check C26 corroborates (c) numerically. par

The basis-measurement interface outside the eigenvector condition

Notation of Theorem and Corollary : $\mathcal{T}_\sigma(X) = \sum_x \langle x|X|x\rangle\, \sigma_x$, $q_x = \langle x|\sigma_x|x\rangle$, $S_q = \sum_x q_x^{-1}$, and, when every $\sigma_x > 0$, $S_r = \sum_x \langle x|\sigma_x^{-1}|x\rangle$ and the $d \times d$ matrix $G_{xy} = \langle y|\sigma_x^{-1}|x\rangle$.

Lemma. Let ψ be a unit vector and ρ a state with $1 - \langle\psi|\rho|\psi\rangle > 0$. Then $\frac{1}{2}\|proj\,\psi - \rho\|_1 \ge 1 - \langle\psi|\rho|\psi\rangle$, with equality if and only if $\rho|\psi\rangle = \langle\psi|\rho|\psi\rangle|\psi\rangle$.

Proof. $\Delta = proj\,\psi - \rho$ is traceless, and $\langle v|\Delta|v\rangle = -\langle v|\rho|v\rangle \le 0$ for $v \perp \psi$; by the min–max principle its second eigenvalue is ≤ 0, so $\frac{1}{2}\|\Delta\|_1 = \lambda_1$, its largest eigenvalue, and $\langle\psi|\Delta|\psi\rangle = 1 - \langle\psi|\rho|\psi\rangle \le \lambda_1$. If equality holds, then $\lambda_1 > 0$ and ψ attains the maximum of the Rayleigh quotient, so $\Delta\psi = \lambda_1\psi$, i.e. $\rho\psi = (1 - \lambda_1)\psi$. Conversely, if $\rho\psi = \mu\psi$ then Δ leaves $\psi^\perp$ invariant and equals $-\rho \le 0$ there, so $\lambda_1 = 1 - \mu = \langle\psi|\Delta|\psi\rangle$.

par *Does not imply.* An elementary lemma about one input; it implies nothing about other inputs or channels. par

Theorem. Suppose every $q_x > 0$. Then $\varepsilon(\mathcal{T}_\sigma) = 1 - 1/S_q$ if and only if $\sigma_x|x\rangle = q_x|x\rangle$ for every x.

Proof. *If* is Corollary (c). *Only if.* Put $a = 1 - 1/S_q \ge 1 - 1/d > 0$, $p_x = q_x^{-1}/S_q$ and $\psi_p = \sum_x \sqrt{(p_x)}|x\rangle|x\rangle$. Then $\rho' = (id \otimes \mathcal{T}_\sigma)(|\psi_p\rangle\langle\psi_p|) = \sum_x p_x proj\, x \otimes \sigma_x$ and $1 - \langle\psi_p|\rho'|\psi_p\rangle = 1 - \sum_x p_x^2 q_x = a$ (the primal of Theorem (a)). The half-diamond distance is at least the trace distance on the input ψ_p, so

$$a = 1 - \langle\psi_p|\rho'|\psi_p\rangle \le \tfrac{1}{2}\||\psi_p\rangle\langle\psi_p| - \rho'\|_1 \le \varepsilon(\mathcal{T}_\sigma) = a.$$

Lemma gives $\rho^{'}\psi_p = (1 - a)\psi_p$, i.e. $p_x^{3/2}\sigma_x|x\rangle = (1 - a)p_x^{1/2}|x\rangle$ for every x. So $|x\rangle$ is an eigenvector of σ_x, with eigenvalue $\langle x|\sigma_x|x\rangle = q_x$.

par *Does not imply*. The theorem settles when the version-2 lower bound is exact. It does not give the error when the condition fails. Check C28 (computer-assisted) corroborates both directions on random instances, $d = 2, 3$. par

Corollary. For outputs (σ_x) let $\sigma_x^D = \sum_y \langle y|\sigma_x|y\rangle proj\ y$, the outputs of the classical decoder with $M_{yx} = \langle y|\sigma_x|y\rangle$. Then $\varepsilon(\mathcal{T}_{\sigma^D}) = 1 - 1/S_q \le \varepsilon(\mathcal{T}_{\sigma})$, with equality if and only if some $q_x = 0$ or $\sigma_x|x\rangle = q_x|x\rangle$ for every x.

Proof. $M_{xx} = q_x$, so Theorem (c) gives the value and Theorem (a) the inequality. If some $q_x = 0$, both sides are 1. Otherwise Theorem gives the equality case.

par *Does not imply*. On this interface coherence in the decoder's outputs never helps and strictly hurts outside the eigenvector condition. This concerns the stated half-diamond error on this interface only; it does not imply anything about other figures of merit, other interfaces or a physical decoder. par

Corollary. Suppose every $\sigma_x > 0$.

• $\varepsilon(\mathcal{T}_{\sigma}) = 1 - 1/S_r$ if and only if G is Hermitian and positive semidefinite. Then $R = G/S_r$ is a state with $(J - \Omega/S_r)vec\ (R) = 0$, and $(vec\ (R^{1/2}), (1 - 1/S_r)\Omega)$ is an exact dual certificate.

• Under the eigenvector condition $G = diag\ (q_x^{-1})$ and $S_r = S_q$, so both bounds are exact.

• If G is not Hermitian positive semidefinite and the eigenvector condition fails, then $1 - 1/S_q < \varepsilon(\mathcal{T}_{\sigma}) < 1 - 1/S_r$.

• *(Example.)* For $d = 2$, $\sigma_0 = (1/2, -1/4; -1/4, 1/2)$ and $\sigma_1 = 1/26(9, -6; -6, 17)$ one has $\sigma_0^{-1} = (8/3, 4/3; 4/3, 8/3)$, $\sigma_1^{-1} = (34/9, 4/3; 4/3, 2)$, $G = (8/3, 4/3; 4/3, 2) \ge 0$, $S_r = 14/3$ and $R = (4/7, 2/7; 2/7, 3/7)$. So $\varepsilon(\mathcal{T}_{\sigma}) = 11/14$ exactly, while $\sigma_0|0\rangle$ is not an eigenvector and the lower bound $1 - 1/S_q = 43/60$ is strict.

Proof. $J = \sum_x proj\ x \otimes \sigma_x > 0$, $t_{*} = 1/S_r$ (Corollary (b)) and $J^{-1}|\Omega\rangle = \sum_x |x\rangle \otimes \sigma_x^{-1}|x\rangle = vec\ (G)$, so $M_{\mathcal{T}_{\sigma}} = G$; (a) is Theorem (c),(b). (b) $\sigma_x|x\rangle = q_x|x\rangle$ gives $\sigma_x^{-1}|x\rangle = q_x^{-1}|x\rangle$. (c) Theorem , Theorem (a),(b) and (a). (d) The inverses and G are direct 2×2 computations; $Tr\ \sigma_0 = Tr\ \sigma_1 = 1$, $det\ \sigma_0 = 3/16$, $det\ \sigma_1 = 9/52$, $det\ G = 32/9$, and $Tr\ R = 1$; $q_0 = 1/2$, $q_1 = 17/26$, $S_q = 60/17$. Apply (a).

par *Does not imply*. Part (d) shows that the exact law $1 - 1/S_r$ holds for decoders that are neither classical, nor covariant, nor in the eigenvector class. The exact error in case (c) stays *open*; the bracket is the partial result. Check C27 verifies (d) in exact rational arithmetic and by semidefinite programming (value $11/14$ to solver tolerance); check C28 corroborates (c) on random instances. par

Documentary readings of newly delivered companions

PG-OB-1-CERT-v3. (sha256 8c0a7e2fdbb7e4558bad17bc35df10a66feea6ba4959af524921393a61eafbcc.) PG-OB-1-CERT-v3 prop:v3-Z(c) states that the sign flip of its lem:v3-sandwich carries its version-2 rank-one witness $c_{\sigma}\Omega$ to its version-1 block witness. That is the content of Corollary of this sheet, proved here in version 4; the two were obtained separately and nothing is transferred. Its lem:v3-sandwich admits lower witnesses $0 \le W \le \rho \otimes I$ (a mixed reference); this sheet uses only pure primal witnesses. PG-OB-1-CERT-v3 cor:v3-decoder: optimizing a decoder after a replacement channel gives $1 - d^{-2}$ for every σ, while the fixed-output distance $1 - 1/\Im$ equals $1 - d^{-2}$ only at $\sigma = I/d$. On this sheet the record-ignoring composites are exactly the replacement channels, with minimum $1 - 1/d^2$ attained only at I/d (Theorem); the numbers agree and the tasks are stated separately. PG-OB-1-CERT-v3 prop:v3-fibre (an orbit supremum is at most the class supremum, with equality iff the reachable values approach it) has the shape of Theorem (c), which in addition proves attainment. No Page curve.

Hawk07-A/B/C-CERT-v3. (sha256 1e91c9fb7bb97c16d9b4b0d48e821d511badea6e4ace77fc6643a61c76eee776, 2639d64e7ec3340017a91ed62e487b857317f2cef51966a2086fd016ffcf4176, 4352ee5e6a754a153631ab755a44e16ea9273fb1b6b9950e8fc29711b983a416.) The new Hawk07-A-CERT-v3 prop:v3-drule (identical in B and C) records that $1 - 1/\Im(\sigma)$ takes exactly the values $[1 - 1/d^2, 1]$, and that a rule given only d has worst-case error at least $1/(2d^2)$. The range agrees with Theorem ($S \in [d^2, \infty]$); the rule bound

is half the length of that range. The Hawk07 sheets cite lem:cert-sandwich of I-C-CERT-v2 and related labels as source records; those readings match the statements here (checked).

En08-ST-DSS-CERT-v4. (sha256 98c42ee7397263bc694ebed7d66c682b090df437a3230f294b3245f4bd01bee9.) doc:EC-icsandwich reads lem:cert-sandwich of the I-C certificate sheets (version 2) as a feasible-lower/feasible-upper bracket and refuses any identification of targets; that is an accurate reading of Lemma . Its lem:EC-replacement proves decoder-optimized replacement recovery $1 - d^{-2}$ by a Weyl average, the value of PG-OB-1-CERT-v3 cor:v3-decoder. Its doc:EC-icreorg reads I-C-CERT-v2 thm:capacity as $v \le 1/d$ for all accounts and $v \le 1/d^2$ for record-ignoring accounts under record-ignoring reorganizations, and warns that a value ceiling is an upper bound and an error floor a lower bound; that matches Theorem (c) (checked). No Page curve or interior on either side.

Version 6: the fidelity primal, generic non-exactness, singular outputs, and the companion readings

This section is new in version 6 (request t268u; no plan was attached). Everything in it is proved locally from the earlier sections of this sheet, including Sections and ; computer-assisted checks are labelled as such, and standard facts of real analysis and of linear algebra are named where they are used. The newly delivered companions (DS-, VC- and PG-OB-1-CERT-v4 and v5, Hawk07-*-CERT-v4 and v5, and the DSS-CERT-v5 and v6 sheets) are cited by sha256 as source records only, never as premises; I-C-CERT-v5 is the previous version of this sheet. The companion files were verified before use against MANIFEST-CertPlan-S2v4-OB.sha256 and MANIFEST-CertPlan-S2v4-Hawk07.sha256, and, for the second wave read in this version, MANIFEST-CertPlan-S2v5-OB.sha256 and MANIFEST-CertPlan-S2v5-Hawk07.sha256 (in CertPlan/deliver), and MANIFEST-DSS-CERT-v5.sha256 and MANIFEST-DSS-CERT-v6.sha256 (in DSS/deliver); every listed hash matched.

The fidelity primal and exactness as a convex program

For a channel $\mathcal{T}$ on $\mathbb{C}^d$ with $J = J(\mathcal{T})$ put

$$L_F(\mathcal{T}) = 1 - \min\{\langle \mathrm{vec}\,(R) \mathrm{rvert} J\, |\mathrm{vec}\,(R)\rangle : R \text{ a state on} E\},$$

with *vec* as in Section . The minimum exists, since the states form a compact set and the function is continuous.

Proposition.

• $1 - F_e(\mathcal{T}) \le L_F(\mathcal{T}) \le \varepsilon(\mathcal{T}) \le 1 - t_{*}(\mathcal{T})$.

• $t_{*}(\mathcal{T}) = \inf\{\langle v|J|v\rangle : v \in E \otimes Q, \langle \Omega, v\rangle = 1\}$, the infimum over all vectors, not only over vectors $vec\,(R)$ of states.

• $L_F(\mathcal{T}) = 1 - t_{*}(\mathcal{T})$ if and only if $\varepsilon(\mathcal{T}) = 1 - t_{*}(\mathcal{T})$, if and only if some state R satisfies $(J - t_{*}\Omega)vec\,(R) = 0$.

• $R \mapsto \langle vec\,(R) rvert J\, |vec\,(R)\rangle$ is a convex quadratic function on the convex set of states, so L_F is the value of a convex program, and (c) decides exactness of the best rank-one dual by that program.

Proof. (a) Let R be a state, $A = R^{1/2}$ and $\psi = (A \otimes I)|\Omega\rangle$, a unit vector since $\|\psi\|^2 = Tr\, A^{\dagger}A = 1$. Then $(id \otimes \mathcal{T})(proj\ \psi) = (A \otimes I)J(A \otimes I)^{\dagger}$ and $(A \otimes I)^{\dagger}\psi = (R \otimes I)|\Omega\rangle = vec\,(R)$, so the primal value of Lemma (a) at ψ is $1 - \langle vec\,(R) rvert J\, |vec\,(R)\rangle$. Hence $L_F \le \varepsilon$. The state $R = I/d$ has $vec\,(R) = |\Omega\rangle/d$ and gives $1 - F_e$, so $1 - F_e \le L_F$. The last inequality is Theorem (b). (b) If $|\Omega\rangle \notin ran\, J$, then $|\Omega\rangle$ is not orthogonal to $ker\, J = (ran\, J)^{\perp}$, so some $v \in ker\, J$ has $\langle \Omega, v\rangle = 1$ and $\langle v|J|v\rangle = 0 = t_{*}$ (Theorem (a)). If $|\Omega\rangle \in ran\, J$, then $|\Omega\rangle = J^{1/2}(J^{+})^{1/2}|\Omega\rangle$, and for $\langle \Omega, v\rangle = 1$ the Cauchy–Schwarz inequality gives $1 = |\langle (J^{+})^{1/2}\Omega, J^{1/2}v\rangle|^2 \le \langle \Omega|J^{+}|\Omega\rangle\, \langle v|J|v\rangle$, so $\langle v|J|v\rangle \ge t_{*}$, with equality at $v = J^{+}|\Omega\rangle/\langle \Omega|J^{+}|\Omega\rangle$. (c) Put $K_0 = J - t_{*}\Omega \ge 0$. For a state R, $\langle vec\,(R)|\Omega|vec\,(R)\rangle = |Tr\, R|^2 = 1$, so $\langle vec\,(R) rvert J\, |vec\,(R)\rangle = t_{*} + \langle vec\,(R) rvert K_0\, |vec\,(R)\rangle \ge t_{*}$. Thus $L_F = 1 - t_{*}$ if and only if the minimum of $\langle vec\,(R) rvert K_0\, |vec\,(R)\rangle$ over states is 0, if and only if (as $K_0 \ge 0$) some state has $K_0 vec\,(R) = 0$. By Theorem (a) this is equivalent to $\varepsilon = 1 - t_{*}$. (d) $J \ge 0$ and *vec* is linear.

par *Does not imply*. The proposition brackets the error by two programs and identifies when the outer bracket is tight. It does not say that $\varepsilon = L_F$ in general: check C30 (computer-assisted) finds $L_F < \varepsilon$ strictly on most random

instances and equality for replacement channels. A closed error formula for channels that fail the criterion stays *open*. par

On the basis-measurement interface, the state $R = diag\ (p_x)$ with $p_x = q_x^{-1}/S_q$ gives $\langle vec\ (R) rvert J\ |vec\ (R)\rangle = \sum_x p_x^2 q_x = 1/S_q$, so $1 - 1/S_q \le L_F(\mathcal{T}_\{\sigma\}) \le \varepsilon(\mathcal{T}_\{\sigma\})$: the fidelity primal is never below the version-2 lower bound of Theorem (a).

Depolarized unitaries: an explicit exactness boundary

For a unitary U on $\mathbb{C}^d$, $d \ge 2$, and $0 < p < 1$ let $\mathcal{T}_\{U, p\} = p\ Ad_U + (1 - p)\mathcal{R}_\{I/d\}$, and put $c = (1 - p)/d$ and $\tau = Tr\ U$.

Proposition.

• $J(\mathcal{T}_\{U, p\}) = c\ I + p\ vv^\{\dagger\}$ with $v = vec\ (U^T)$, $\|v\|^2 = d$ and $\langle v, \Omega\rangle = \overline{\tau}$. In particular $J > 0$ and $F_e(\mathcal{T}_\{U, p\}) = (1 - p + p|\tau|^2)/d^2$.

• $M_\{\mathcal{T}_\{U, p\}\} = 1/c(I - (p\overline{\tau})/(c + pd)U^T)$ and $t_\{*\}(\mathcal{T}_\{U, p\}) = (c(c + pd))/(d(c + pd) - p|\tau|^2)$.

• $\varepsilon(\mathcal{T}_\{U, p\}) = 1 - t_\{*\}(\mathcal{T}_\{U, p\})$ if and only if $\tau = 0$ or $e^\{-iarg\ \tau\}\ U$ is Hermitian. For $\tau = 0$ the Bell witness is exact and $t_\{*\} = F_e$.

• For $U = diag\ (1, i, 1, \ldots, 1)$ the criterion fails, so $\varepsilon(\mathcal{T}_\{U, p\}) < 1 - t_\{*\}(\mathcal{T}_\{U, p\})$ for every $0 < p < 1$. For $d = 2$, $p = 1/2$: $t_\{*\} = 5/24$, so $\varepsilon < 19/24$.

Proof. (a) $J(Ad_U) = (I \otimes U)\Omega(I \otimes U)^\{\dagger\}$ and $(I \otimes U)vec\ (I) = vec\ (U^T)$; $J(\mathcal{R}_\{I/d\}) = I \otimes I/d$. Also $\|v\|^2 = Tr\ (UU^T) = d$ and $\langle v, \Omega\rangle = Tr\ U = \overline{\tau}$. Since $c > 0$, $J \ge cI > 0$, and $\langle\Omega|J|\Omega\rangle = cd + p|\tau|^2$. (b) By the Sherman–Morrison formula, $J^\{-1\} = 1/c(I - (p\ vv^\{\dagger\})/(c + pd))$, so $J^\{-1\}|\Omega\rangle = 1/c(|\Omega\rangle - (p\overline{\tau})/(c + pd)v) = vec\ (1/c(I - (p\overline{\tau})/(c + pd)U^T))$. Then $1/t_\{*\} = Tr\ M_\{\mathcal{T}_\{U, p\}\} = 1/c(d - (p|\tau|^2)/(c + pd))$ by Theorem (c). (c) By Theorem (c), exactness holds if and only if $M = M_\{\mathcal{T}_\{U, p\}\}$ is Hermitian and positive semidefinite. M is Hermitian if and only if $X = \overline{\tau}U^T$ is, if and only if $\overline{\tau}U$ is (a matrix is Hermitian if and only if its transpose is), that is, $\tau = 0$ or $e^\{-iarg\ \tau\}U$ Hermitian. If X is Hermitian, then $X^2 = XX^\{\dagger\} = |\tau|^2 I$, so its eigenvalues are $\pm|\tau|$, and those of M are at least $1/c(1 - (p|\tau|)/(c + pd)) > 0$, because $|\tau| \le d$ and $c > 0$; so $M > 0$ automatically. If $\tau = 0$, then $J|\Omega\rangle = c|\Omega\rangle$, and with $F_e = c/d$ this is the Bell condition of Theorem ; Theorem (c) gives $t_\{*\} = F_e$. (d) $\tau = d - 1 + i \ne 0$, and $e^\{-iarg\ \tau\}U$ has the eigenvalues $e^\{-iarg\ \tau\}$ and $ie^\{-iarg\ \tau\}$, which cannot both be real; so (c) fails, and $\varepsilon < 1 - t_\{*\}$ by Theorem (a) and Theorem (b). For $d = 2$, $p = 1/2$: $c = 1/4$, $c + pd = 5/4$, $|\tau|^2 = 2$, and (b) gives $t_\{*\} = (5/16)/(3/2) = 5/24$.

par *Does not imply*. The proposition decides exactness on one explicit family; it does not give the error when the criterion fails. Check C32 (computer-assisted: exact sympy for M, semidefinite programming for the error) confirms (b)–(d) at $p = 1/2$ for $U = Z, X, e^\{7i/10\}diag\ (1, 1, -1)$ (exact) and $diag\ (1, i)$, $diag\ (1, i, 1)$ (not exact; for $d = 2$ the solver value is $0.\ 68402$). par

Non-exactness is generic

Let $A = \{J = J^\{\dagger\}onE \otimes Q: Tr_Q\ J = I_E\}$, a real affine space of dimension $d^4 - d^2$, let $C = \{J \in A: J \ge 0\}$ be the set of Choi operators of channels on $\mathbb{C}^d$ (Choi's theorem), and $C^\{\circ\} = \{J \in A: J > 0\}$.

Theorem. Let $d \ge 2$, and let $N \subseteq C^\{\circ\}$ be the set of J for which $M_\{\mathcal{T}\}$ is not Hermitian.

• $C^\{\circ\}$ is convex, open in A and dense in C.

• N is open in A and dense in C, and $C \setminus N$ has Lebesgue measure zero in A.

• Every channel with $J \in N$ has $\varepsilon(\mathcal{T}) < 1 - t_\{*\}(\mathcal{T})$ strictly. Hence the channels for which the best rank-one dual is exact form a subset of $C \setminus N$, which is closed and nowhere dense in C and has measure zero.

Proof. (a) Openness and convexity are clear, and $J_0 = I \otimes I/d \in C^\{\circ\}$. For $J \in C$ and $0 < s \le 1$, $(1 - s)J + sJ_0 \in C^\{\circ\}$ and tends to J as $s \to 0$. (b) On $C^\{\circ\}$ the map $J \mapsto M_\{\mathcal{T}\} - M_\{\mathcal{T}\}^\{\dagger\}$, with $vec\ (M_\{\mathcal{T}\}) = J^\{-1\}|\Omega\rangle$, is real-analytic in affine coordinates of A (the entries of $J^\{-1\}$ are rational functions with nonvanishing denominator $det\ J$). It is not identically zero: the depolarized unitary of Proposition (d) lies in $C^\{\circ\}$ and has non-Hermitian $M_\{\mathcal{T}\}$. So some real or imaginary part f of one entry is a real-analytic function on the connected open set $C^\{\circ\}$ that is not identically zero. By the standard fact that the zero set of such a function is closed in $C^\{\circ\}$, has empty interior and has Lebesgue measure zero, $C^\{\circ\} \setminus N \subseteq \{f = 0\}$ is a closed null set with empty interior in $C^\{\circ\}$.

Hence N is open and dense in C°, so dense in C by (a). Finally $C \setminus C^{\circ}$ lies in the boundary of the convex set C, which has nonempty interior in A; such a boundary has measure zero. So $C \setminus N$ is null. (c) If $J \in N$, then $M_{\mathcal{T}}$ is not Hermitian, and Theorem (c) and Theorem (b) give $\varepsilon < 1 - t_{*}$. The set $C \setminus N$ is closed in C (as N is open) and has empty interior in C (as N is dense).

par *Does not imply*. This settles the version-5 item ``generic non-exactness is not proved". The measure is Lebesgue measure on the affine hull of the channel set, not a physical prior; the theorem says nothing about any particular channel beyond Theorem , and it does not compute the error of non-exact channels, which stays the value of a finite semidefinite program. The exact channels include the identity, the replacement channels and the examples of Section . Check C33 (computer-assisted) finds no exact channel among *30* random full-rank channels, and a non-Hermitian $M_{\mathcal{T}}$ along the open segment from a replacement channel to the example of Proposition (d). par

The basis-measurement interface with singular outputs

Notation of Corollary . When $|x\rangle \in \operatorname{ran} \sigma_x$ for every x, put $S^{+} = \sum_x \langle x|\sigma_x^{+}|x\rangle$ (so $t_{*} = 1/S^{+}$ by Corollary (a)) and $G_{xj}^{+} = \langle j|\sigma_x^{+}|x\rangle$.

Theorem. Suppose $|x\rangle \in \operatorname{ran} \sigma_x$ for every x.

• $\ker (J - t_{*}\Omega) = \{\sum_x |x\rangle \otimes (t_{*}\, \gamma\, \sigma_x^{+}|x\rangle + k_x) : \gamma \in \mathbb{C},\ k_x \in \ker \sigma_x\}$.

• $\varepsilon(\mathcal{T}_{\sigma}) = 1 - 1/S^{+}$ if and only if there is a state $R = t_{*}G^{+} + K$, where for each x the vector $\sum_j K_{xj}|j\rangle$ lies in $\ker \sigma_x$. This is a finite linear matrix inequality feasibility problem. When every $\sigma_x > 0$ it reduces to Corollary (a) ($K = 0$, $G^{+} = G$).

• *($d = 2$, one pure output.)* Let $\sigma_0 = \text{proj } 0$ and $\sigma_1 = (\alpha, \beta; \bar{\beta}, \gamma) > 0$ with $D = \det \sigma_1$. Then $\varepsilon(\mathcal{T}_{\sigma}) = 1 - t_{*} = \alpha/(\alpha + D)$ if and only if $\alpha D \geq |\beta|^2$; otherwise $\varepsilon(\mathcal{T}_{\sigma}) < \alpha/(\alpha + D)$.

• *(Examples.)* $\sigma_1 = \frac{1}{6}(2, -1; -1, 4)$ gives $R = \frac{1}{19}(7, 6; 6, 12)$ and $\varepsilon = 12/19$ exactly, with $K = \frac{6}{19}|0\rangle\langle 1| \neq 0$, while the lower bound $1 - 1/S_q = 3/5$ is strict. $\sigma_1 = \frac{1}{9}(1, -2; -2, 8)$ fails the condition, so $\varepsilon < 9/13$.

Proof. (a) Write $v = \sum_x |x\rangle \otimes r_x$ and $\gamma = \langle \Omega, v\rangle = \sum_x \langle x, r_x\rangle$. Then $(J - t_{*}\Omega)v = \sum_x |x\rangle \otimes (\sigma_x r_x - t_{*}\gamma|x\rangle)$, so v is in the kernel if and only if $\sigma_x r_x = t_{*}\gamma|x\rangle$ for every x. As $|x\rangle \in \operatorname{ran} \sigma_x$, $\sigma_x \sigma_x^{+}|x\rangle = |x\rangle$, and the solutions are $r_x = t_{*}\gamma\sigma_x^{+}|x\rangle + k_x$ with $k_x \in \ker \sigma_x$. Conversely such r_x give $\sum_x \langle x, r_x\rangle = t_{*}\gamma S^{+} + \sum_x \langle x, k_x\rangle = \gamma$, since $t_{*}S^{+} = 1$ and $|x\rangle \in \operatorname{ran} \sigma_x \perp \ker \sigma_x$; so the parameter γ is consistent. (b) $\text{vec } (R) = \sum_x |x\rangle \otimes r_x$ with $r_x = \sum_j R_{xj}|j\rangle$, and $\gamma = \operatorname{Tr} R = 1$ for a state. By (a), $\text{vec } (R) \in \ker (J - t_{*}\Omega)$ if and only if $R_{xj} = t_{*}G_{xj}^{+} + K_{xj}$ with rows of K as stated. Apply Theorem (a). If every $\sigma_x > 0$, all kernels are 0. (c) $\ker \sigma_0 = \mathbb{C}|1\rangle$ and $\ker \sigma_1 = 0$, so $K = \kappa|0\rangle\langle 1|$. With $\sigma_1^{-1} = D^{-1}(\gamma, -\beta; -\bar{\beta}, \alpha)$ one has $G^{+} = (1, 0; a, b)$, $a = -\beta/D$, $b = \alpha/D$, and $t_{*} = 1/(1 + b) = D/(D + \alpha)$. $R = t_{*}G^{+} + K$ is Hermitian if and only if $\kappa = t_{*}\bar{a}$, and then $R = t_{*}(1, \bar{a}; a, b)$ is positive semidefinite if and only if $b \geq |a|^2$, i.e. $\alpha D \geq |\beta|^2$. The strict inequality otherwise is Theorem (b) with (b). (d) First example: $\alpha = 1/3$, $\beta = -1/6$, $D = 7/36$, $\alpha D = 7/108 \geq 1/36$, $t_{*} = 7/19$, $a = 6/7$, $b = 12/7$; $q_0 = 1$, $q_1 = 2/3$, $S_q = 5/2$. Second: $\alpha = 1/9$, $\beta = -2/9$, $D = 4/81$, $\alpha D = 4/729 < 4/81$, $t_{*} = 4/13$.

par *Does not imply*. The theorem extends the version-5 exactness criterion on this interface to singular outputs; it is still a criterion, not an error formula, and the exact error when it fails stays *open* (partial: the bracket of Theorem). Check C31 (computer-assisted) verifies (d) in exact rationals and by semidefinite programming (values $12/19$ and $0.62700 < 9/13$), and finds agreement of the feasibility test in (b), and of the condition in (c), with the solver on random instances with one rank-deficient output. par

Documentary readings of newly delivered companions

PG-OB-1-CERT-v4. (sha256 3dc921457bd781606e9e673231a8c832be104435950913521fdfb2ed737fc1c1.) PG-OB-1-CERT-v4 thm:v4-PG-floor: through an m-dimensional register, $F_e(\mathcal{D}\mathcal{T}) \leq \min\{1, m^2/d^2\}$, so the error is at least $1 - m^2/d^2$, attained for $m \leq d$; PG-OB-1-CERT-v4 cor:v4-PG-register turns this into $m \geq d\sqrt{(1 - \varepsilon)}$. At $m = 1$ the composite is a replacement channel and the floor $1 - 1/d^2$ is the minimum of Theorem , attained only at I/d; the statements agree, and nothing is transferred. Its documentary reading of thm:v4-rankone(d) and cor:v4-translate of I-C-CERT-v4 ($t_{*}(\mathcal{R}_{\sigma}) = 1/\Xi(\sigma)$; corresponding witnesses, not the same operator) is accurate; those statements are identical in Theorem and Corollary of this sheet.

Hawk07-A-CERT-v4. (sha256 a88a2430a9e3672a9cbdb7c661dc021154dd3ac4237a1040b4eab097194fceb5.) Hawk07-A-CERT-v4 prop:v4-rankone(b),(c) are Lemma and Corollary written output-first, proved there independently. Its reading says that the same labels appear with the same statements in I-C-, I-C-Ex- and I-C-Ex2-CERT-v4; that is correct (the statements of lem:v4-convention, thm:v4-rankone and cor:v4-translate are character-identical in the three sheets, checked). Hawk07-A-CERT-v4 cor:v4-dim (decoder errors after a replacement channel fill *[1 − d^{−2}, 1]*) concerns optimization over decoders, a different task from the fixed composites here.

En08-ST-DSS-CERT-v5. (sha256 9be4fe4fdd9cf1164a75f0218035f0ee3a4c340ed7f15787551de3997fe5af64.) En08-ST-DSS-CERT-v5 cor:EC-decoders(a): after a replacement channel, the optimal decoders are exactly those with output *I/d*, and a decoder with output *σ* has error at least *1 − 1/S_{σ}*. On this sheet a record-ignoring composite with output *σ* has error exactly *1 − 1/S_{σ}* (Theorem); the statements agree. En08-ST-DSS-CERT-v5 prop:EC-regfloor is the register floor of PG-OB-1-CERT-v4, proved there separately. En08-ST-DSS-CERT-v5 doc:EC-pgic5 reads lem:v4-convention and cor:v4-translate of I-C-CERT-v4 as ``the same values with different operators"; that is accurate. Its doc:EC-icrow5 is treated in Section (E6.2).

Red06-G-DSS-CERT-v5 and V-DST-DSS-CERT-v5. (sha256 e4a1b02552c138b8f9f009f2e71e391825ad0014f717b43f96883c445605213a, 4133961f566c059c22054b5922423d1ff94cb0d1c888ca353a7c143bba1541be.) Red06-G-DSS-CERT-v5 doc:RC-icrow5 and V-DST-DSS-CERT-v5 doc:VDC-icrow5 repeat the qualification of the version-4 plan-verification row; the errata they cite were confirmed in the version-4 error lists and are recorded in Section . Their other new items (prop:RC-ends, prop:VDC-odd, prop:VDC-E1) concern the de Sitter source map and are not used.

VC-OB-1-CERT-v4 and DS-OB-1-CERT-v4. (sha256 f9802da66896159503132b736164016c1d06d2a6530bbbc06cb4ad3af0256387, 0b5b86f87f91503ba9e96cf221b2ef96a4b4e96e3f27489336eb2b89848463c6.) Both cite lem:v4-convention, cor:v4-translate and thm:v4-rankone of I-C-CERT-v4 as statements about finite channels that do not bear on their sheets; that is accurate. DS-OB-1-CERT-v4 notes that the radius lemma is lem:v4-radius of I-C-Th-CERT-v4, not of I-C-CERT-v4; that is correct. Their curvature results are read in I-C-Th-CERT-v6.

Documentary readings of the second wave of companions

The following files were delivered during the preparation of this version and verified against their manifests (Section , opening paragraph). They are source records only.

PG-OB-1-CERT-v5. (sha256 c96ea4631de51c635521ca42fd24337728591676ee9019a24f3ad9fcdcdeb8a2.) It reads I-C-CERT-v5 thm:v5-exact as the criterion ``exact if and only if a state *R* satisfies *(J − t_{*}Ω)vec (R) = 0*" and I-C-CERT-v5 cor:v5-exact-cases(a) as the Bell-saturated case; both readings are accurate (Theorem , Corollary). Its PG-OB-1-CERT-v5 lem:v5-PG-sandwich, PG-OB-1-CERT-v5 thm:v5-PG-shared and PG-OB-1-CERT-v5 thm:v5-PG-divisible concern recovery through a register with shared randomness, a task not treated here; nothing is transferred.

Hawk07-A-CERT-v5. (sha256 9db43c8772a84fcf2fcf8d3674b86781ab407b93de05d370740e89f3f00b7add.) Hawk07-A-CERT-v5 prop:v5-kernel describes, output factor first, the kernel of the block dual $\sigma \otimes I - \Xi^{-1}\Omega$ of a replacement channel: for *σ > 0* it is spanned by *vec (σ^{−1})*, and for singular *σ* the operator is $\sigma \otimes I$. Its reading of I-C-CERT-v5 cor:v5-exact-cases(b) (*M = (σ^{−1})^T ≥ 0, R = (σ^{−1})^T/S_{σ}*, input factor first) is accurate; in the input-first order of this sheet the same kernel statement is the case $J = I \otimes \sigma$ of Theorem . For singular *σ*, *Ω ∉ ran J* and *t_{*} = 0* here, consistent with its part (c).

En08-ST-DSS-CERT-v6. (sha256 7d821d99ced3a9d265d5437f5c1466d8f524a09884730d13662278c31b1da086.) En08-ST-DSS-CERT-v6 doc:EC-v6read reads I-C-CERT-v5 thm:v5-exact, I-C-CERT-v5 thm:v5-lower and I-C-CERT-v5 cor:v5-upper as deciding when *1 − 1/S_q* and *1 − 1/S_r* are attained and leaving the value strictly between them open (partial). That is accurate for version 5; version 6 adds the lower bound *L_F* (Proposition) and exact values for some singular outputs (Theorem), and the general value between the bounds stays open. It also reads this sheet's version-5 reading of En08-ST-DSS-CERT-v4 lem:EC-replacement as accurate. Its En08-ST-DSS-CERT-v6

prop:EC-assisted (register recovery with shared randomness) is not used.

DS-OB-1-CERT-v5, VC-OB-1-CERT-v5, Hawk07-F/H-CERT-v5, Red06-G-DSS-CERT-v6 and V-DST-DSS-CERT-v6. (sha256 a0805f8eb081a250f78e8074b9de5986ccd23106749df9e0fe5fac2f71e157f9, d3a93a3afb0e8803d18f230ce824f78c19253f6a4bc841f8ceef763ce14f7cac, 2cfa6282f30e25ac57188298f2e16d3b05a66fff4440204e6f11acce352f5bc3, 74a70015a03906e76cac8fde88a082da9789ea6e863d987be1dbba67f08bb5dd, c3f04faab2c76fb661054f2c1393bd71f484a1bfdfeeace52e5c612a34fc4599, 710c656210aab6dd708cd9f830ab7bf59df592f0887ab168b1e7fa7be2df95f9.) The two OB sheets cite I-C-CERT-v5 thm:v5-exact as a finite-channel criterion that does not bear on their labels; that is accurate. The other files do not cite this sheet; their curvature results are read in I-C-Th-CERT-v6. None of the second-wave files records an error in version 5 of this sheet, and none was found in their readings.

Verification of plan t256u against the sheets (version 1, retained)

The plan's statements for I-C were checked against the parent I-C-v2 and against Big01-A version 5 (source records). toprule Plan statement Finding Status

midrule Big01 uses the floor $1 - 1/d$ Big01-A lem:shredder and Big01-A prop:shredsharp use $1 - 1/d$, citing I-C as I-C-v2. confirmed

Gap $(d - 1)/d^2$ Big01-A prop:shredsharp displays $(1 - 1/d^2) - (1 - 1/d) = (d - 1)/d^2$. confirmed

``Horizon law" In Big01-A, $1 - 1/d^2$ is the horizon *floor* (Big01-A thm:B, Big01-A thm:Bsharp); $(d - 1)/d^2$ is its gap to the shredder floor. wording corrected (PC1.2)

Error of every decoder equals $1 - 1/S$ True in Big01-A (Big01-A thm:Bexact) for decoders after a replacement channel; false for the full classical-record class of I-C (Proposition). corrected (PC1.1)

Witness = Schmidt vector as in Big01 The vector ψ_p, $p_i \propto \lambda_i^{-1}$, of Big01-A thm:Bexact is the primal witness of Theorem . confirmed

Lower bound $1 - 1/d$ I-C-v2 thm:A; re-proved as Lemma . confirmed

bottomrule

Plan corrections of version 1 (retained)

• The plan's ``error of every decoder equals $1 - 1/S$ for explicit S from output spectrum" is false for the classical-record class of I-C: Δ and $Ad_X \circ \Delta$ have the same output spectrum and errors $1 - 1/d$ and 1 (Proposition). Proved instead: exact $1 - 1/S$ for every record-ignoring decoder (Theorem); exact optimum $1 - 1/d$ with dual operator $(1 - 1/d)\Omega$ (Theorem); errors filling $[1 - 1/d, 1]$.

• ``Horizon law $((d - 1)/d^2)$" conflates two Big01-A quantities: the horizon floor $1 - 1/d^2$ and its gap $(d - 1)/d^2$ to the shredder floor. Corollary certifies the gap inside I-C without a horizon.

• ``Not only lower bound $1 - 1/d$": the parent already proved that $1 - 1/d$ is sharp, by a primal construction and the triangle inequality. This sheet adds the explicit dual operator; the value is unchanged.

• (Amendment t257u) The plan's original ``new version of the existing sheet" is superseded: this is a new standalone sheet, and I-C is not given a new version.

Verification of plan t258u against the sources (version 2, retained)

Every label named by plan t258u was searched in the read-only copies (automated log plan_label_audit.json of this run). For this sheet: toprule Plan statement Finding Status

midrule Add the dephasing case of Big01-A thm:Bdephase The label exists; its part (a) is complete dephasing as a thermal operation at the shredder floor. Version 1 already certified complete dephasing (the pinching, Theorem); new here are the thermal-operation realization and the named floors (Theorem). proved; PC1.5

Thermal replacement strictly above the floor, as in Big01-A thm:Bthermal True for the shredder floor (Theorem). Relative to the record-ignoring floor $1 - 1/d^2$ it is not strict in general: equality iff H is scalar, which is Big01-A thm:Bthermal(c). proved; PC1.6

Capacity reading via Big01-A def:reorg and Big01-A thm:tick Both labels exist; used as patterns only (Theorem). proved; PC1.8

Extremes of prop:counter on a continuous path checked by C3 Confirmed: Proposition (b) proves continuity along $(1 - s)\Delta + sAd_X \Delta$, and C3 samples six points by SDP ($d = 3$). Now exact: $1 - (1 - s)/d$ (Theorem (e)). confirmed

Classification: error depends on the action on the record True on the basis-measurement interface, exactly for classical decoders and up to two-sided bounds otherwise (Theorem). proved (scoped); PC1.7

Cross-reference of the sandwich lemma Added (Section); the labels exist in the companions. done

bottomrule

Plan corrections of version 2 (retained)

• ``Add the dephasing case … complete dephasing attains the shredder floor exactly'': already proved in version 1 (Theorem ; the pinching is complete dephasing). Version 2 adds the thermal-operation realization (Theorem (c)).

• ``A thermal replacement channel on the record has error strictly above the floor'': true for the shredder floor $1 - 1/d$; for the record-ignoring floor $1 - 1/d^2$ equality holds exactly when the Hamiltonian is scalar (Theorem (b)).

• ``Strengthen prop:counter into a classification'': an exact classification is proved for classical decoders on the basis-measurement interface. For general decoders only two-sided bounds are proved, and numerically the bounds differ in general (check C9). No classification over all measurements is claimed.

• ``No reorganization of the classical record closes the gap'': a reorganization can move an account from the record-ignoring floor to the shredder floor; that is reading the record. What no reorganization does is exceed retained value $1/d$, and no record-ignoring reorganization exceeds $1/d^2$. The gap is the exact gain (Theorem).

Verification of plan t262u against the sources (version 3, retained)

Every label named by plan t262u was searched in the delivered read-only files, pinned by sha256 (automated log plan_label_audit.json of this run; v1 and v2 of each CERT sheet, and v2 and v3 of each DSS CERT sheet). The plan's own ``insertion status'' says that it does not authenticate the other sheets; here they were checked against the delivered files. Findings for this sheet: toprule Plan statement Finding Status

midrule All optimal measure-and-prepare composites True as stated; proved (Theorem), with the Bell-saturation criterion proved locally (Theorem). proved

Exact non-classical subclass True as stated (Corollary); the non-covariance proof is replaced by an explicit two-dimensional rotation. proved

I-C-CERT-v2 thm:capacity(c) too strong Confirmed: the increase from a record-ignoring account with output σ is $1/d - 1/S_{\sigma}$, up to $1/d$ (Proposition). v2 error E1.1

Hawk07 thm:s2-exact: ``matching Schmidt and rank-one dual witnesses'' The label exists in Hawk07-A/B/C-CERT and PG-OB-1-CERT (v1 and v2), and is a replacement-recovery statement with the same value and primal witness. Its dual operator is not rank-one and not of the form $(1 - t)\Omega$; it is written output-first, for $\mathscr{R}_{\sigma} - id$ (opposite sign convention to this sheet). corrected; PC1.9

Big01-A prop:shredsharp: same number $(d - 1)/d^2$ Confirmed (Big01-A v5). confirmed

En08-ST-DSS-CERT-v2 thm:EC-reorg: preservation of a declared class Confirmed, with the precision that its part (c) shows the ceiling failing off the declared class. confirmed

Local-proof cross-reference Added (typed cross-reference, Section). done

bottomrule

Plan corrections of version 3, and errors found in version 2 (retained)

• ``The Hawk07 replacement certificates share its witness form'' and ``matching Schmidt and rank-one dual witnesses'': true for the primal (Schmidt) witness and for the value $1 - 1/\Xi$. False for the dual: Hawk07-A-CERT-v2 thm:s2-exact(b) uses a block operator Y with $Y \geq \sigma \otimes I - \Omega$ (output-first, Choi operator of $\mathscr{R}_{\sigma} - id$), not $(1 - t)\Omega$. The rank-one dual $(1 - 1/S)\Omega$ is this sheet's (Theorem). The sign convention of this

sheet is opposite to Hawk07's, as reported by earlier streams; confirmed.

• The plan's Lemma ``rank-one certificate'' is Corollary (a),(b) of version 1; it is not new and is not restated.

• The plan presents its corollary ``covariance is sufficient, not necessary'' without a proof of sufficiency; the proof of Theorem supplies it.

Errors found in version 2.

• I-C-CERT-v2 thm:capacity(c), last sentence, and item (iv) of the version-2 abstract: ``the largest gain … from a record-ignoring account to any account is exactly *(d − 1)/d^2*'' is false for an arbitrary starting account; corrected in Section (Proposition). No other statement of version 2 was found to be false; the version-2 file is not edited.

Verification of plan t266u against the sources

Every label named by plan t266u (sha256 39aea6f1117318f9517712567fc87db46dd188176498fd71ff637076175f0621) was searched in the delivered read-only files, pinned by sha256 (automated log plan_label_audit.json of this run: Big01-A v5; v1 and v2 of DS-OB-1-CERT, VC-OB-1-CERT and PG-OB-1-CERT; v2 and v3 of Hawk07-A/B/C/F/G/H-CERT; v2, v3 and v4 of the three DSS CERT sheets; v3 of the five sheets of this family; the OpenAI catalogue). The OB sheets' version 3, in preparation elsewhere, was not read. Findings for this sheet: toprule Plan statement Finding Status

midrule Big01-A: thm:Hcert, prop:Dyncert, def:reorg, thm:tick, thm:Bthermal, thm:Bdephase; also prop:shredsharp, thm:D, thm:D4 All present in Big01-A v5 (sha256 e1160613…). confirmed

I-C family v3: lem:cert-sandwich, thm:bell-iff, thm:v3-floor The first two are in all three v3 sheets; thm:v3-floor (and thm:kdreorg, since v2) only in I-C-Ex-CERT and I-C-Ex2-CERT, not in I-C-CERT. corrected; T266.1

PG-OB-1-CERT-v2: thm:s2-exact, prop:s2-ceiling, lem:v2-sandwich; second witness *Z_{σ} = (1 − 1/϶)Ω* All present in v2 (lem:v2-sandwich not in v1). *Z_{σ}* is in v2 as thm:v2-Z, a label the plan does not name; it has the shape of this sheet's rank-one dual. confirmed

PG reading: exact dual for replacement recovery; block dual in the opposite sign convention True. Made precise here: the block dual minus the Choi difference is exactly *Z_{σ}* (Corollary , Lemma). proved

Hawk07 thm:s2-exact of A/B/C gives ``the same dual certificate'' Qualified: same value and primal; the dual operators differ in form (block versus rank one) and sign convention, and correspond exactly under the convention translation. corrected; T266.2

Hawk07 CERT: thm:s2-exact, prop:s2-ceiling, prop:s2-vac, thm:s2-sep, thm:s2-cas A union, not one sheet's list: thm:s2-exact, prop:s2-ceiling only in A/B/C; prop:s2-vac, thm:s2-sep only in C/F/G/H; thm:s2-cas only in F/G/H. All cited statements are identical in v2 and v3; v3 (newest) is cited. corrected; T266.3

prop:s2-ceiling ``the same hard ceiling'' False: an entropy bound *S(ρ_R) ≤ min (log d_R, log d_{R^c} + S(ρ))*, a fixed-resource bound of a different quantity (as recorded in version 3). corrected; T266.4

En08-ST-DSS-CERT-v3 lem:EC-sandwich ``re-proves the primal/dual bracket *τ_j*'' Present in v3 and, unchanged, in v4; absent in v2. A bracket for the lattice hull deficit *τ_j*, exact when both witnesses are optimal; not a channel statement. confirmed, typed

DSS CERT-v3 labels (thm:EC-dual, thm:EC-reorg, thm:RC-coef, prop:RC-dyn, thm:VDC-dyn, prop:VDC-coef) All present in v3; statements unchanged in the newer v4. confirmed

OpenAI catalogue: no isotropic-channel or Choi entries Re-searched: 0 whole-word ``Choi'', 0 ``isotropic channel''. confirmed

Big01-A prop:shredsharp records *(d − 1)/d^2*; ``both record exact gain from reading the record'' The number is there, as the difference of two exact floors (*1 − 1/d^2* and *1 − 1/d*). It is not a gain from every start; on this interface it is the best-against-best difference (Proposition). corrected; T266.5

Exact error of non-classical decoders outside the eigenvector condition open Stays open; Corollary (c) extends the exact law to decoders with *σ_x|x⟩ = q_x|x⟩* and unequal *q_x*. open, narrowed

bottomrule

Plan corrections of version 4, and errors found in version 3

T266.1 The plan's ``I-C family v3: thm:v3-floor'' is not in I-C-CERT; it is in I-C-Ex-CERT and I-C-Ex2-CERT, as is thm:kdreorg.

T266.2 ``thm:s2-exact of A/B/C gives the same dual certificate'': same value and primal; the dual operators differ in form and sign convention and correspond under $W \mapsto W - J(\mathscr{T}) + \Omega$ (Corollary).

T266.3 The Hawk07 label list is a union over sheets (see the table).

T266.4 prop:s2-ceiling is not ``the same hard ceiling'': it bounds an entropy.

T266.5 prop:shredsharp records $(d-1)/d^2$ as a difference of two floors, not as ``the exact gain from reading the record''; on this interface the number is the best-against-best gap, and the correction of version 3 stays.

Errors found in version 3.

V3E.1 The version-3 abstract repeated the version-2 sentence ``the gap $(d-1)/d^2$ is exactly the gain from reading the record'' without the quantifier that version 3 itself corrected (Proposition); the version-4 abstract adds it. The title of Theorem is retained and is to be read with the same quantifier.

V3E.2 Version 3 said that the PG replacement certificate's dual is not of the form $(1-t)\Omega$. That is true of PG-OB-1-CERT-v2 thm:s2-exact, but the same sheet contains the rank-one witness Z_{σ} (PG-OB-1-CERT-v2 thm:v2-Z), which version 3 omitted. The version-3 file is not edited.

Version 5: the items of request t267u, and errors found in version 4

No plan was attached to request t267u, so version 5 has no plan audit. The request asked that the items left open by the version-4 report be addressed honestly; this table records where each is treated and with what status (proved, partial, open, or not applicable to this sheet). Missing is never zero. toprule Item Treatment here Status

midrule Exact error of non-classical decoders outside the eigenvector condition Lower bound exact only under the eigenvector condition (Theorem); upper bound exact iff G is Hermitian positive semidefinite, with an explicit example outside the eigenvector condition (Corollary); otherwise strictly between the bounds proved; exact value in the remaining case *open* (partial)

Channels with an exact best rank-one dual beyond the two proved cases Criterion for every channel (Theorem); unitary orbits (Corollary); new exact decoders (Corollary (a),(d)) proved

VC-OB-1-CERT fibre statements with corrected tensors Not a statement of this sheet; treated in I-C-Th-CERT-v5 n/a here

I-B-G-Q-C: Δ_4 and the conical running constant Treated in I-B-G-Q-C-CERT-v5 n/a here

Readings: Hawk07 CERT v3, PG-OB-1-CERT-v3, En08-ST-DSS-CERT-v4 Section , documentary done

Readings: DS-OB-1-CERT-v3, VC-OB-1-CERT-v3, Red06/V-DST-DSS-CERT-v4 Not cited by this sheet's statements; read in I-C-Th-CERT-v5 and I-B-G-Q-C-CERT-v5 n/a here

bottomrule

Errors found in version 4.

V4E.1 The non-claim of Theorem said ``for generic channels $1-t_{*}$ is strictly above the error; check C23''. Check C23 tested only $\varepsilon \le 1-t_{*}$ (the strict gaps are visible in its recorded values but were not part of its pass criterion), and genericity was not proved. Version 5 replaces the statement by the exact criterion of Theorem , adds the strict test C26, and leaves generic non-exactness unproved.

V4E.2 The non-claim of Corollary said that outside the eigenvector condition ``in the instances of check C23 the error lies strictly below the best rank-one value''. C23 tested only $\le$, and the implied picture is false in general: Corollary (d) gives a decoder outside the eigenvector condition whose error equals the best rank-one value $1-1/S_r$ exactly. The correct statement is Corollary (a),(c). The version-4 file is not edited.

Version 6: the items of request t268u, and errors of earlier versions

No plan was attached to request t268u, so version 6 has no plan audit. The request asked that the items left open

by version 5 be targeted and that the newest companion sheets be read; this table records where each item is treated and with what status (proved, partial, open, conditional, or not applicable to this sheet). Missing is never zero. toprule Item Treatment here Status

midrule Generic non-exactness of the best rank-one dual (version 5: not proved) Theorem , with the explicit non-exact family of Proposition proved

Exact error of channels failing the criterion New lower program L_F with $1 - F_e \le L_F \le \varepsilon$ and the exactness test $L_F = 1 - t_{*}$ (Proposition); $L_F < \varepsilon$ in general (C30) *open* (partial: bracket)

Decoders with singular outputs outside both conditions (version 5: open) Exactness criterion as a linear matrix inequality, closed form for $d = 2$ with one pure output, exact and non-exact examples (Theorem) criterion proved; error when it fails *open*

Decoders with every $\sigma_x > 0$ outside the eigenvector condition and outside $G \ge 0$ Bracket of Corollary (c); $L_F \ge 1 - 1/S_q$ added *open* (partial)

Conservation of $E^{(2)}$; (Hdil) Treated in I-C-Th-CERT-v6 and I-B-G-Q-C-CERT-v6 n/a here

Readings: OB CERT v4, Hawk07-A-CERT-v4, DSS CERT v5 Section , documentary; E6.2 below done

Readings, second wave: OB-1-CERT-v5, Hawk07-*-CERT-v5, DSS CERT v6 Section , documentary done

bottomrule

Errors of earlier versions found in version 6.

E6.1 (Version 5, abstract.) Item (ii) of the version-5 abstract says that dephasing the decoder's outputs ``strictly lowers" the error outside the eigenvector condition. That is imprecise when some $q_x = 0$: then both errors equal $1$. Corollary itself states the equality case correctly (``some $q_x = 0$ or $\sigma_x|x\rangle = q_x|x\rangle$ for every x"); the abstract is to be read with that proviso.

E6.2 The plan-verification table of version 4 (retained) records the DSS version-3 labels thm:EC-dual, thm:EC-reorg, thm:RC-coef, prop:RC-dyn, thm:VDC-dyn, prop:VDC-coef as ``all present in v3; statements unchanged in the newer v4". That is true of the text, but it omits the errata of DSS CERT version 4, which qualify how three of them are read: prop:RC-dyn(d) needs the normalization $\varphi(0) = 0$ or the stated rescaling (Red06-G-DSS-CERT-v4 app:RC-errors); thm:VDC-dyn(c) fails at a prescribed event without the pointwise qualification, and prop:VDC-coef(a) needs a non-zero component (V-DST-DSS-CERT-v4 app:VDC-errors). The omission was pointed out by Red06-G-DSS-CERT-v5 doc:RC-icrow5 (identical text in En08-ST-DSS-CERT-v5 doc:EC-icrow5 and V-DST-DSS-CERT-v5 doc:VDC-icrow5) and confirmed here against the version-4 error lists. The row is to be read together with those errata. No statement of this sheet uses the three labels as premises. No other statement of versions 1–5 was found false. Resolved rather than corrected: the version-5 non-claim ``generic non-exactness is not proved" (Theorem). The second-wave companions (OB-1-CERT-v5, Hawk07-*-CERT-v5, DSS CERT v6) record no error in version 5 of this sheet; their readings of it were checked and found accurate. The earlier files are not edited.

Audit of the parent, checks, and non-claims

Audit of the parent. The parts of I-C-v2 used for comparison (I-C-v2 lem:EB, I-C-v2 thm:A and its proof) were re-read; no mathematical error was found. The parent is not revised.

Computer-assisted checks. Appendix lists floating-point and semidefinite-programming checks of the dual witnesses, the Schmidt overlap, the two counterexample errors, the interpolating path, the classification, the thermal replacement and the dephasing unitary, and (new in version 3) the Bell-saturation criterion, the floor attainers, the non-classical subclass and the optimal retained values (C14, C16–C18). They are computer-assisted, corroborate the proofs, and no proof depends on them. No independent review has been performed. Checks C21–C23 are new in version 4 (convention translation, the Hawk07/PG block dual, the best rank-one dual). Checks C26–C28 are new in version 5 (exactness criterion, the explicit exact decoder, the lower-bound criterion); C25 and C29 belong to the companion sheets. Version 6: checks C30–C33 are new (fidelity primal; singular outputs; depolarized unitaries; generic non-exactness); C34 and C35 belong to the companion sheets.

Open. An exact error formula for non-classical decoders outside the eigenvector condition of Corollary , and for

non-attaining composites of general measurements (finite semidefinite programs compute each instance [Watrous2013]; no closed formula is claimed); any physical identification of the interface. O.1, Gate 0, XV-B, J2, U6-sel and Host C are unchanged. Version 4: the exact error outside the eigenvector condition of Corollary (c); a characterization of the channels whose best rank-one dual is exact beyond Theorem (c),(d). Version 5: the exact error of decoders with every $\sigma_x > 0$ that satisfy neither the eigenvector condition nor $G \geq 0$ (bracket proved, value open), and of decoders with singular outputs outside both conditions; generic non-exactness of the rank-one dual (not proved). Version 6: the exact error of channels failing the criterion of Theorem , including decoders outside the eigenvector condition with $G \not\geq 0$ and singular decoders failing Theorem (b) (brackets proved, values open); whether $L_F = \varepsilon$ on any larger class.

Non-claims. This sheet does not imply a horizon, a horizon law, a temperature, a finite Hilbert space of the host, Einstein dynamics, or any quantum coefficient. The exact dual certificates do not imply that every decoded classical-record channel has a spectral error law. The gap $(d - 1)/d^2$ does not imply a physical separation between a shredder and a horizon. Nothing here closes an open item of the parent or any standing gate. Host stays prescribed, labels stay named.

References

J. Watrous, *The Theory of Quantum Information*, Cambridge University Press, 2018.

M. Horodecki, P. W. Shor and M. B. Ruskai, Entanglement breaking channels, *Rev. Math. Phys.* 15 (2003) 629–641.

J. Watrous, Simpler semidefinite programs for completely bounded norms, *Chicago J. Theor. Comput. Sci.* (2013), Article 8; arXiv:1207.5726.

I-C-v2, *The Shredder Interface*; title page names Ngo Cheung. Parent sheet; source record only.

I-C-Th-v1, *Three Ledgers for Destructive Readout*; source record only.

Big01-A (version 5), *Horizon-Bounded Capacity on a Prescribed de Sitter Host*; title page names Ngo Cheung (authorship requires confirmation). Source record only.

appendix

Label audit

Every cited label of a parent or source sheet was found as a verb || in the read-only copy listed in Appendix (automated; 56 labels, 0 missing). Labels that the plan names but that do not exist are recorded as plan corrections. begingroup toprule Sheet Label Found

midrule endhead Big01-A def:reorg yes

Big01-A lem:shredder yes

Big01-A prop:shredsharp yes

Big01-A thm:B yes

Big01-A thm:Bdephase yes

Big01-A thm:Bexact yes

Big01-A thm:Bsharp yes

Big01-A thm:Bthermal yes

Big01-A thm:tick yes

Big01-A thm:tickexact yes

En08-ST-DSS-CERT-v2 thm:EC-reorg yes

En08-ST-DSS-CERT-v3 lem:EC-sandwich yes

En08-ST-DSS-CERT-v3 thm:EC-dual yes

En08-ST-DSS-CERT-v4 lem:EC-replacement yes

En08-ST-DSS-CERT-v5 cor:EC-decoders yes

En08-ST-DSS-CERT-v5 doc:EC-icrow5 yes
En08-ST-DSS-CERT-v5 doc:EC-pgic5 yes
En08-ST-DSS-CERT-v5 prop:EC-regfloor yes
En08-ST-DSS-CERT-v6 doc:EC-v6read yes
En08-ST-DSS-CERT-v6 prop:EC-assisted yes
Hawk07-A-CERT-v2 thm:s2-exact yes
Hawk07-A-CERT-v3 prop:s2-ceiling yes
Hawk07-A-CERT-v3 prop:v3-drule yes
Hawk07-A-CERT-v3 thm:s2-exact yes
Hawk07-A-CERT-v4 cor:v4-dim yes
Hawk07-A-CERT-v4 prop:v4-rankone yes
Hawk07-A-CERT-v5 prop:v5-kernel yes
Hawk07-B-CERT-v2 thm:s2-exact yes
Hawk07-C-CERT-v2 thm:s2-exact yes
I-C-CERT-v2 thm:capacity yes
I-C-CERT-v5 cor:v5-exact-cases yes
I-C-CERT-v5 cor:v5-upper yes
I-C-CERT-v5 thm:v5-exact yes
I-C-CERT-v5 thm:v5-lower yes
I-C-Ex-CERT-v1 cor:cert-bell yes
I-C-Ex-CERT-v1 lem:cert-sandwich yes
I-C-Ex2-CERT-v1 cor:cert-bell yes
I-C-Ex2-CERT-v1 lem:cert-sandwich yes
I-C-Th-v1 thm:underdetermination yes
I-C-v2 eq:qc yes
I-C-v2 lem:EB yes
I-C-v2 thm:A yes
PG-OB-1-CERT-v2 thm:s2-exact yes
PG-OB-1-CERT-v2 thm:v2-Z yes
PG-OB-1-CERT-v3 cor:v3-decoder yes
PG-OB-1-CERT-v3 prop:v3-Z yes
PG-OB-1-CERT-v3 prop:v3-fibre yes
PG-OB-1-CERT-v4 cor:v4-PG-register yes
PG-OB-1-CERT-v4 thm:v4-PG-floor yes
PG-OB-1-CERT-v5 lem:v5-PG-sandwich yes
PG-OB-1-CERT-v5 thm:v5-PG-divisible yes
PG-OB-1-CERT-v5 thm:v5-PG-shared yes
Red06-G-DSS-CERT-v4 app:RC-errors yes
Red06-G-DSS-CERT-v5 doc:RC-icrow5 yes
V-DST-DSS-CERT-v4 app:VDC-errors yes
V-DST-DSS-CERT-v5 doc:VDC-icrow5 yes
bottomrule endgroup

Computer-assisted checks

All checks in this appendix are computer-assisted (floating point, mpmath, sympy, scipy). They corroborate the proofs, and no proof depends on them. The selection is made by C-common/code/ic_select.py from C-common/code/ic_checks.py (C1–C8, version 1), ic_checks2.py (C9–C13, version 2), ic_checks3.py (C14–C19, version 3) and ic_checks4.py with ic_checks4_curv.py (C20–C24, version 4) and ic_checks5.py with ic_checks5_curv.py (C25–C29, version 5) and ic_checks6.py with ic_checks6_curv.py (C30–C35, new in version 6). Script code/checks.py, output logs/checks.json. begingroup toprule Id Content Result

midrule endhead C1 replacement composites, d=2,3,4, 36 random full-rank outputs: dual Z=(1-1/S)Omega feasible (min eigenvalue of Z-(Omega-I x s')), Schmidt overlap 1/S, SDP value 1-1/S for d<=3 OK ({"max_primal_dev": 1.3183898417423734e-16, "max_sdp_dev": 7.866270634870887e-09, "min_eig_dual": -1.8841072718232774e-16, "samples": 36})

C1b S=sum 1/lambda_i >= d\{}2 on 50 random states, d=3 OK ({})

C2 J(pinching)-Omega/d >= 0 (d=2,3,4); SDP errors of pinching 1-1/d and of X o pinching 1 (d=2,3); equal output spectra OK ({})

C3 errors along (1-s) pinching + s X o pinching, d=3, s=0,0.2,...,1: from 2/3 to 1, monotone (SDP) OK ({"values": [0.6666666668480365, 0.7333333337900749, 0.8000000000750112, 0.8666666668298688, 0.9333333342242476, 1.0000000004932152]})

C7 random channels, d=2,3: SDP error >= 1-F_e OK ({"min_gap": 0.07354203921983427})

C9 basis-measurement interface, d=2,3: stochastic decoders (8 random) SDP error = 1-1/sum_x M_xx\{}-1; general outputs (8 random) between 1-1/sum q_x\{}-1 and 1-1/sum <x|s_x\{}-1|x>; all relabellings (d=2,3): error 1-1/d for the identity, 1 otherwise; path (1-s)I+sX (d=3, 6 points): error 1-(1-s)/d OK ({"general_bounds_ok": true, "max_dev_path": 8.909142312774065e-10, "max_dev_stochastic": 9.919848720230107e-09, "permutations_ok": true, "samples": 8})

C10 thermal replacement R_gamma, d=2,3, two beta each, random spectra: SDP error = 1-1/(Z_b Z_-b) > 1-1/d; dual (1-1/S)Omega feasible OK ({"max_dev": 1.4398845338270405e-08, "min_eig_dual": -1.43982048506075e-16, "qubit_gap1_beta1": 0.8033880667585181})

C11 U=sum_a Z\{}a x |a><a| (d=2,3,4): unitary, commutes with (diagonal H) x I, and with bath I/d gives the complete dephasing OK ({})

C14 Bell-saturation criterion, d=2,3, 12 channels (6 non-covariant projection channels from two random bases, 6 generic random channels): SDP error equals 1-F_e exactly when J Omega = d F_e Omega; error >= 1-F_e always OK ({"rows": 12, "sample": [[2, 0, 0.5, 0.5, 0.0], [2, 1, 0.5, 0.5, 0.0], [2, 2, 0.5, 0.5, 0.0], [2, 3, 0.947168, 0.816969, 0.58751141]]})

C16 measure-and-prepare, d=2,3: rank-one effects from two random bases with outputs the normalized effects give SDP error 1-1/d; perturbed outputs, and full-rank effects with pure outputs, lie strictly above OK ({"excess": [0.02499999728133906, 0.4116805362111944, 0.022222221395135966, 0.33333333080488015], "max_dev": 7.884211061792712e-10})

C17 d=3, q=0.3,0.7: outputs q|x><x|+(1-q)|v_x><v_x| give SDP error 1-q/d; covariance defect under a random unitary is nonzero OK ({"covariance_defect": [0.4471352521368626, 0.3596169421607667], "max_dev": 3.5439751133736763e-10})

C18 optimal retained value over all decoders (one SDP over decoder outputs), d=2,3: basis measurement 1/d, constant record 1/d\{}2, rank-one mixed measurement 1/d, flat effects 1/d\{}2, a rank-2 effect (d=3) strictly between; gain 1/d-1/S for a random record-ignoring output exceeds (d-1)/d\{}2 OK ({"v_star": {"2": [0.5, 0.25, 0.5, 0.25], "3": [0.333333, 0.111111, 0.333333, 0.111111], "gain_d2": [0.428272, 0.25], "gain_d3": [0.328669, 0.222222], "rank2_d3": 0.2}})

C21 random channels, d=2,3: W -> W-J_Delta maps witnesses W>=0, W>=J_Delta onto I-C witnesses Z>=0, Z >=Omega-J with the same Tr_Q; the two SDP optima agree with the SDP error OK ({"max_dev": 8.88262416449235e-16, "max_gap_optima": 3.513781043906761e-08})

C22 d=2,3,4, regular and singular sigma: the block operator Y of the Hawk07/PG replacement certificate, in input-first order, satisfies Y-J_Delta=(1-1/S)Omega, Y>=0, Y>=J_Delta, Tr_Q Y=(1-1/S)I OK ({"max_dev": 1.1102230246251565e-16, "min_eig": -5.857851386599364e-16})

C23 best rank-one dual t*=1/<Omega|J\{}+|Omega>: J-t*Omega>=0 and J-(t*+1e-5)Omega not, t*<=F_e, SDP error <= 1-t* (generic channels d=2,3); t*=1/S for replacement; t*=1/sum_x <x|s_x\{}-1|x> for basis composites; t*=0 for the shifted relabelling OK ({"sample": [[2, "gen", 0.14276, 0.2642, 0.78058], [2, "gen", 0.04157, 0.19296, 0.83706], [2, "gen", 0.13377, 0.34612, 0.73055], [2, "repl", 0.05943, 0.05943], [2, "basis", 0.1603, 0.1603, 0.81476], [2, "shift", 0.0], [3, "gen", 0.00206, 0.10585, 0.92403], [3, "gen", 0.02907, 0.10389, 0.93231]]})

C26 exactness of the best rank-one dual, d=2,3: for full-rank Choi operators M=mat(J\{}-1 Omega); generic channels: M not Hermitian PSD and SDP error < 1-t* strictly; replacement and isotropic channels: M Hermitian PSD and SDP error = 1-t*; unitary conjugation preserves error, t* and the criterion OK ({"sample": [[2, "gen", "False", 0.87, 0.98912], [2, "gen", "False", 0.80999, 0.95917], [2, "gen", "False", 0.84136, 0.99216], [2, "repl", "True", 0.79092, 0.79092], [2, "iso", "True", 0.3, 0.3], [2, "conj", 0.9129, 0.9129], [3, "gen", "False", 0.97269, 0.99797], [3, "gen", "False", 0.93915, 0.99108], [3, "gen", "False", 0.90122, 0.99625], [3, "repl", "True", 0.9938, 0.9938], [3, "iso", "True", 0.3...)

C27 exact rationals and SDP: basis composite with sigma_0=[[1/2,-1/4],[-1/4,1/2]], sigma_1=(1/26)[[9,-6],[-6,17]]: G Hermitian PSD, R=t*G a state with (J-t*Omega)vec R=0, error 11/14 (SDP), lower bound 43/60, sigma_0|0> not an eigenvector OK ({"R": "[[4/7, 2/7], [2/7, 3/7]]", "dual_mineig": -0.0, "lower": "43/60", "sdp": 0.7857143, "sigma0": "[[1/2, -1/4], [-1/4, 1/2]]", "sigma1": "[[9/26, -3/13], [-3/13, 17/26]]", "value": "11/14"})

C28 basis composites d=2,3: random outputs have SDP error > 1-1/S_q strictly, and < 1-t* strictly when G is not Hermitian PSD; outputs with sigma_x|x>=q_x|x> (unequal q_x) have SDP error 1-1/S_q OK ({"sample": [[2, "gen", 0.77016, 0.85849, 0.8971, "False"], [2, "gen", 0.90782, 0.92734, 0.93616, "False"], [2, "gen", 0.69398, 0.79094, 0.81658, "False"], [2, "eig", 0.84828, 0.84828], [3, "gen", 0.9323, 0.96292, 0.98063, "False"], [3, "gen", 0.86234, 0.92182, 0.96059, "False"], [3, "gen", 0.94512, 0.96082, 0.98308, "False"], [3, "eig", 0.77629, 0.77629]]})

C30 fidelity primal L_F=1-min_R <vec R|J|vec R> over states (convex program), d=2,3: random channels of Kraus rank d\{}2 and 2, basis composites and replacement channels satisfy 1-F_e <= L_F <= SDP error <= 1-t*; replacement: L_F = error = 1-t*; number of samples with L_F < error strictly (margin 1e-4) recorded OK ({"sample": [[2, "gen", 0.82969, 0.83872, 0.86287, 0.93016], [2, "gen", 0.67064, 0.70025, 0.7384, 0.93938], [2, "gen", 0.83836, 0.84574, 0.89111, 0.97477], [2, "gen2", 0.76736, 0.78948, 0.87041, 1.0], [2, "gen2", 0.61157, 0.8419, 0.85224, 1.0], [2, "basis", 0.76087, 0.91111, 0.91184, 0.9178], [2, "basis", 0.74406, 0.78708, 0.80939, 0.9551], [2, "repl", 0.75, 0.91332, 0.91332, 0.91332], [3, "gen", 0...)

C31 singular outputs on the basis interface: exact rationals sigma_0=|0><0|, sigma_1=(1/6)[[2,-1],[-1,4]]: R=t*G+ + K a state in ker(J-t*Omega), t*=7/19, SDP error 12/19 > lower bound 3/5; sigma_1=(1/9)[[1,-2],[-2,8]]: the candidate is not a state, t*=4/13, SDP error < 9/13; random instances with one rank-deficient output (d=2,3) and two phase-conjugated exact instances: feasibility of the criterion (LMI) agrees with SDP error = 1-t*; for d=2 with sigma_0=|0><0| the condition alpha det(sigma_1) >= |beta|\{}2 and the value 1-t*=alpha/(alpha+det sigma_1) agree with the solver OK ({"exact_example": {"R": "[[7/19, 6/19], [6/19, 12/19]]", "kernel": true, "lower": "3/5", "sdp": 0.6315789, "state": true, "tstar": "7/19", "upper": "12/19"}, "nonexact_example": {"R": "[[4/13, 18/13], [18/13, 9/13]]", "kernel": true, "lower": "9/17", "sdp": 0.6269976, "state": false, "tstar": "4/13", "upper": "9/13"}, "random": [[2, 0.57208, 0.57208, true], [2, 0.64494, 0.64523, false], [2, 0.7046...)

C32 depolarized unitary T=p Ad_U+(1-p)R_{I/d}, p=1/2, exact sympy: M_T equals (I-p conj(Tr U) U\{}T/(c+pd))/c (c=(1-p)/d); Hermitian for U=Z, X, e\{}{7i/10}diag(1,1,-1) with SDP error = 1-t*; not Hermitian for diag(1,i), diag(1,i,1) with SDP error < 1-t* strictly; closed form of M_T on 6 random unitaries (d=2,3, random p) OK ({"p": "1/2"})

C33 30 random full-rank channels (d=2,3): none has M_T Hermitian PSD; on the segment from a replacement channel (exact) to the depolarized diag(1,i) channel the anti-Hermitian part of M_T vanishes only at the replacement end (11 points) OK ({"random_exact": "0/30", "segment_skew_norm": [0.0, 0.559053, 0.959603,

1.273439, 1.538442, 1.777889, 2.008691, 2.245867, 2.506053, 2.811884, 3.2]})
bottomrule endgroup Result: 22 of 22 checks OK.

Artifact hashes (sha256)

begingroup toprule File sha256
midrule endhead inputs/Big01-A-v5.api.tex
e1160613ea8e02b37fb45954fe677d89f5fa7cca90062d42b68973b03a2bff1d
inputs/DS-OB-1-CERT-v4.tex 0b5b86f87f91503ba9e96cf221b2ef96a4b4e96e3f27489336eb2b89848463c6
inputs/DS-OB-1-CERT-v5.tex a0805f8eb081a250f78e8074b9de5986ccd23106749df9e0fe5fac2f71e157f9
inputs/En08-ST-DSS-CERT-v2.tex 40d0dfb5adda8e3279bdb814d8d7912c0e94be51965927a057a5c5f63ac0ebc2
inputs/En08-ST-DSS-CERT-v3.tex 3882f4e56296b5fa8aecc5cb359bbba8e0b72dac7d2ccc6f2d040c631e7271ff
inputs/En08-ST-DSS-CERT-v4.tex 98c42ee7397263bc694ebed7d66c682b090df437a3230f294b3245f4bd01bee9
inputs/En08-ST-DSS-CERT-v5.tex 9be4fe4fdd9cf1164a75f0218035f0ee3a4c340ed7f15787551de3997fe5af64
inputs/En08-ST-DSS-CERT-v6.tex 7d821d99ced3a9d265d5437f5c1466d8f524a09884730d13662278c31b1da086
inputs/Hawk07-A-CERT-v2.tex 9f4a813e5ed721ac074c05dfd6316a591bac28ccb1be7e23c4972cc32cab0cca
inputs/Hawk07-A-CERT-v3.tex 1e91c9fb7bb97c16d9b4b0d48e821d511badea6e4ace77fc6643a61c76eee776
inputs/Hawk07-A-CERT-v4.tex a88a2430a9e3672a9cbdb7c661dc021154dd3ac4237a1040b4eab097194fceb5
inputs/Hawk07-A-CERT-v5.tex 9db43c8772a84fcf2fcf8d3674b86781ab407b93de05d370740e89f3f00b7add
inputs/Hawk07-B-CERT-v2.tex 47d9b5e886faf80023963a32953a899bba46df79f15ac5dbac33a51e41b5ba5b
inputs/Hawk07-B-CERT-v3.tex 2639d64e7ec3340017a91ed62e487b857317f2cef51966a2086fd016ffcf4176
inputs/Hawk07-C-CERT-v2.tex c54297c3f2e64ff35bf5261c259e9172140927502e5868f13a4e9f92a5e2ce25
inputs/Hawk07-C-CERT-v3.tex 4352ee5e6a754a153631ab755a44e16ea9273fb1b6b9950e8fc29711b983a416
inputs/Hawk07-F-CERT-v4.tex 3b4eef32040e7f3aad9766523b6c5deff83c9679fcd5924d3b93f5b110221e63
inputs/Hawk07-F-CERT-v5.tex 2cfa6282f30e25ac57188298f2e16d3b05a66fff4440204e6f11acce352f5bc3
inputs/Hawk07-H-CERT-v5.tex 74a70015a03906e76cac8fde88a082da9789ea6e863d987be1dbba67f08bb5dd
inputs/I-C-CERT-v1.tex 4f932078b3c653f94093b76ee99a5f78990027ff26d1da1b1914a6a12b4c3980
inputs/I-C-CERT-v2.tex a850d04730a82dd0658262fc616abc03a851ecddd83625c10458af416658512c
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inputs/I-C-CERT-v4.tex 9917ea20e07a3483e08fbd1a7de01eb2084eb3f7f5be23f2474a7c6283496487
inputs/I-C-CERT-v5.tex 56c30a1a434ce3cec589929093d7764a97330fbc27748afdf8e6f5bc82b984bf
inputs/I-C-Ex-CERT-v1.tex 2b3e5ef48225271bbc8ad56bb798c7d138ecd5afd20fe31682b5f105f11f568f
inputs/I-C-Ex2-CERT-v1.tex 8cc8832536c8cae006822f5f35e88c008a9b45dde9cb845c364fae3ec81c35b3
inputs/I-C-Th-v1.tex 3d8d2a76deea9469326a2fc864368c9503eb93acea792787eb45adc4610e3f60
inputs/I-C-v2.tex 6684c95253a6b61d5690ad3374c6a38779c11d569fcc5d4c45a6bea639c5d187
inputs/PG-OB-1-CERT-v2.tex 015f230247fc86885e278ecf2b796b14652f7742c57e200fde8e670fbaf09a40
inputs/PG-OB-1-CERT-v3.tex 8c0a7e2fdbb7e4558bad17bc35df10a66feea6ba4959af524921393a61eafbcc
inputs/PG-OB-1-CERT-v4.tex 3dc921457bd781606e9e673231a8c832be104435950913521fdfb2ed737fc1c1
inputs/PG-OB-1-CERT-v5.tex c96ea4631de51c635521ca42fd2433728591676ee9019a24f3ad9fcdcdeb8a2
inputs/PLAN_t258u.md a5984d52062bafe5e031047d509b7cbccf0f5ca537965d258c2ba1ca5edf6f49
inputs/PLAN_t262u.tex 630a938e80b215d75906abf685d62aa99ec48e89783a14395a3f25d3d5fdacbc
inputs/PLAN_t266u.md 39aea6f1117318f9517712567fc87db46dd188176498fd71ff637076175f0621
inputs/Red06-G-DSS-CERT-v4.tex 574d8abfdcd620476e8054092bce4323652afd03740ceb917a165798bc653ebc
inputs/Red06-G-DSS-CERT-v5.tex e4a1b02552c138b8f9f009f2e71e391825ad0014f717b43f96883c445605213a

inputs/Red06-G-DSS-CERT-v6.tex c3f04faab2c76fb661054f2c1393bd71f484a1bfdfeeace52e5c612a34fc4599

inputs/V-DST-DSS-CERT-v4.tex 550372abe2117364ec4e3f62e8840d66570c8845ad27e842d6ed1b74b5321302

inputs/V-DST-DSS-CERT-v5.tex 4133961f566c059c22054b5922423d1ff94cb0d1c888ca353a7c143bba1541be

inputs/V-DST-DSS-CERT-v6.tex 710c656210aab6dd708cd9f830ab7bf59df592f0887ab168b1e7fa7be2df95f9

inputs/VC-OB-1-CERT-v4.tex f9802da66896159503132b736164016c1d06d2a6530bbbc06cb4ad3af0256387

inputs/VC-OB-1-CERT-v5.tex d3a93a3afb0e8803d18f230ce824f78c19253f6a4bc841f8ceef763ce14f7cac

../C-common/code/preamble_template.tex
2b3b5de7495a5e175a4545b6c4a01927c9b879d373a0a3b14a0e17947cd9d3b5

../C-common/code/build.py 5dbc919f5773abb3bf12f3e56aacdab7a8161d61f05035945fd40204d536a88f

code/sheet.json 589b81c12a328f7ec8064c9ddf0a8f616d0425132f2a338018e10f40eeb98f5b

code/checks.py c2bb3de7e5b344dcb0034e326cf3df01673f205e33ea00130235badc5c45acc0

parts/01_front.tex e299817023394f4aa0bddf71f97d382367540405c242daba339fbd8b30bc5d21

parts/02_sandwich.tex 2e39bc2113ded9ad148d20b4e6d863e62a89be19ae0b8993a300be9e6ce07292

parts/03_setting.tex 9e0f5d8d07c984ea4cd3169413f35670d218e88f9249f740ece2ca37ccf9ac9f

parts/04_cert.tex c0553f24380a7e6e6af15e3c9cdf631d3d7fef18ddef992f7b77cd2a3d76bde9

parts/04b_v2.tex 086addaa3c1e7bb9f22932281ba398fa4033cb221a89dd442abcc60f0cd3f854

parts/04c_v3.tex 4c3b3ec2d3fa143e724d9b73cfc42d941ce1350f8ee809d1c3e7fd562fc8eb0a

parts/04d_v4.tex f9221c8432f58e606010bba2a1a97d319a865c26244b5d247b748420f6f12d17

parts/04e_v5.tex 66f5be5032c3602569c983610b8900dc55c44db87472208ffe44c230c0003bba

parts/04f_v6.tex b76f83274dc6c2288ec24c29a410c2aa5798fffd9af717828de13ff78c0e0b7b

parts/05_plan.tex 7e172f3c6aa1e553b2321c32cb094a3744a27facfd9ad84d47b85bd15eb21194

logs/checks.json 9ae67fdd846dde1c22cbe28f4362b6500b747429bb12b02bbc512ba071dc5708

logs/inputs.sha256 9e065c61f58ed61a40cde81924155e2d098f227170761e22de5f0e9285b9c120

../C-common/code/ic_checks.py 912c95e4b1db7998f312426da001034368de6c3dd64e0fb4a9a42b7e53021bbf

../C-common/code/ic_checks2.py 1f5f5c348b1c711e6466347e12904db433a889a4247c94d378fb9275e38c2da7

../C-common/code/ic_select.py 8bc20e667e8571b0f6f78b5e6ea3168fb9e815186613d2c07735690b4ec5729e

../logs/get_log_S1.txt f4ad94933a9086eed368e50d2b7a5d9ea1b22ded9a1a07ec55ad12d5fb4872ac

../logs/get_log.txt f639ab99a753eed5217b1d001fe5cfa88a1c4191f4c4c7b89decb05f71514108

../logs/plan_label_audit.json f59bffdb66c63c8e64734a5490205357ea03fe78ea6108abed0d05b9365ef86b

../C-common/code/ic_checks3.py d2661722e5dfa7a3493a3d1a7f7b15db2f20db64e6fca34eb6641d62d90b4fca

../logs/get_log_S1v2.txt 470a182c91cdb81697779de8edc5498d31b4806ce398ca65e9481e46d6214683

../C-common/code/ic_checks4.py 5e573a4fa9fdee85ae57225cad0d652b0ae87e26a0213902433278467228b204

../C-common/code/ic_checks4_curv.py
355487e6b9e2d97d373cef95ab4849b155069a05b6357c19427c8517adab0365

../C-common/code/ic_checks5.py fb661fa24dca783522dd2a86bd39a4b5d178f840d6539c2a865ba730e4e532bb

../C-common/code/ic_checks5_curv.py
3c08d86437024cb0ed1f1031fb7409b055de461517d0b92e21f16a27f484b13d

../C-common/code/ic_checks6.py d883c43acada82668fa5dc4f51d8f9437aafe1d701fa88b11855df075a7db81e

../C-common/code/ic_checks6_curv.py
5436c6c1011618c22e8ac3e57b13c865108b0a7def56cf1f842d8d3efbe1d7c1

midrule Frozen tree (before)

XVI-P ec965c15ae25cd3bb9cc2b3a134e966b19a1803c78d1fe5bf7554da2a8b3aaa6 OK

XVI-Q 0df9a0d10409d31ac0c2d0a47197e3f6e03c7ae258ec5fb775137f66cb0493c4 OK

XVI-R 57187bc6825f5c6ae88633b7279c6c0be57f7c4918fdbc525da4aaf958331edb OK

bottomrule endgroup The hashes of the delivered .tex and PDF are recorded in MANIFEST-CertPlan-S1v6.sha256, not here, because a file cannot contain its own hash.