I-C-Ex2-CERT-v6: Exact Dual Certificates for the Access Law 1 − k/d

Written dual witnesses, the memory model, the covariant class, the Bell-saturation criterion, all channels at the access floor, the best rank-one dual, when it is exact, and why it generically is not

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Abstract

This is version 6 of the standalone certificate sheet I-C-Ex2-CERT. It contains version 5 (I-C-Ex2-CERT-v5, which contains versions 4, 3, 2 and 1; all stay untouched) and adds the items of request t268u (no plan was attached), superseding version 5 by addition; errors of earlier versions found in version 6 are recorded in Section. The parent sheet I-C-Ex2 is cited as a source record only and is not revised. For a cut whose accessible output is a classical record plus quantum blocks of dimension at most k, followed by any decoder on a d-dimensional code, the entanglement fidelity is at most k/d and the half-diamond error at least 1 − k/d. Here the optimum 1 − k/d is certified on both sides: the Bell vector is the primal witness and (1 − k/d)Ω is a written dual witness for the attaining channel T_{d, k}. For the parent's finite memory model (d = qk, classical label of dimension q, quantum memory of dimension k), the dual witness (1 − 1/q)Ω certifies the recovery error 1 − k/d on both sides. For every object of the class the error is at least 1 − F_e ≥ 1 − k/d, and it equals 1 − F_e exactly on the covariant class. Exact recovery forces k = d. The plan's statement that the error of every such channel equals the formula is false: the errors of the class fill [1 − k/d, 1]. This is recorded as a plan correction. New in version 2: the twirl of any channel is covariant, so its error is exactly 1 − F_e by the written dual, and the twirl is a replacement channel exactly when F_e = 1/d^2, where the spectral law gives the same value; every replacement channel twirls to $\mathscr{R}$_{I/d}. The class is closed under unitary pre-encoding, decoder changes, block-local refinement and coin mixing, so 1 − k/d is a hard floor (retained value at most k/d) along every reorganization. The commutant step of the attainer rests on a local Schur–Weyl argument: the OpenAI catalogue, which the plan proposed as a source record for it, contains no Choi-operator or isotropic-channel entries (plan correction). New in version 3. (i) For every channel, the error equals 1 − F_e if and only if J|Ω⟩ = dF_e|Ω⟩, if and only if J ≥ F_eΩ; covariance is sufficient, not necessary. (ii) The composites of the class are exactly the channels with a Kraus family of rank at most k, and such a channel is at the floor 1 − k/d exactly when it has the form X ↦ ∑_i λ_iP_iXP_i with rank-k projections and ∑_i λ_iP_i = I; each has the certificate (Φ_d, (1 − k/d)Ω), with no covariance assumed. (iii) The memory-model attainer and the covariant attainer T_{d, k} are two members of this class. (iv) The Hawk07 and PG replacement certificates are comparisons whose dual operator is not of the form (1 − t)Ω (plan correction; qualified in version 4, since PG-OB-1-CERT-v2 also has the rank-one witness and the block dual corresponds to it under a sign change). No version-2 statement was found to be false. New in version 4. (i) A witness in the Hawk07/PG convention is turned into one of this sheet by W ↦ W − J($\mathscr{T}$) + Ω, with the same partial trace; the Hawk07/PG block dual of a replacement channel becomes exactly the rank-one witness (1 − 1/S)Ω. (ii) The best dual of the form (1 − t)Ω has t_{*} = 1/⟨Ω|J^{+}|Ω⟩, exact for replacement channels and on the Bell-saturated class. (iii) Enlarging the accessible block from k to k^{′} lowers the floor by (k^{′} − k)/d; scaling the code and the block by the same factor leaves it unchanged; so thm:kdreorg is closure under the local moves of Definition, not under Big01-A's def:reorg (plan correction). (iv) Readings of PG-OB-1-CERT-v2, Hawk07-A/B/C-CERT-v3, En08-ST-DSS-CERT-v3 and Big01-A. One version-3 error is recorded. New in version 5. (i) For every channel, the best rank-one dual (1 − t_{*})Ω is exact if and only if ker (J − t_{*}Ω) contains vec (R) for a state R; for full-rank J, if and only if the matrix of J^{−1}|Ω⟩ is Hermitian positive semidefinite. Exactness is invariant under unitary conjugation. (ii) Pinching onto m orthogonal blocks of arbitrary sizes has error exactly 1 − 1/m, certified by the rank-one dual; for unequal sizes this channel is neither covariant nor Bell-saturated, and its error lies strictly above the floor 1 − k/d. (iii) Readings of PG-OB-1-CERT-v3, Hawk07-A/B/C-CERT-v3 and En08-ST-DSS-CERT-v4. One version-4 error is recorded (Section). New in version 6. (i) The fidelity primal L_F,

a convex program over states, satisfies $1 - F_e \le L_F \le \varepsilon \le 1 - t_{*}$, and the best rank-one dual is exact if and only if $L_F = 1 - t_{*}$. (ii) Non-exactness is generic: the channels whose matrix $M_{\mathscr{T}}$ is not Hermitian form an open dense set of full measure, on which $\varepsilon < 1 - t_{*}$ strictly. A depolarized unitary $p\mathrm{Ad}_U + (1-p)\mathscr{R}_{I/d}$ is exact if and only if $\mathrm{Tr}\, U = 0$ or U is a phase times a Hermitian unitary. (iii) The mixtures $p\,\mathrm{id} + (1-p)\cdot$(pinching onto m blocks of any sizes) have error exactly $(1-p)(1-1/m)$, with a witness independent of p. (iv) Readings of the newly delivered OB CERT v4, Hawk07 CERT v4 and DSS CERT v5 sheets; one omission of version 4 is recorded (Section). The exact error of non-exact channels stays open. All proofs are local; the parent and Big01-A, Hawk07-A/B/C-CERT, PG-OB-1-CERT and En08-ST-DSS-CERT-v2 are source records, not premises. No quantum coefficient is admitted. Host C is not created; Host A and Host B stay separate. O.1 stays NOT_CLOSED. XVI-N is not discharged (N-Ins.4 unresolved), and there is no Route-B successor. XV-B is not opened. H2, I3 and T3 stay unresolved; J2 stays Unresolved. U6-sel is not moved. Res02-A Q1–Q4 and A1–A2 stay open. The last reading bars (quantized observable, no external clock) stay open. No Einstein dynamics, derived metric, spacetime emergence, external-clock removal or Page curve is claimed. Gate 0 stays NOT_SUPPLIED, and thm:J is not substituted. Continuum acquisition stays a negative certificate. No compilation is asserted. Citations identify source records only; no proof transfer. Authorship requires confirmation. Nothing is pushed or registered.

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I-C-Ex2-CERT-v6: Exact Dual Certificates for the Access Law *1 − k/d* Written dual witnesses, the memory model, the covariant class, the Bell-saturation criterion, all channels at the access floor, the best rank-one dual, when it is exact, and why it generically is not Ngo Cheung (authorship requires confirmation)

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Parent, sources and scope

Previous version. I-C-Ex2-CERT-v1 (sha256 8cc8832536c8cae006822f5f35e88c008a9b45dde9cb845c364fae3ec81c35b3) is retained in the sections before Section , unchanged except for the cross-reference paragraph at the end of Section ; the file I-C-Ex2-CERT-v1 itself is not edited.

Version 2. I-C-Ex2-CERT-v2 (sha256 76cb43695a7ca4e98333ccea318fb6a2b118246b1a52b830317427ab45ac256b) is retained in the sections before Section and in its plan audits; Section was new in version 3. The file I-C-Ex2-CERT-v2 itself is not edited.

Version 3. I-C-Ex2-CERT-v3 (sha256 d3a5ddcf5dae6b48f5bdf031c5016e506466580629f0e7defd4003dca065a52b) is retained in the sections before Section and in its plan audits; Section and the plan-t266u audit were new in version 4. The file I-C-Ex2-CERT-v3 itself is not edited.

Version 4. I-C-Ex2-CERT-v4 (sha256 62dd45f881ff66fd6722b23c8e1cca42131d5ea5910fe7ccaccfc6d927806818) is retained in the sections before Section and in its plan audits; Section and Section were new in version 5. The file I-C-Ex2-CERT-v4 itself is not edited.

Version 5. I-C-Ex2-CERT-v5 (sha256 b147e794d9836bae2c7622bbdc0e07ccb6006adb3f755b3037ae0904525dd5b1) is retained in the sections before Section and in its plan audits; Section and Section are new. The file I-C-Ex2-CERT-v5 itself is not edited.

Parent. The parent sheet is I-C-Ex2; its latest version, I-C-Ex2 (sha256 698337771275c0d804ce51948910abe930d1c30b65c10ad0d6e254ccd3d7d403), was read by read-only retrieval and is a source record [Parent]. Its access cut is I-C-Ex2 eq:cut, its sharp law I-C-Ex2 thm:sharp, and its complement criterion I-C-Ex2 thm:complement. What this sheet needs is re-proved in Section . The parent stays untouched; this sheet is not a version of it.

Other source records. Big01-A version 5 (sha256 e1160613ea8e02b37fb45954fe677d89f5fa7cca90062d42b68973b03a2bff1d) records the bound $1 - k/d$ without attainment (Big01-A prop:Bblocks). It also records the reorganization monoid Big01-A def:reorg, used as a pattern. The companion sheets I-C-CERT-v1 and I-C-Ex-CERT-v1 are cited for cross-references, and the OpenAI catalogue (sha256 76314b644e51864fa1818f28f49f32a2ce2e06bdd32364ad939595c792cbf99d) was searched. New in version 3: Hawk07-A-CERT-v2 (sha256 9f4a813e5ed721ac074c05dfd6316a591bac28ccb1be7e23c4972cc32cab0cca), Hawk07-B-CERT-v2 (47d9b5e886faf80023963a32953a899bba46df79f15ac5dbac33a51e41b5ba5b), Hawk07-C-CERT-v2 (c54297c3f2e64ff35bf5261c259e9172140927502e5868f13a4e9f92a5e2ce25), PG-OB-1-CERT-v2 (015f230247fc86885e278ecf2b796b14652f7742c57e200fde8e670fbaf09a40) and En08-ST-DSS-CERT-v2 (40d0dfb5adda8e3279bdb814d8d7912c0e94be51965927a057a5c5f63ac0ebc2); their v1 files (Hawk07, PG) and v3 file (En08) were also checked and contain the cited labels. None is a premise [BigA]. New in version 4: Hawk07-A-CERT-v3 (sha256 1e91c9fb7bb97c16d9b4b0d48e821d511badea6e4ace77fc6643a61c76eee776), Hawk07-B-CERT-v3 (2639d64e7ec3340017a91ed62e487b857317f2cef51966a2086fd016ffcf4176), Hawk07-C-CERT-v3 (4352ee5e6a754a153631ab755a44e16ea9273fb1b6b9950e8fc29711b983a416), the newest delivered Hawk07 CERT versions; the cited statements are unchanged from v2 (checked). Hawk07 CERT v3 corrects the version-2 Definition def:v2-H and the name ``conformal warp in the sheets that contain them; neither is cited here. En08-ST-DSS-CERT-v3 (sha256 3882f4e56296b5fa8aecc5cb359bbba8e0b72dac7d2ccc6f2d040c631e7271ff), the version named by the plan; En08-ST-DSS-CERT-v4 (98c42ee7397263bc694ebed7d66c682b090df437a3230f294b3245f4bd01bee9) is newer and has the cited statements unchanged. PG-OB-1-CERT-v2 as above (only v2 is delivered; its v3, in preparation elsewhere, was not read). None is a premise. New in version 5: PG-OB-1-CERT-v3 (sha256 8c0a7e2fdbb7e4558bad17bc35df10a66feea6ba4959af524921393a61eafbcc), newest delivered, superseding the PG-OB-1-CERT-v2 cited above; Hawk07-A/B/C-CERT-v3 as above; En08-ST-DSS-CERT-v4 (sha256 98c42ee7397263bc694ebed7d66c682b090df437a3230f294b3245f4bd01bee9). Newer versions of Hawk07 CERT (version 4), of the OB CERT sheets (version 4) and of DSS CERT (version 5) are being built concurrently and were not read. None is a premise. New in version 6, each verified against its delivery manifest: PG-OB-1-CERT-v4 (sha256 3dc921457bd781606e9e673231a8c832be104435950913521fdfb2ed737fc1c1); Hawk07-A-CERT-v4 (sha256 a88a2430a9e3672a9cbdb7c661dc021154dd3ac4237a1040b4eab097194fceb5); En08-ST-DSS-CERT-v5 (sha256 9be4fe4fdd9cf1164a75f0218035f0ee3a4c340ed7f15787551de3997fe5af64); Red06-G-DSS-CERT-v5 (sha256 e4a1b02552c138b8f9f009f2e71e391825ad0014f717b43f96883c445605213a); V-DST-DSS-CERT-v5 (sha256 4133961f566c059c22054b5922423d1ff94cb0d1c888ca353a7c143bba1541be); VC-OB-1-CERT-v4 (sha256 f9802da66896159503132b736164016c1d06d2a6530bbbc06cb4ad3af0256387); DS-OB-1-CERT-v4 (sha256 0b5b86f87f91503ba9e96cf221b2ef96a4b4e96e3f27489336eb2b89848463c6); Red06-G-DSS-CERT-v4 and V-DST-DSS-CERT-v4 are read for their error lists. None is a premise. Second wave read in version 6: DS-, VC- and PG-OB-1-CERT-v5, Hawk07-A/F/H-CERT-v5 and En08-ST/Red06-G/V-DST-DSS-CERT-v6, verified against their manifests and cited by sha256 in Section as source records only.

Standing ledger (verbatim). No quantum coefficient is admitted. Host C is not created; Host A and Host B stay separate. O.1 stays NOT_CLOSED. XVI-N is not discharged (N-Ins.4 unresolved), and there is no Route-B successor. XV-B is not opened. H2, I3 and T3 stay unresolved; J2 stays Unresolved. U6-sel is not moved. Res02-A Q1–Q4 and A1–A2 stay open. The last reading bars (quantized observable, no external clock) stay open.

No Einstein dynamics, derived metric, spacetime emergence, external-clock removal or Page curve is claimed. Gate 0 stays NOT_SUPPLIED, and thm:J is not substituted. Continuum acquisition stays a negative certificate. No compilation is asserted. Citations identify source records only; no proof transfer. Authorship requires confirmation. Nothing is pushed or registered.

Plan refusal lines. Host stays prescribed, labels stay named. No finite Hilbert space, temperature, or Einstein dynamics derived. None imports Big01-A or any other sheet as a premise. The certificates are documentary separations: they do not discharge O.1, Gate 0, XV-B, J2, U6-sel or any standing gate, and they do not create Host C. No quantum coefficient is admitted. No derived metric, spacetime emergence or Page curve is claimed. This sheet revises no other sheet.

Authorship. The title page names Ngo Cheung. Authorship requires confirmation. No compilation is asserted.

The primal–dual sandwich

Throughout this sheet $Q \cong Q^{\{'\}} \cong E \cong \mathbb{C}^d$ with $d \geq 2$, and a fixed intended identification of $Q^{\{'\}}$ with Q. Put

$$|\Omega\rangle = \sum_{j=0}^{d-1} |j\rangle_E |j\rangle_Q = \sqrt{d}\,|\Phi_d\rangle, \qquad \Omega = |\Omega\rangle\langle\Omega|, \qquad J(\mathscr{T}) = (\mathrm{id}_E \otimes \mathscr{T})(\Omega),$$

so $J(\mathscr{T})$ is the unnormalized Choi operator of a channel $\mathscr{T}: Q \to Q^{\{'\}}$, $J(id) = \Omega$, $Tr_Q\, J(\mathscr{T}) = I_E$, and $F_e(\mathscr{T}) = \langle\Omega|J(\mathscr{T})|\Omega\rangle/d^2$. The error of $\mathscr{T}$ is

$$\varepsilon(\mathscr{T}) = d_\diamond(\mathscr{T}, \mathrm{id}_Q) = \tfrac{1}{2}\|\mathscr{T} - \mathrm{id}_Q\|_\diamond.$$

Lemma. Let $\mathscr{T}: Q \to Q^{\{'\}}$ be a channel.

• *(Primal witness.)* For every unit vector $\psi \in E \otimes Q$, $\varepsilon(\mathscr{T}) \geq 1 - \langle\psi|(id_E \otimes \mathscr{T})(proj\ \psi)|\psi\rangle$.

• *(Dual witness.)* For every operator Z on $E \otimes Q$ with

$$Z \geq 0, \qquad Z \geq \Omega - J(\mathscr{T}),$$

one has $\varepsilon(\mathscr{T}) \leq \|Tr_Q\, Z\|_{\infty}$.

• *(Exact dual certificate.)* If a unit vector ψ and an operator Z as in (a) and (b) give the same value t, that is $1 - \langle\psi|(id_E \otimes \mathscr{T})(proj\ \psi)|\psi\rangle = t = \|Tr_Q\, Z\|_{\infty}$, then $\varepsilon(\mathscr{T}) = t$ exactly. The pair (ψ, Z) is then called an *exact dual certificate* for $\mathscr{T}$.

Proof. (a) Put $\rho = proj\ \psi$ and $\rho^{\{'\}} = (id_E \otimes \mathscr{T})(\rho)$. Testing $\rho - \rho^{\{'\}}$ with the projector ρ gives $1/2\|\rho - \rho^{\{'\}}\|_1 \geq Tr\,[\rho(\rho - \rho^{\{'\}})] = 1 - \langle\psi|\rho^{\{'\}}|\psi\rangle$. The half-diamond distance is at least this trace distance.

(b) The half-diamond norm of the difference of two channels on a d-dimensional input is attained on pure inputs with a reference of dimension d [Watrous]; we use this standard fact only. Every unit vector of $E \otimes Q$ has the form $|\psi\rangle = (A \otimes I_Q)|\Omega\rangle$ with $Tr\, A^{\dagger}A = 1$. Since $A \otimes I_Q$ acts on the untouched reference,

$$\mathrm{proj}\,\psi - (\mathrm{id}_E \otimes \mathscr{T})(\mathrm{proj}\,\psi) = (A \otimes I)(\Omega - J(\mathscr{T}))(A \otimes I)^{\dagger} =: \Delta_\psi,$$

which is Hermitian and traceless. Hence $1/2\|\Delta_{\psi}\|_1 = \max_{0 \leq P \leq I} Tr\,(P\Delta_{\psi})$. For every such P, using $Z - (\Omega - J(\mathscr{T})) \geq 0$, then $P \leq I$ and $Z \geq 0$,

$$\mathrm{Tr}\,(P\Delta_\psi) \leq \mathrm{Tr}\,(P(A \otimes I)Z(A \otimes I)^{\dagger}) \leq \mathrm{Tr}\,((A^{\dagger}A \otimes I)Z) = \mathrm{Tr}\,(A^{\dagger}A\,\mathrm{Tr}_Q Z) \leq \|\mathrm{Tr}_Q Z\|_{\infty}.$$

Taking the maximum over ψ proves (b). (c) is (a) and (b) together.

par *Does not imply*. The lemma is the standard primal–dual pair for the half-diamond distance to the identity; it is proved here, not imported from any cited sheet, and it does not imply any physical recovery protocol. par

Corollary. Let $0 \leq t \leq 1$.

• If $J(\mathscr{T}) \geq t\,\Omega$, then $Z = (1 - t)\Omega$ is a dual witness and $\varepsilon(\mathscr{T}) \leq 1 - t$.

• For every channel, $\varepsilon(\mathscr{T}) \geq 1 - F_e(\mathscr{T})$, with the Bell vector Φ_d as primal witness.

• If $J(\mathscr{T}) \geq F_e(\mathscr{T})\,\Omega$, then $(\Phi_d, (1 - F_e(\mathscr{T}))\Omega)$ is an exact dual certificate and $\varepsilon(\mathscr{T}) = 1 - F_e(\mathscr{T})$.

Proof. (a) $Z \geq 0$, and $Z - (\Omega - J(\mathscr{T})) = J(\mathscr{T}) - t\Omega \geq 0$. Moreover $Tr_Q\, \Omega = I_E$, so $Tr_Q\, Z = (1 - t)I_E$. (b) is Lemma (a) with $\psi = \Phi_d$, since $\langle\Phi_d|(id \otimes \mathscr{T})(\Phi_d)|\Phi_d\rangle = F_e(\mathscr{T})$. (c) combines (a) with $t = F_e(\mathscr{T})$ and (b).

par *Does not imply*. The corollary gives explicit constants only. It does not imply that every channel satisfies the Bell criterion of part (c). par

Cross-reference. The same lemma and corollary, with the same local proofs, are in the companion sheets I-C-CERT and I-C-Ex-CERT (I-C-CERT-v1 lem:cert-sandwich, I-C-CERT-v1 cor:cert-bell; I-C-Ex-CERT-v1 lem:cert-sandwich, I-C-Ex-CERT-v1 cor:cert-bell; unchanged in their versions 2). Each sheet proves them independently; nothing here depends on the companions.

Setting: cuts of quantum block dimension at most k

Definition. Let $Q \cong Q^{'} \cong \mathbb{C}^d$, $d \geq 2$. A finite cut of quantum block dimension at most k is a channel $\mathcal{M}(X) = \bigoplus_r \mathcal{N}_r(X)$ with completely positive branches $\mathcal{N}_r: \mathcal{B}(Q) \to \mathcal{B}(K_r)$, $k_r = \dim K_r \leq k$, $\sum_r \mathrm{Tr}\, \mathcal{N}_r(X) = \mathrm{Tr}\, X$; the direct-sum label is a classical record. A decoder is a channel on the record and blocks; the composite is $\mathcal{T} = \mathcal{D}\mathcal{M}: Q \to Q^{'}$. Put $p_r = \mathrm{Tr}\, \mathcal{N}_r(I/d)$. The case $k = 1$ is classical-only.

par *Does not imply*. The definition restates the parent's access cut (I-C-Ex2 eq:cut). It does not bound an encoder's working memory and does not model a physical memory. par

Lemma. For every cut and decoder, $F_e(\mathcal{D}\mathcal{M}) \leq 1/d \sum_r p_{r\min}(k_r, d) \leq k/d$, hence $\varepsilon(\mathcal{D}\mathcal{M}) \geq 1 - k/d$.

Proof. A unit vector of $\mathbb{C}^d \otimes \mathbb{C}^d$ with coefficient matrix A, $\|A\|_2 = 1$, $\mathrm{rank}\, A \leq m$, has Bell overlap $1/d|\mathrm{Tr}\, A|^2 \leq 1/d\|A\|_1^2 \leq m/d$. By convexity the same bound holds for states of Schmidt number at most m, and local completely positive maps (each Kraus operator multiplies the coefficient matrix on one side) do not increase the Schmidt number. The r-block of $(\mathrm{id} \otimes \mathcal{M})(\Phi_d)$ has trace p_r and lives on $E \otimes K_r$, so its normalized state has Schmidt number at most $\min(k_r, d)$; after decoding, its Bell overlap is at most $p_{r\min}(k_r, d)/d$. Sum over r; then use Corollary (b).

par *Does not imply*. This re-proves the parent's resource bound (I-C-Ex2 thm:sharp) for local use; it is the Schmidt-number mechanism of the literature [TerhalHorodecki] and does not imply attainment, which is Lemma . par

Lemma. Let $1 \leq k \leq d$, P_0 a rank-k projection, $P_U = UP_0U^{\dagger}$, and $T_{d,k}(X) = d/k \int P_U X P_U \, dU$ (Haar measure). Then:

- $T_{d,k}$ is a channel, covariant under every unitary conjugation, with $F_e(T_{d,k}) = k/d$;
- its Choi operator is isotropic: $J(T_{d,k}) = k\,\Phi_d + \frac{d(1 - k/d)}{(d^2 - 1)}(I - \Phi_d)$;
- $T_{d,k} = \mathcal{D}\mathcal{N}$ for a finite cut with blocks of dimension k.

Proof. (a) $\int P_U \, dU$ commutes with every unitary, so it is $(k/d)I$ by its trace; hence $T_{d,k}$ is trace preserving. Covariance follows from invariance of Haar measure. The Kraus formula $F_e = d^{-2} \sum |\mathrm{Tr}\, K|^2$ gives $F_e = d^{-2} d/k \int |\mathrm{Tr}\, P_U|^2 \, dU = k/d$. (b) Covariance gives $(\bar{U} \otimes U) J (\bar{U} \otimes U)^{\dagger} = J$ for all U, because $(\bar{U} \otimes U)|\Omega\rangle = |\Omega\rangle$. The commutant of $\{\bar{U} \otimes U\}$ is spanned by I and Φ_d (the standard isotropic twirl [Watrous]), so $J = a\Phi_d + b(I - \Phi_d)$; $\mathrm{Tr}\, J = d$ and $\langle \Phi_d | J | \Phi_d \rangle = dF_e = k$ fix $a = k$, $b = d(1 - k/d)/(d^2 - 1)$. (c) The maps $X \mapsto P_U X P_U$ form a compact subset of a real vector space of dimension d^4; the Haar average lies in its convex hull, which is compact, and by Carathéodory it is a convex combination $\sum_j w_j P_j X P_j$ of at most $d^4 + 1$ of them. With $P_j = V_j V_j^{\dagger}$, $V_j: \mathbb{C}^k \to \mathbb{C}^d$ isometric, the branches $A_j = \sqrt{(d w_j/k)}\, V_j^{\dagger}$ satisfy $\sum_j A_j^{\dagger} A_j = d/k \sum_j w_j P_j = I$; return j and $A_j X A_j^{\dagger}$, and decode block j by $Y \mapsto V_j Y V_j^{\dagger}$.

par *Does not imply*. The lemma re-proves the parent's finite attainment for local use. It does not imply an efficient synthesis of $T_{d,k}$. par

The attainer has an exact dual certificate

Theorem. Let $1 \leq k \leq d$ and let $T_{d,k}$ be the channel of Lemma . Then

$$J(T_{d,k}) - \frac{k}{d}\Omega = \frac{d(1 - k/d)}{d^2 - 1}(I - \Phi_d) \;\geq 0,$$

so $(\Phi_d, Z_{d,k} = (1 - k/d)\Omega)$ is an exact dual certificate and

$$\varepsilon(T_{d,k}) = 1 - \frac{k}{d}.$$

Hence the minimum of $d_{\diamond}(\mathcal{D}\mathcal{N}, id_Q)$ over all cuts of quantum block dimension at most k and all decoders equals $1 - k/d$, certified from below by the Bell witness and from above by $Z_{d, k}$.

Proof. By Lemma (b), $J(T_{d, k}) = k\,\Phi_d + (d(1 - k/d))/(d^2 - 1)(I - \Phi_d)$, while $k/d\Omega = k\,\Phi_d$. The difference is a nonnegative multiple of a projector, and $F_e(T_{d, k}) = k/d$. Apply Corollary (c). The lower bound for every cut and decoder is Lemma , and Lemma (c) puts $T_{d, k}$ in the class.

par *Does not imply*. The dual operator replaces the triangle-inequality upper bound of the earlier proof by an explicit semidefinite witness. It does not change the value $1 - k/d$ and does not imply an efficient implementation of $T_{d, k}$. par

What holds for every object of the class

Theorem. Let $\mathcal{N}$ be a cut as in Definition , with block dimensions k_r and weights $p_r = Tr\,\mathcal{N}_r(I/d)$, and let $\mathcal{D}$ be any decoder; put $\mathcal{T} = \mathcal{D}\mathcal{N}$.

• $\varepsilon(\mathcal{T}) \geq 1 - F_e(\mathcal{T}) \geq 1 - 1/d\sum_r p_r \min(k_r, d) \geq 1 - k/d$.

• If $\mathcal{T}$ is covariant, $\mathcal{T} \circ Ad_U = Ad_U \circ \mathcal{T}$ for every unitary U (equivalently, its Choi operator is isotropic), then $(\Phi_d, (1 - F_e(\mathcal{T}))\Omega)$ is an exact dual certificate and $\varepsilon(\mathcal{T}) = 1 - F_e(\mathcal{T})$.

• For every channel $\mathcal{T}$, its twirl $\bar{\mathcal{T}} = \int Ad_{U^{\dagger}} \circ \mathcal{T} \circ Ad_U\, dU$ is covariant, has $F_e(\bar{\mathcal{T}}) = F_e(\mathcal{T})$, and satisfies $\varepsilon(\mathcal{T}) \geq \varepsilon(\bar{\mathcal{T}}) = 1 - F_e(\mathcal{T})$.

Proof. (a) The first inequality is Corollary (b); the others are Lemma .

(b) Covariance forces the isotropic form $J(\mathcal{T}) = d[F\Phi_d + (1 - F)/(d^2 - 1)(I - \Phi_d)]$ with $F = F_e(\mathcal{T})$, as in the proof of Lemma (b). Then $J(\mathcal{T}) - F\Omega = (d(1 - F))/(d^2 - 1)(I - \Phi_d) \geq 0$, and Corollary (c) applies.

(c) Since $(I \otimes U)|\Phi_d\rangle = (U^T \otimes I)|\Phi_d\rangle$, each conjugate $Ad_{U^{\dagger}}\,\mathcal{T}Ad_U$ has the same entanglement fidelity as $\mathcal{T}$, and so does the average. Unitary pre- and post-composition does not change the half-diamond distance to $id = Ad_{U^{\dagger}}\,idAd_U$, and the distance is convex. The Haar average is a finite convex combination, as in the proof of Lemma (c), so $\varepsilon(\bar{\mathcal{T}}) \leq \varepsilon(\mathcal{T})$. The twirl is covariant by invariance of Haar measure; apply (b).

par *Does not imply*. Part (c) is a twirling inequality, not an exact formula for $\mathcal{T}$. It does not imply that the error of a non-covariant object equals $1 - F_e$. par

Proposition. Let $1 \leq k \leq d$ and $X|j\rangle = |j + 1 \bmod d\rangle$.

• The classical cut ($k_r = 1 \leq k$) with the pinching decoder and with the relabelled decoder give Δ and $Ad_X \circ \Delta$. Both have the same blocks and weights, hence the same value of $1 - 1/d\sum_r p_r k_r = 1 - 1/d$, but $\varepsilon(\Delta) = 1 - 1/d$ and $\varepsilon(Ad_X \circ \Delta) = 1$.

• The set of errors of the class is exactly $[1 - k/d, 1]$.

• In particular, for every k with $1 \leq k \leq d$, the statement ``the error of every such channel equals $1 - k/d$" is false; at $k = d$ the channel Ad_X has error 1 while the class minimum is 0.

Proof. (a) $J(\Delta) = \sum_j |jj\rangle\langle jj| \geq \Omega/d$ gives $\varepsilon(\Delta) = 1 - 1/d$ by Corollary (c). The input $|0\rangle|0\rangle$ is sent by $Ad_X \circ \Delta$ to an orthogonal state, so Lemma (a) gives error 1. (b) Two cuts of quantum block dimension at most k with their decoders can be mixed by a classical coin: the mixed instrument has the union of the branches, relabelled by the coin outcome, and the decoder acts blockwise. So the class of composites is convex. The error is 1-Lipschitz for $d_{\diamond}$, hence continuous along the segment from $T_{d, k}$ to $Ad_X \circ \Delta$. The intermediate value theorem, Theorem and (a) give every value in $[1 - k/d, 1]$; $\varepsilon \leq 1$ and Theorem (a) give the reverse inclusion. (c) follows from (a), (b), and $\varepsilon(Ad_X) = 1$ for $d \geq 2$ (test $|0\rangle|0\rangle$ again).

par *Does not imply*. The proposition does not imply that the exact optimum or the covariant-class formula fails; it shows only that the resource formula is not the error of every object. par

Corollary. If $\varepsilon(\mathcal{D}\mathcal{N}) = 0$, then every block of positive weight has $k_r \geq d$; in particular $k \geq d$, so within $k \leq d$ exact recovery forces $k = d$. At $k = d$ the bound $1 - k/d = 0$ is attained by the trivial cut with the identity decoder, with the exact dual certificate $(\Phi_d, 0)$.

Proof. $\varepsilon = 0$ forces $F_e = 1$ by Theorem (a), so $\sum_r p_r \min(k_r, d) \geq d$ with $\sum_r p_r = 1$; hence $\min(k_r, d) = d$ whenever $p_r > 0$. For the identity, $J = \Omega$, and $Z = 0$ satisfies $Z \geq \Omega - J = 0$.

par *Does not imply*. The corollary is the finite access-dimension statement only. It does not imply a horizon, a soft-charge sector, or any physical memory, and it does not replace the parent's complete-complement criterion (I-C-Ex2 thm:complement), which is a different (rank) statement. par

The parent's finite memory model: dual certificate

The parent's declared finite memory candidate (I-C-Ex2 eq:softmodel) is restated here. Let $d = qk$ with $q \geq 2$, identify $Q = \mathbb{C}^q \otimes \mathbb{C}^k$, fix a unitary U on Q, and put

$$A_r = (\langle r| \otimes I_k)U, \qquad \mathcal{N}_{\mathrm{mem}}(X) = \mathrm{bigoplus}_{r=0}^{q-1} A_r X A_r^{\dagger},$$

a cut of quantum block dimension k with a classical label r. Let $e_{rec}(\mathcal{N}) = \inf_{\mathcal{D}} d_{\diamond}(\mathcal{D}\mathcal{N}, id_Q)$.

Theorem. In this model, let $\mathcal{D}_U$ reinsert $|r\rangle$ and apply $U^{\dagger}$, as in the parent's proof of I-C-Ex2 thm:softhair. Then

$$J(\mathcal{D}_U \mathcal{N}_{\mathrm{mem}}) - \frac{1}{q}\Omega \geq 0, \qquad F_{\mathrm{e}}(\mathcal{D}_U \mathcal{N}_{\mathrm{mem}}) = \frac{1}{q},$$

so $(\Phi_d, (1 - 1/q)\Omega)$ is an exact dual certificate and $\varepsilon(\mathcal{D}_U\mathcal{N}_{mem}) = 1 - 1/q = 1 - k/d$, for every unitary U. Hence $e_{rec}(\mathcal{N}_{mem}) = 1 - k/d$ is certified on both sides by explicit witnesses; the lower bound for every decoder is Lemma .

Proof. Write $\Phi = \Delta_q \otimes id_k$, where Δ_q is the pinching of $\mathbb{C}^q$ in the basis $(|r\rangle)$, so that $\mathcal{D}_U\mathcal{N}_{mem} = Ad_{U^{\dagger}} \circ \Phi \circ Ad_U$. Order $E \otimes Q$ as $(E_q \otimes Q_q) \otimes (E_k \otimes Q_k)$; then $\Omega_d = \Omega_q \otimes \Omega_k$ and $J(\Phi) = J(\Delta_q) \otimes \Omega_k$. On $span\{|rr\rangle\}$, $J(\Delta_q) = \sum_r |rr\rangle\langle rr|$ is the identity and Ω_q has norm q, so $J(\Delta_q) \geq \Omega_q/q$; tensoring with $\Omega_k \geq 0$ gives $J(\Phi) \geq \Omega_d/q$. For channels, $J(Ad_V \circ \Phi \circ Ad_U) = WJ(\Phi)W^{\dagger}$ with $W = U^T \otimes V$, because $(I \otimes U)|\Omega\rangle = (U^T \otimes I)|\Omega\rangle$. With $V = U^{\dagger}$, $W|\Omega\rangle = (I \otimes U^{\dagger}U)|\Omega\rangle = |\Omega\rangle$. Hence $J(\mathcal{D}_U\mathcal{N}_{mem}) - \Omega/q = W(J(\Phi) - \Omega/q)W^{\dagger} \geq 0$. Also $\langle\Omega|J(\mathcal{D}_U\mathcal{N}_{mem})|\Omega\rangle = \langle\Omega_{qrvert}J(\Delta_q)|\Omega_q\rangle\ |\langle\Omega_k, \Omega_k\rangle|^2 = q\,k^2$, so $F_e = qk^2/d^2 = 1/q$. Apply Corollary (c); the identity $1/q = k/d$ is $d = qk$.

par *Does not imply*. The theorem is a certificate for the declared finite model only. The words ``memory candidate" remain an access label. It does not imply a soft-charge sector, soft hair, a horizon, or any physical memory, and it does not identify U with any dynamics. par

Version 2: the twirl reduction, the reorganization reading, and the commutant

This section is new in version 2 (plan t258u). Everything in it is proved locally.

Theorem. Let $\mathcal{T}$ be any channel on $\mathbb{C}^d$, $F = F_e(\mathcal{T})$, and $\mathcal{F}$ its twirl (Theorem (c)).

• $\mathcal{F}(X) = a\,X + (1 - a)Tr\,(X)\,I/d$ with $a = (d^2F - 1)/(d^2 - 1)$, and $\varepsilon(\mathcal{F}) = 1 - F$ with the exact dual certificate $(\Phi_d, (1 - F)\Omega)$; $\varepsilon(\mathcal{T}) \geq 1 - F$.

• $\mathcal{T}$ is a replacement channel if and only if $F = 1/d^2$, and then $\mathcal{T} = \mathcal{R}_{I/d}$, whose error by the spectral law ($S = d^2$) is $1 - 1/d^2 = 1 - F$: the two certificates agree.

• For every replacement channel $\mathcal{R}_{\sigma}$, $F_e(\mathcal{R}_{\sigma}) = 1/d^2$, $\mathcal{R}_{\sigma} = \mathcal{R}_{I/d}$, and $\varepsilon(\mathcal{R}_{\sigma}) = 1 - 1/S \geq 1 - 1/d^2 = \varepsilon(\mathcal{R}_{\sigma})$, $S = Tr\,\sigma^{-1}$, with equality iff $\sigma = I/d$.

Proof. (a) By the proof of Theorem (b), $J(\mathcal{F}) = d[F\Phi_d + (1 - F)/(d^2 - 1)(I - \Phi_d)]$. The channel $X \mapsto aX + (1 - a)Tr\,(X)I/d$ has Choi operator $a\,d\,\Phi_d + (1 - a)I/d$; comparing the coefficients of Φ_d and $I - \Phi_d$ gives $(1 - a)/d = d(1 - F)/(d^2 - 1)$, which is the stated a. The rest is Theorem (b),(c). (b) A channel of the form in (a) is a replacement channel iff $a = 0$ (apply it to X and to $X^{'}$ with $Tr\,X = Tr\,X^{'}$), iff $F = 1/d^2$; the output is then I/d. For $\mathcal{R}_{I/d}$, $J = I \otimes I/d \geq \Omega/d^2$ because $\Omega \leq d\,I$, so Corollary (c) gives $1 - 1/d^2$, which is the spectral value $1 - 1/S$ at $S = d^2$. (c) $J(\mathcal{R}_{\sigma}) = I \otimes \sigma$, so $\langle\Omega|J|\Omega\rangle = Tr\,\sigma = 1$ and $F_e = 1/d^2$; apply (b). The exact value: for $\sigma > 0$, Cauchy–Schwarz gives $|\langle\Omega, z\rangle|^2 \leq \langle\Omega, (I \otimes \sigma^{-1})\Omega\rangle\langle z, (I \otimes \sigma)z\rangle = S\langle z, Jz\rangle$, so $J \geq \Omega/S$ and $\varepsilon \leq 1 - 1/S$ (Corollary (a)); the Schmidt vector with weights $p_i \propto \lambda_i^{-1}$ in an eigenbasis of σ has overlap $\sum_i p_i^2\lambda_i = 1/S$, so $\varepsilon \geq 1 - 1/S$ (Lemma (a)); if σ is singular, a product vector on its kernel gives $\varepsilon = 1 = 1 - 1/S$. Finally $S \geq d^2$ with equality iff $\sigma = I/d$ (Cauchy–Schwarz).

par *Does not imply*. The reduction connects three certificates (spectral, covariant, resource) inside this sheet; the same spectral law is proved in the companion I-C-CERT (I-C-CERT-v1 thm:replacement), which is not used.

Twirling loses information: it does not imply that $\varepsilon(\mathcal{T}) = 1 - F_e(\mathcal{T})$ for non-covariant $\mathcal{T}$, and the twirl of a channel is a replacement only in the case $F = 1/d^2$. Check C12 corroborates (a)–(c) with the single-qubit Clifford group. par

Definition. For fixed d and k, a *reorganization* of an account $(\mathcal{N}, \mathcal{D})$ of Definition is any combination of: (i) a unitary pre-encoding $\mathcal{N} \mapsto \mathcal{N} \circ Ad_V$; (ii) a change of decoder, including any classical post-processing of the record; (iii) a block-local refinement, replacing a branch $\mathcal{N}_r$ by branches $\mathcal{M}_{r,s} \circ \mathcal{N}_r$ for an instrument $(\mathcal{M}_{r,s})_s$ on K_r with output blocks of dimension at most k; (iv) a classical coin mixture of two accounts. The *retained value* is $v = 1 - \varepsilon(\mathcal{DN})$.

par *Does not imply*. This is the local pattern of Big01-A's reorganization monoid (Big01-A def:reorg) for a finite cut; no host is introduced. par

Theorem. The class of cuts of quantum block dimension at most k, with their decoders, is closed under every reorganization. Hence $F_e \le k/d$, $\varepsilon \ge 1 - k/d$ and $v \le k/d$ hold along every orbit, and the ceiling $v = k/d$ is attained (by $T_{d,k}$, Theorem) with an exact dual certificate. A retained value above k/d requires an accessible block of dimension larger than k.

Proof. (i) $\mathcal{N}_r \circ Ad_V$ maps into the same K_r and the branch traces still sum to the trace. (ii) A classical post-processing followed by a decoder is a decoder. (iii) The new branches are completely positive into blocks of dimension at most k, and $\sum_s Tr\, \mathcal{M}_{r,s}(Y) = Tr\, Y$ keeps the trace. (iv) is the convexity argument of Proposition (b). The bounds are Lemma on every element of the class. For the last sentence, apply Lemma with $max_r\, k_r$ in place of k.

par *Does not imply*. The ceiling is a statement about finite cuts. It does not imply a capacity of any host or horizon, a soft-charge sector, or a physical memory. par

Remark. The proof of Lemma (b) uses that the commutant of $\{\bar{U} \otimes U\}$ is spanned by I and Φ_d. A local proof: an operator A commuting with all $\bar{U} \otimes U$ corresponds, under the partial transpose on the first factor, to an operator commuting with all $U \otimes U$, which by Schur–Weyl duality for two tensor factors is spanned by I and the swap; the partial transpose of the swap is $\Omega = d\,\Phi_d$. The plan proposed to cite OpenAI catalogue entries on Choi operators and isotropic channels as source records for this step; the fetched catalogue contains no such entries (plan correction), so the step rests on this local argument and on the standard reference [Watrous] only.

Version 3: Bell saturation and all channels at the access floor

This section is new in version 3 (plan t262u). Everything in it is proved locally from the earlier sections. Hawk07-A/B/C-CERT, PG-OB-1-CERT, En08-ST-DSS-CERT-v2 and Big01-A are cited as source records only. The plan's ``local rank-one certificate'' is Corollary (a),(b) of version 1 and is not restated.

The Bell-saturation criterion

Theorem. Let $\mathcal{T}$ be a channel on $\mathbb{C}^d$ and $F = F_e(\mathcal{T})$. The following are equivalent:

- $\varepsilon(\mathcal{T}) = 1 - F$;
- $J(\mathcal{T})|\Omega\rangle = dF|\Omega\rangle$;
- $J(\mathcal{T}) \ge F\,\Omega$. When they hold, $(\Phi_d, (1-F)\Omega)$ is an exact dual certificate. In particular unitary covariance is sufficient, but not necessary, for $\varepsilon = 1 - F_e$.

Proof. Put $P = |\Phi_d\rangle\langle\Phi_d| = \Omega/d$, $B = J(\mathcal{T})/d$ (a state) and $H = P - B$, the output difference on the Bell input; $Tr\, H = 0$ and $Tr\,(PH) = 1 - F$. Write $H = H_{+} - H_{-}$ with $H_{\pm} \ge 0$, $H_{+}H_{-} = 0$, so $Tr\, H_{+} = 1/2\|H\|_1$.

(i)⇒(ii). By the definition of the diamond norm, $1/2\|H\|_1 \le \varepsilon(\mathcal{T}) = 1 - F$. Hence

$$\operatorname{Tr} H_{+} = \tfrac{1}{2}\|H\|_1 \le 1 - F = \operatorname{Tr}(PH) = \operatorname{Tr}(PH_{+}) - \operatorname{Tr}(PH_{-}) \le \operatorname{Tr}(PH_{+}) \le \operatorname{Tr} H_{+},$$

so all are equalities: $Tr\,[(I-P)H_{+}] = 0$ and $Tr\,(PH_{-}) = 0$. Since $(I-P)H_{+}(I-P) \ge 0$ has trace zero, $H_{+}^{1/2}(I-P) = 0$, so $H_{+} = PH_{+} = (Tr\, H_{+})P = (1-F)P$; likewise $H_{-}^{1/2}P = 0$, so $H_{-}|\Phi_d\rangle = 0$. Therefore $H|\Phi_d\rangle = (1-F)|\Phi_d\rangle$, i.e. $B|\Phi_d\rangle = F|\Phi_d\rangle$, which is equivalent to (ii) because $J = dB$ and $|\Omega\rangle = \sqrt{(d)}|\Phi_d\rangle$.

(ii)⇒(iii). $J = J(\mathcal{T}) \geq 0$ is Hermitian with eigenvector $|\Omega\rangle$ (norm $\sqrt{(d)}$) and eigenvalue dF, so $J = F\Omega + J^{'}$ with $J^{'} = (I - P)J(I - P) \geq 0$.

(iii)⇒(i). Corollary (a) with $t = F$ gives $\varepsilon \leq 1 - F$, and Corollary (b) gives $\varepsilon \geq 1 - F$. The same two statements give the exact certificate. A unitarily covariant channel satisfies $(U \otimes U)J(U \otimes U)^{\dagger} = J$ for every unitary U, and $|\Omega\rangle$ spans the joint fixed vectors of these operators; since J commutes with all of them, it maps $|\Omega\rangle$ to a multiple of $|\Omega\rangle$, which is (ii). Non-covariant channels satisfying (ii) are given below.

par *Does not imply*. The criterion characterizes when the scalar $1 - F_e$ is the error; it does not give a scalar error law for channels that fail (ii), and the companion sheets, where the same criterion is proved independently, are not used. Check C14 corroborates the equivalence by semidefinite programming. par

All channels at the access floor

Theorem. Fix $1 \leq k \leq d$ and let $\mathcal{A}_{d, k}$ be the set of composites $\mathcal{DN}$ of Definition .

• $\mathcal{A}_{d, k}$ is exactly the set of channels on $\mathbb{C}^d$ that have a finite Kraus family of operators of rank at most k.

• For $\mathcal{T} \in \mathcal{A}_{d, k}$ the following are equivalent: (1) $\varepsilon(\mathcal{T}) = 1 - k/d$; (2) $F_e(\mathcal{T}) = k/d$; (3) there are finitely many rank-k orthogonal projections P_i and numbers $\lambda_i > 0$ with

$$\sum_i \lambda_i P_i = I, \qquad \mathcal{T}(X) = \sum_i \lambda_i P_i X P_i.$$

• Every such channel has $J(\mathcal{T})|\Omega\rangle = k|\Omega\rangle$, $J(\mathcal{T}) \geq k/d\Omega$, and the exact certificate $(\Phi_d, (1 - k/d)\Omega)$. No covariance is assumed.

Proof. (a) A branch $\mathcal{N}_r$ has Kraus operators $A_{r\alpha}: \mathbb{C}^d \to K_r$. The record makes the input of the decoder block diagonal, so on block r the decoder is a channel $\mathcal{D}_r$ from $\mathcal{B}(K_r)$ with Kraus operators $B_{r\beta}: K_r \to \mathbb{C}^d$; the composite has the Kraus family $(B_{r\beta}A_{r\alpha})$, of ranks at most $k_r \leq k$. Conversely, let $\mathcal{T}$ have Kraus operators K_i of ranks $r_i \leq k$. Choose isometries $W_i: \mathbb{C}^{r_i} \to \mathbb{C}^d$ onto the ranges and put $A_i = W_i^{\dagger}K_i$. Then $\sum_i A_i^{\dagger}A_i = \sum_i K_i^{\dagger}K_i = I$, so $\mathcal{M}(X) = \bigoplus_i A_iXA_i^{\dagger}$ is a cut with blocks of dimension $r_i \leq k$, and the decoder $Y_i \mapsto W_iY_iW_i^{\dagger}$ on block i gives $\mathcal{T}$.

(b) Let (K_i) be as in (a), $K_i \neq 0$, and R_i the projection onto the orthogonal complement of $\ker K_i$, of rank $r_i \leq k$; then $K_i = K_iR_i$. By the Hilbert–Schmidt Cauchy–Schwarz inequality,

$$|\mathrm{Tr}\, K_i|^2 = |\mathrm{Tr}\,(R_iR_iK_i)|^2 \leq \|R_i\|_2^2\|R_iK_i\|_2^2 \leq r_i\|K_i\|_2^2 \leq k\|K_i\|_2^2.$$

With $F_e(\mathcal{T}) = d^{-2}\sum_i |Tr\ K_i|^2$ and $\sum_i \|K_i\|_2^2 = Tr \sum_i K_i^{\dagger}K_i = d$ this gives $F_e \leq k/d$, the Kraus-level form of Lemma . (1)⇒(2): $1 - k/d = \varepsilon \geq 1 - F_e \geq 1 - k/d$ (Corollary (b)). (2)⇒(3): all inequalities above are equalities for every i with $K_i \neq 0$. Hence $r_i = k$; $\|R_iK_i\|_2 = \|K_i\|_2$, so $(I - R_i)K_i = 0$ and $K_i = R_iK_i$; and equality in Cauchy–Schwarz gives $R_iK_i = z_iR_i$ with $z_i \neq 0$. So $K_i = z_iP_i$ with $P_i = R_i$ of rank k; put $\lambda_i = |z_i|^2$, and $\sum_i K_i^{\dagger}K_i = I$ gives $\sum_i \lambda_iP_i = I$. (3)⇒(1) is (c).

(c) With $|P_i\rangle\rangle = (I \otimes P_i)|\Omega\rangle$ one has $J(\mathcal{T}) = \sum_i \lambda_i|P_i\rangle\rangle\langle\langle P_i|$ and $\langle\langle P_i|\Omega\rangle\rangle = Tr\ P_i = k$, so

$$J(\mathcal{T})|\Omega\rangle = k\sum_i \lambda_i(I \otimes P_i)|\Omega\rangle = k|\Omega\rangle.$$

So $F_e = k/d$ and (ii) of Theorem holds; that theorem gives $J \geq k/d\Omega$, $\varepsilon = 1 - k/d$ and the certificate. The channels in (3) have Kraus operators $\sqrt{(\lambda_i)}P_i$ of rank k, so they lie in $\mathcal{A}_{d, k}$ by (a).

par *Does not imply*. The theorem classifies the attainers of the access floor. It does not give the error of a non-attaining composite, and class-wide attainability does not mean that a fixed cut can reach the floor with some decoder. It does not model a physical memory. Check C15 corroborates both directions numerically. par

Corollary. The composite $\mathcal{D}_U\mathcal{N}_{mem}$ of Theorem is $X \mapsto \sum_r P_rXP_r$ with $P_r = U^{\dagger}(\mathrm{proj}\ r \otimes I_k)U$, rank k, $\sum_r P_r = I$; it is of the form (3) with $\lambda_r = 1$. The attainer $T_{d, k}$ has the finite form $\sum_j w_jP_jXP_j$ of Lemma (c), of the form (3) with $\lambda_j = dw_j/k$.

Proof. For the memory model, $A_r = (\langle r| \otimes I_k)U$ and $\mathcal{D}_U$ maps block r by $Y \mapsto U^{\dagger}(\mathrm{proj}\ r \otimes Y)U$, so the composite has Kraus operators $U^{\dagger}(|r\rangle \otimes I_k)(\langle r| \otimes I_k)U = P_r$. For $T_{d, k}$, $d/k\sum_j w_jP_j = I$ by Lemma (c).

par *Does not imply*. These are two implementations of the same equality conditions; they do not show that every access-limited composite is at the floor (Proposition). par

Documentary comparisons

Hawk07 and PG. Hawk07-A-CERT-v2 thm:s2-exact (identically in Hawk07-B-CERT-v2 thm:s2-exact, Hawk07-C-CERT-v2 thm:s2-exact and PG-OB-1-CERT-v2 thm:s2-exact) is a replacement-recovery statement: the half-diamond error of $\mathscr{R}_{\sigma}$ is $1 - 1/\Xi(\sigma)$. Its primal witness is a Schmidt vector, which is the Bell vector only for $\sigma = I/d$. Its dual witness is a block operator $Y \geq \sigma \otimes I - \Omega$, written output-first for $\mathscr{R}_{\sigma} - id$, i.e. with the opposite sign convention to Lemma (b); it is not of the form $(1 - t)\Omega$. The access certificates of this sheet are of the form $(1 - t)\Omega$. Hawk07-A-CERT-v2 prop:s2-ceiling (and its copies in Hawk07-B/C-CERT and PG-OB-1-CERT) is an entropy bound $S(\rho_R) \leq \min(\log d_R, \log d_{R^c} + S(\rho))$, a fixed-resource bound of a different kind from the access ceiling k/d; neither identifies an interior or gives a Page curve.

En08. En08-ST-DSS-CERT-v2 thm:EC-dual is a pair of matching lower and upper witnesses for a hull deficit. The present target is a half-diamond distance. No Bell vector, Choi operator, lattice facet or prescribed host is transferred between them.

Big01 reorganizations. Big01-A def:reorg and Big01-A thm:tick identify the fixed-resource pattern only. Under the reorganizations of Definition , $\varepsilon \geq 1 - k/d$ and $\nu = 1 - \varepsilon \leq k/d$ (Theorem). The error stays above its floor and the retained value below its ceiling; these are not monotonicity statements for an individual account, since decoder changes can raise or lower the error.

Non-covariant objects. The scalar formula $1 - F_e$ is characterized exactly by Theorem ; it is not confined to covariant channels. This does not give one scalar error law for every channel. For a fixed finite channel the error is the value of a finite semidefinite program [Watrous2013]; the open item is a closed formula or classification beyond Theorem . The twirl does not keep the error of the original channel in general (Theorem (c)).

Catalogue. The OpenAI catalogue copy of version 2 (sha256 76314b644e51864fa1818f28f49f32a2ce2e06bdd32364ad939595c792cbf99d) was searched again in this run: no whole-word ``Choi'' entry; ``isotropic'' occurs only in unrelated entries (convex bodies, inverse problems, Heisenberg ferromagnets, Chern classes, anisotropic groups). This is a search of one copy, not an exhaustive audit.

Typed cross-reference. The memory-model and general access certificates satisfy the locally proved projection-saturation conditions (Theorem , Corollary); the companion I-C-Ex-CERT proof is not imported, and neither certificate supplies a Page curve or an interior.

Version 4: sign conventions, the best rank-one dual, and what a reorganization may change

This section is new in version 4 (plan t266u). Everything in it is proved locally from Sections –. Hawk07-A/B/C-CERT (v3 cited; v2 also read), PG-OB-1-CERT-v2, En08-ST-DSS-CERT-v3 and Big01-A are cited as source records only; I-C-Ex2-CERT-v3 is the previous version of this sheet.

Sign conventions and the best rank-one dual

Throughout this subsection $\mathscr{T}$ is a channel on $\mathbb{C}^d$, $J = J(\mathscr{T})$ is its Choi operator in the order $E \otimes Q$ of Section , and $J_{\Delta} := J - \Omega$ is the Choi operator of $\mathscr{T} - id$. The swap $F|e\rangle|q\rangle = |q\rangle|e\rangle$ fixes Ω, and $F(\cdot)F$ turns an operator written with the output factor first into one written in the order $E \otimes Q$, carrying the partial trace over the output factor to Tr_Q.

Lemma. For an operator W on $E \otimes Q$ put $Z = W - J_{\Delta} = W + \Omega - J$. Then

$$(W \geq 0 \text{ and } W \geq J_{\Delta}) \Leftrightarrow (Z \geq 0 \text{ and } Z \geq \Omega - J), \qquad \mathrm{Tr}_Q Z = \mathrm{Tr}_Q W.$$

Hence every $W \geq 0$ with $W \geq J_{\Delta}$ gives $\varepsilon(\mathscr{T}) \leq \|Tr_Q W\|_{\infty}$, and the two witness sets, $\{W \geq 0, W \geq J_{\Delta}\}$ and $\{Z \geq 0, Z \geq \Omega - J\}$ of Lemma (b), have the same infimum of $\|Tr_Q \cdot\|_{\infty}$; $W \mapsto W - J_{\Delta}$ is a bijection between them that preserves Tr_Q.

Proof. $W \geq J_{\Delta}$ is $Z \geq 0$, and $W \geq 0$ is $Z \geq -J_{\Delta} = \Omega - J$. Since $Tr_Q J = I_E = Tr_Q \Omega$, $Tr_Q J_{\Delta} = 0$, so $Tr_Q Z = Tr_Q W$. The bound is Lemma (b) applied to Z. The inverse map is $Z \mapsto Z + J_{\Delta}$.

par *Does not imply*. The lemma is bookkeeping between two sign conventions for one semidefinite bound. It does not say that witnesses written in the two conventions are equal operators, it does not use any companion sheet, and

it implies no recovery protocol. Check C21 corroborates the bijection and the equality of the two optima. par

Theorem. Put $t_{*}(\mathscr{T}) = sup \{t \geq 0: J \geq t\, \Omega\}$, and let J^{+} be the Moore–Penrose inverse of J.

• $t_{*}(\mathscr{T}) = 1/\langle\Omega|J^{+}|\Omega\rangle$ if $|\Omega\rangle$ lies in the range of J, and $t_{*}(\mathscr{T}) = 0$ otherwise; the supremum is attained.

• $t_{*}(\mathscr{T}) leF_e(\mathscr{T}) \leq 1$ and $\varepsilon(\mathscr{T}) \leq 1 - t_{*}(\mathscr{T})$: among dual witnesses of the form $(1 - t)\Omega$, the best value is $1 - t_{*}$.

• $t_{*}(\mathscr{T}) = F_e(\mathscr{T})$ if and only if the equivalent conditions of Theorem hold; then $\varepsilon(\mathscr{T}) = 1 - t_{*}$ with the Bell primal witness.

• *(Replacement channels.)* Let $\mathscr{R}_{\sigma}(X) = Tr(X)\sigma$, with eigenvalues λ_i of σ and $S_{\sigma} = \sum_i \lambda_i^{-1}$ ($1/S_{\sigma} = 0$ if σ is singular). Then $t_{*}(\mathscr{R}_{\sigma}) = 1/S_{\sigma}$, and $(\psi_{\sigma}, (1 - 1/S_{\sigma})\Omega)$ is an exact dual certificate, with $\psi_{\sigma} = \sum_i \sqrt{(p_i)}|i\rangle|i\rangle$ in an eigenbasis of σ (taken as the basis defining Ω), $p_i = \lambda_i^{-1}/S_{\sigma}$ if $\sigma > 0$, and $\psi_{\sigma} = |k\rangle|k\rangle$ with $\lambda_k = 0$ otherwise. So $\varepsilon(\mathscr{R}_{\sigma}) = 1 - 1/S_{\sigma}$, while $t_{*}(\mathscr{R}_{\sigma}) < F_e(\mathscr{R}_{\sigma}) = 1/d^2$ unless $\sigma = I/d$: the best rank-one dual can be exact without Bell saturation.

Proof. (a) We use: *for $J \geq 0$ and a vector v, $J \geq |v\rangle\langle v|$ iff $v \in ran\, J$ and $\langle v|J^{+}|v\rangle \leq 1$.* If $J \geq |v\rangle\langle v|$ and $Jx = 0$, then $0 \leq |\langle x, v\rangle|^2 \leq \langle x|J|x\rangle = 0$, so $v \perp ker\, J$, i.e. $v \in ran\, J$; with $x = J^{+}v$, $\langle v|J^{+}|v\rangle^2 = |\langle x, v\rangle|^2 \leq \langle x|J|x\rangle = \langle v|J^{+}JJ^{+}|v\rangle = \langle v|J^{+}|v\rangle$, so $\langle v|J^{+}|v\rangle \leq 1$. Conversely, if $v = JJ^{+}v$ then for every x, by Cauchy–Schwarz for the positive form $\langle\cdot|J|\cdot\rangle$, $|\langle x, v\rangle|^2 = |\langle x|J|J^{+}v\rangle|^2 \leq \langle x|J|x\rangle\, \langle v|J^{+}JJ^{+}|v\rangle = \langle x|J|x\rangle\, \langle v|J^{+}|v\rangle \leq \langle x|J|x\rangle$. Apply this to $v = \sqrt{(t)}\, |\Omega\rangle$: if $|\Omega\rangle \notin ran\, J$ only $t = 0$ is allowed; otherwise $\langle\Omega|J^{+}|\Omega\rangle > 0$ (J^{+} is positive definite on $ran\, J$) and the allowed t form the closed interval $[0, 1/\langle\Omega|J^{+}|\Omega\rangle]$. (b) $J \geq t_{*}\Omega$ gives $d^2F_e = \langle\Omega|J|\Omega\rangle \geq t_{*}d^2$. Corollary (a) with $t = t_{*}$ gives $\varepsilon \leq 1 - t_{*}$, and $(1 - t)\Omega$ is a witness exactly when $J \geq t\Omega$. (c) If $t_{*} = F_e$, then $JgeF_e\, \Omega$, which is (iii) of Theorem . Conversely (iii) gives $t_{*}geF_e$, hence $t_{*} = F_e$ by (b); Theorem gives $\varepsilon = 1 - F_e$ with the Bell primal. (d) $J(\mathscr{R}_{\sigma}) = \sum_{i,j} |i\rangle\langle j| \otimes Tr(|i\rangle\langle j|)\sigma = I \otimes \sigma$. Its range is $\mathbb{C}^d \otimes ran\, \sigma$, which contains $|\Omega\rangle = \sum_j |j\rangle|j\rangle$ iff $ran\, \sigma = \mathbb{C}^d$. If $\sigma > 0$, $J^{+} = I \otimes \sigma^{-1}$ and $\langle\Omega|J^{+}|\Omega\rangle = Tr\, \sigma^{-1} = S_{\sigma}$; otherwise $t_{*} = 0 = 1/S_{\sigma}$. So $Z = (1 - 1/S_{\sigma})\Omega$ is a witness with $Tr_Q\, Z = (1 - 1/S_{\sigma})I$. Primal: if $\sigma > 0$, $(id \otimes \mathscr{R}_{\sigma})(|\psi_{\sigma}\rangle\langle\psi_{\sigma}|) = diag(p) \otimes \sigma$, whose overlap with ψ_{σ} is $\sum_i p_i^2\lambda_i = S_{\sigma}^{-2}\sum_i \lambda_i^{-1} = 1/S_{\sigma}$; if $\lambda_k = 0$, the overlap of $proj\, k \otimes \sigma$ with $|k\rangle|k\rangle$ is $\lambda_k = 0$. Lemma (c) gives exactness. Finally $F_e(\mathscr{R}_{\sigma}) = \langle\Omega|I \otimes \sigma|\Omega\rangle/d^2 = 1/d^2$ and $d^2 = (\sum_i \lambda_i^{1/2}\lambda_i^{-1/2})^2 \leq \sum_i \lambda_i \sum_i \lambda_i^{-1} = S_{\sigma}$, with equality iff all λ_i are equal.

par *Does not imply.* The theorem identifies the best certificate within one family of duals. It does not say that the best rank-one dual is exact in general (for generic channels $1 - t_{*}$ is strictly above the error; check C23), and it gives no closed error formula outside (c) and (d). Check C23 corroborates (a), (b) and (d). par

Corollary. Let σ have eigenvalues λ_i in the basis $|i\rangle$ that defines Ω, $S = S_{\sigma}$, $V = span\{|i\rangle|i\rangle\}$, and define, in the order $E \otimes Q$,

$$Y_{\sigma} = \sum_{j \neq i} \lambda_i\, \mathrm{proj}\, j \otimes \mathrm{proj}\, i \;+ \sum_{i,i'} (\lambda_i \delta_{ii'} - S^{-1})|ii\rangle\langle i'i'|.$$

Then $Y_{\sigma} - J_{\Delta}(\mathscr{R}_{\sigma}) = (1 - 1/S)\, \Omega$. Consequently $Y_{\sigma} \geq 0$, $Y_{\sigma} \geq J_{\Delta}(\mathscr{R}_{\sigma})$, $Tr_Q\, Y_{\sigma} = (1 - 1/S)I$, and under the bijection of Lemma the witness Y_{σ} corresponds exactly to the rank-one witness $(1 - 1/S)\Omega$ of Theorem (d). If $\sigma > 0$ and $d \geq 2$, Y_{σ} has rank at least $d(d - 1) \geq 2$, so the two witnesses are different operators.

Proof. $J_{\Delta}(\mathscr{R}_{\sigma}) = I \otimes \sigma - \Omega$ with $I \otimes \sigma = \sum_{j,\, i} \lambda_i proj\, j \otimes proj\, i$. All three of Y_{σ}, $I \otimes \sigma$ and Ω are reduced by V and $V^{\perp} = span\{|j\rangle|i\rangle: j \neq i\}$. On $V^{\perp}$, $Y_{\sigma} = I \otimes \sigma$ and $\Omega = 0$, so $Y_{\sigma} - J_{\Delta} = 0$. On V, in the basis $|ii\rangle$, $Y_{\sigma} = D_{\lambda} - S^{-1}11^T$, $I \otimes \sigma = D_{\lambda}$ and $\Omega = 11^T$, so $Y_{\sigma} - J_{\Delta} = (1 - S^{-1})11^T = (1 - 1/S)\Omega$. Now $Z = (1 - 1/S)\Omega$ satisfies $Z \geq 0$ and $Z \geq \Omega - I \otimes \sigma$ (the latter is $I \otimes \sigma \geq \Omega/S$, Theorem (a),(d)); Lemma gives the three properties of $Y_{\sigma} = Z + J_{\Delta}$. On $V^{\perp}$, Y_{σ} is diagonal with the $d(d - 1)$ entries $\lambda_i > 0$.

par *Does not imply.* Written output-first, Y_{σ} is the operator displayed in Hawk07-A-CERT-v3 thm:s2-exact(b) and PG-OB-1-CERT-v2 thm:s2-exact(b) (source records; the formula is restated and verified here, not imported), and $(1 - 1/S)\Omega$ is the witness of PG-OB-1-CERT-v2 thm:v2-Z. The corollary makes precise in which sense these are ``the same dual certificate'': same value, same primal, corresponding duals under a convention change; not the

same operator. No Page curve, interior or physical recovery protocol is implied. Check C22 corroborates the identity, including singular σ. par

What a reorganization may and may not change

Proposition. Let $\mathscr{A}_{d, k}$ be as in Theorem , and write $\varphi(d, k) = \min \{\varepsilon(\mathscr{T}): \mathscr{T} \in \mathscr{A}_{d, k}\}$.

• $\varphi(d, k) = 1 - k/d$ for $1 \le k \le d$, attained by $T_{d, k}$. Hence a map that replaces the block bound k by $k^{'} > k$ leaves $\mathscr{A}_{d, k}$ and lowers the floor by exactly $(k^{'} - k)/d$.

• For $m \ge 1$, $\varphi(dm, km) = 1 - k/d$: multiplying the code dimension and the block bound by the same factor leaves the floor unchanged.

• If only the block bound is multiplied, by 2^n, the floor becomes $1 - \min (2^nk, d)/d$, which is 0 once $2^nk \ge d$.

• Every reorganization of Definition keeps $\mathscr{A}_{d, k}$ (Theorem); a block-enlarging map is not among them.

Proof. (a) Theorem (a) gives $\varepsilon \ge 1 - k/d$ on $\mathscr{A}_{d, k}$, and Theorem gives $\varepsilon(T_{d, k}) = 1 - k/d$; since $T_{d, k^{'}} \in \mathscr{A}_{d, k^{'}}$ has error $1 - k^{'}/d < 1 - k/d$ for $k^{'} > k$, it is not in $\mathscr{A}_{d, k}$. (b) is (a) with (dm, km). (c) is (a) with $k^{'} = \min (2^nk, d)$; for $k^{'} = d$, the identity channel is a cut with one block of dimension d and has error 0 . (d) is Theorem : its moves (i)–(iv) keep every block dimension at most k.

par *Does not imply*. The proposition is about finite cuts. The factor 2^n only mirrors the arithmetic of Big01-A's monoid; no declared register, host functional or horizon label is introduced, and no physical memory is modelled. par

Readings required by plan t266u

PG-OB-1-CERT-v2. PG-OB-1-CERT-v2 thm:s2-exact is an exact dual certificate for replacement recovery: value $1 - 1/\Xi$, Schmidt primal, block dual Y_{σ} in the opposite sign convention. The second witness $Z_{\sigma} = (1 - 1/\Xi)\Omega$ of PG-OB-1-CERT-v2 thm:v2-Z is of the shape $(1 - t)\Omega$ of this sheet (verified in the file). By Corollary the two PG witnesses correspond under the convention translation of Lemma , and Z_{σ} is the best rank-one dual of $\mathscr{R}_{\sigma}$ (Theorem (d)). No Page curve.

Hawk07 recovery certificates. The plan's ``thm:s2-exact of A/B/C gives the same dual certificate" is qualified: same value and primal; dual operators differ in form and sign convention; they correspond exactly under Lemma (Corollary). The statements of Hawk07-A-CERT-v3 thm:s2-exact, Hawk07-B-CERT-v3 thm:s2-exact, Hawk07-C-CERT-v3 thm:s2-exact and of prop:s2-ceiling are unchanged from v2 (checked). Hawk07-A-CERT-v3 prop:s2-ceiling is an entropy ceiling, not ``the same hard ceiling" as the access floor $1 - k/d$; both are fixed-resource bounds of different quantities.

En08-ST-DSS-CERT-v3. En08-ST-DSS-CERT-v3 lem:EC-sandwich brackets the hull deficit τ_j between a primal and a dual witness and is exact when both are optimal. It is an exact certificate with written witnesses for a lattice-hull quantity, not for a channel; no Page curve or interior. Its statement is unchanged in En08-ST-DSS-CERT-v4.

Big01-A reorganization ceiling. thm:kdreorg exists in this sheet (since version 2) and in its companion, not in I-C-CERT. It records closure of $\mathscr{A}_{d, k}$ under the moves of Definition , the local finite-cut pattern of Big01-A def:reorg; it does not record closure under Big01-A's reorganizations, which are arbitrary host-fixing maps of accounts and include the dimension-growing maps of Big01-A thm:D4. With that correction, the $1 - k/d$ floor is a hard floor on $\mathscr{A}_{d, k}$ (Proposition (a),(d)). Big01-A thm:D(a) is a fibre-supremum ceiling for a fixed host; Theorem is of the same shape for a fixed block bound. Big01-A thm:D4 multiplies a declared dimension by 2^n; the analogous move on a cut is excluded by hypothesis and, if admitted, lowers the floor (Proposition (c)). No Page curve.

Typed cross-reference. Lemma is a source record for the dual-certificate pattern only. The Hawk07 and PG certificates correspond to rank-one certificates under a sign change; En08 shares only written witnesses; Big01-A shares the fixed-resource shape, with def:reorg replaced by Definition .

Version 5: exactness of the best rank-one dual, and the companion readings

This section is new in version 5 (request t267u; no plan was attached). Everything in it is proved locally from the

earlier sections, including Section ; computer-assisted checks are labelled as such. PG-OB-1-CERT-v3, Hawk07-A/B/C-CERT-v3 and En08-ST-DSS-CERT-v4 are cited by sha256 as source records only; I-C-Ex2-CERT-v4 is the previous version of this sheet. Other streams are concurrently building Hawk07 CERT version 4, the OB CERT sheets version 4 and DSS CERT version 5; their files were not read, and any statement here about the companions refers only to the delivered versions cited by sha256.

When the best rank-one dual is exact

For an operator M on $E = \mathbb{C}^d$ write $vec(M) = (M \otimes I_Q)|\Omega\rangle = \sum_{i,j} M_{ij}|i\rangle|j\rangle$, in the basis that defines Ω. Then vec is a linear bijection, $\langle vec(M), vec(N)\rangle = Tr(M^\dagger N)$, $\langle\Omega|vec(M)\rangle = Tr\, M$, and $(X \otimes Y)vec(M) = vec(XMY^T)$.

Theorem. Let $\mathcal{T}$ be a channel on $\mathbb{C}^d$, $J = J(\mathcal{T})$, and $t_* = t_*(\mathcal{T})$ as in Theorem .

• $\varepsilon(\mathcal{T}) = 1 - t_*$ if and only if there is a state R on E with $(J - t_*\Omega)vec(R) = 0$.

• For such R, $\psi_R = vec(R^{1/2})$ is a unit vector and $(\psi_R, (1 - t_*)\Omega)$ is an exact dual certificate.

• If $J > 0$, then $\ker(J - t_*\Omega) = \mathbb{C}\, J^{-1}|\Omega\rangle$. Define $M_{\mathcal{T}}$ by $vec(M_{\mathcal{T}}) = J^{-1}|\Omega\rangle$. Then $Tr\, M_{\mathcal{T}} = 1/t_*$, and $\varepsilon(\mathcal{T}) = 1 - t_*$ if and only if $M_{\mathcal{T}}$ is Hermitian and positive semidefinite; then $R = t_* M_{\mathcal{T}}$.

Proof. Put $a = 1 - t_*$ and $K_0 = J - t_*\Omega \geq 0$ (Theorem (a)). If $a = 0$, then $J \geq \Omega$ and $Tr\, J = d = Tr\, \Omega$ give $J = \Omega$, $\mathcal{T} = id$, $\varepsilon = 0$, and $R = I/d$ satisfies (a); assume $a > 0$. For $|\psi\rangle = (A \otimes I)|\Omega\rangle$ with $Tr\, A^\dagger A = 1$ the proof of Lemma (b) gives $\Delta_\psi = (A \otimes I)(\Omega - J)(A \otimes I)^\dagger$, and since $(A \otimes I)\Omega(A \otimes I)^\dagger = proj\, \psi$,

$$\Delta_\psi = a\,\mathrm{proj}\,\psi - K_\psi, \qquad K_\psi = (A \otimes I)K_0(A \otimes I)^\dagger \geq 0.$$

If. Let R be a state with $K_0 vec(R) = 0$ and $A = R^{1/2}$. Then $\|\psi_R\|^2 = Tr\, R = 1$ and $(A \otimes I)^\dagger|\psi_R\rangle = vec(R)$, so $\langle\psi_R rvert K_{\psi_R}|\psi_R\rangle = \langle vec(R) rvert K_0|vec(R)\rangle = 0$. The primal value of Lemma (a) is $1 - \langle\psi_R|(id \otimes \mathcal{T})(|\psi_R\rangle\langle\psi_R|)|\psi_R\rangle = \langle\psi_R|\Delta_{\psi_R}|\psi_R\rangle = a$, and $Z = a\Omega$ is a dual witness with $\|Tr_Q Z\|_\infty = a$ (Theorem (b)). Lemma (c) gives $\varepsilon = a$ and (b). *Only if.* By the standard fact used in the proof of Lemma (b), and compactness of the unit sphere, there is $\psi = (A \otimes I)|\Omega\rangle$ with $1/2\|\Delta_\psi\|_1 = \varepsilon = a$. Since Δ_ψ is Hermitian and traceless, $a = \max_{0 \leq P \leq I} Tr(P\Delta_\psi)$; let P attain it. As $Tr(P\Delta_\psi) = a\langle\psi|P|\psi\rangle - Tr(PK_\psi) \leq a$, equality forces $\langle\psi|P|\psi\rangle = 1$ and $Tr(PK_\psi) = 0$. The first gives $(I - P)^{1/2}\psi = 0$, so $P\psi = \psi$; the second gives $K_\psi^{1/2}P^{1/2} = 0$, so $K_\psi\psi = K_\psi P\psi = 0$. Hence $0 = \langle\psi|K_\psi|\psi\rangle = \langle vec(R) rvert K_0|vec(R)\rangle$ with $R = A^\dagger A$, a state, and $K_0 \geq 0$ gives $K_0 vec(R) = 0$. (c) If $K_0 v = 0$ then $Jv = t_*\langle\Omega, v\rangle|\Omega\rangle$, so $v \in \mathbb{C}J^{-1}|\Omega\rangle$ (if $\langle\Omega, v\rangle = 0$ then $Jv = 0$ and $v = 0$); conversely $K_0 J^{-1}|\Omega\rangle = |\Omega\rangle - t_*\langle\Omega|J^{-1}|\Omega\rangle|\Omega\rangle = 0$ by Theorem (a). Also $Tr\, M_{\mathcal{T}} = \langle\Omega|J^{-1}|\Omega\rangle = 1/t_*$. A state R with $vec(R) \in \mathbb{C}J^{-1}|\Omega\rangle$ is $R = sM_{\mathcal{T}}$ with $Tr\, R = 1$, i.e. $s = t_*$; it is a state iff $M_{\mathcal{T}} \geq 0$. Apply (a).

par *Does not imply*. The criterion characterizes when one particular dual family is optimal; it is not a closed error formula for channels that fail it, whose error lies strictly below $1 - t_*$ and is in each instance the value of a finite semidefinite program. Generic non-exactness is not proved: check C26 (computer-assisted) finds $M_{\mathcal{T}}$ not Hermitian positive semidefinite, and the error strictly below $1 - t_*$, for random channels, and confirms the criterion for replacement and isotropic channels. par

Corollary.

• *(Bell saturation.)* If $J|\Omega\rangle = dF_e|\Omega\rangle$, then $t_* = F_e$ and $R = I/d$ satisfies Theorem (a).

• *(Replacement.)* For $\sigma > 0$, $J(\mathcal{R}_\sigma) = I \otimes \sigma$, $M_{\mathcal{R}_\sigma} = (\sigma^{-1})^T \geq 0$ and $R = (\sigma^{-1})^T/S_\sigma$.

• *(Unitary conjugation.)* For a unitary U put $\mathcal{T}^U = Ad_U \circ \mathcal{T} \circ Ad_{U^\dagger}$. Then $\varepsilon(\mathcal{T}^U) = \varepsilon(\mathcal{T})$, $t_*(\mathcal{T}^U) = t_*(\mathcal{T})$, and $\mathcal{T}^U$ satisfies Theorem (a) with $\bar{U}RU^T$ iff $\mathcal{T}$ does with R.

Proof. (a) Theorem and Theorem (c) give $t_* = F_e$; then $(J - F_e\Omega)|\Omega\rangle = dF_e|\Omega\rangle - F_e d|\Omega\rangle = 0$ and $vec(I/d) = |\Omega\rangle/d$. (b) $J^{-1}|\Omega\rangle = (I \otimes \sigma^{-1})vec(I) = vec((\sigma^{-1})^T)$, and $Tr\, \sigma^{-1} = S_\sigma$. (c) $\mathcal{T}^U - id = Ad_U \circ (\mathcal{T} - id) \circ Ad_{U^\dagger}$, and composing with unitary channels on either side does not change the diamond norm (submultiplicativity in both directions). With $W = \bar{U} \otimes U$, $(id \otimes Ad_{U^\dagger})(\Omega) = (\bar{U} \otimes I)\Omega(\bar{U} \otimes I)^\dagger$, hence $J(\mathcal{T}^U) = WJ(\mathcal{T})W^\dagger$, and $W|\Omega\rangle = vec(\bar{U}U^T) = |\Omega\rangle$. So $J(\mathcal{T}^U) - t\Omega = W(J - t\Omega)W^\dagger$ for every t, which gives the same t_*, and $Wvec(R) = vec(\bar{U}RU^T)$, a state iff R is.

par *Does not imply*. The corollary re-derives the two exact cases of Theorem (c),(d) from the criterion and shows

that exactness is a property of unitary orbits. It does not say that these are the only exact channels; Theorem and the examples below show that they are not. Check C26 corroborates (c) numerically. par

Pinching onto blocks of unequal size

Proposition. Let $P_1, \ldots, P_m$ be orthogonal projections on $\mathbb{C}^d$ with $\sum_i P_i = I$, ranks $k_i \geq 1$ and $m \geq 2$, and let $\mathcal{R}(X) = \sum_i P_iXP_i$.

• $t_{*}(\mathcal{P}) = 1/m$, $R = 1/m\sum_i P_i^T/k_i$ satisfies $(J - \Omega/m)vec(R) = 0$, and $\varepsilon(\mathcal{P}) = 1 - 1/m$ exactly, with the exact dual certificate $(vec(R^{1/2}), (1 - 1/m)\Omega)$.

• $F_e(\mathcal{P}) = \sum_i k_i^2/d^2 \geq 1/m$, with equality iff all k_i are equal. So for unequal ranks $\mathcal{P}$ is not Bell-saturated and $1 - F_e < \varepsilon(\mathcal{P})$.

• $\mathcal{P}$ is the composite of the cut with branches $X \mapsto P_iXP_i$ (block dimensions k_i) and the decoder that re-embeds the blocks, so $\mathcal{P} \in \mathcal{A}_{d, k}$ for $k = max_i k_i$; its error $1 - 1/m$ is at least the floor $1 - k/d$, with equality iff all k_i are equal.

Proof. (a) $J = \sum_i |v_i\rangle\langle v_i|$ with $|v_i\rangle = (I \otimes P_i)|\Omega\rangle = vec(P_i^T)$; these are orthogonal, $\|v_i\|^2 = k_i$, and $|\Omega\rangle = \sum_i |v_i\rangle$. So $J^{+} = \sum_i |v_i\rangle\langle v_i|/k_i^2$ and $\langle\Omega|J^{+}|\Omega\rangle = \sum_i k_i^2/k_i^2 = m$, whence $t_{*} = 1/m$ by Theorem (a). With $vec(R) = 1/m\sum_i |v_i\rangle/k_i$: $Jvec(R) = 1/m\sum_i |v_i\rangle = 1/m|\Omega\rangle$, and $1/m\Omega vec(R) = 1/mTr(R)|\Omega\rangle = 1/m|\Omega\rangle$. $R \geq 0$ and $Tr\, R = 1/m\sum_i k_i/k_i = 1$. Theorem (a),(b) give the rest. (b) $\langle\Omega|J|\Omega\rangle = \sum_i |\langle v_i, \Omega\rangle|^2 = \sum_i k_i^2$, and $(\sum_i k_i)^2 \leq m\sum_i k_i^2$ by Cauchy–Schwarz, with equality iff the k_i are equal; then Theorem (c). (c) The branches are completely positive with $\sum_i Tr\,(P_iXP_i) = Tr\, X$, as in Section . $mk \geq d$ gives $1 - 1/m \geq 1 - k/d$, with equality iff $mk = d$, i.e. iff all $k_i = k$.

par *Does not imply*. The proposition adds an exact family that is neither covariant nor Bell-saturated nor a replacement channel; its error depends only on the number of blocks. It does not say that the floor $1 - k/d$ is attained by unequal block pinchings (it is not), and it identifies no physical cut. Check C29 (computer-assisted) corroborates (a) and (b) for four block patterns in random bases. par

Documentary readings of newly delivered companions

PG-OB-1-CERT-v3. (sha256 8c0a7e2fdbb7e4558bad17bc35df10a66feea6ba4959af524921393a61eafbcc.) PG-OB-1-CERT-v3 prop:v3-Z(c) states that the sign flip of its lem:v3-sandwich carries its version-2 rank-one witness $c_{\sigma}\Omega$ to its version-1 block witness. That is the content of Corollary of this sheet, proved here in version 4; the two were obtained separately and nothing is transferred. Its lem:v3-sandwich admits lower witnesses $0 \leq W \leq \rho \otimes I$ (a mixed reference); this sheet uses only pure primal witnesses. PG-OB-1-CERT-v3 cor:v3-decoder: optimizing a decoder after a replacement channel gives $1 - d^{-2}$ for every σ, while the fixed-output distance $1 - 1/\Im$ equals $1 - d^{-2}$ only at $\sigma = I/d$. PG-OB-1-CERT-v3 cor:v3-dimension bounds the code dimension of a replacement-recovery contract by $\lfloor(1 - \varepsilon)^{-1/2}\rfloor$. The analogue here is immediate from Theorem (a): a composite in $\mathcal{A}_{d, k}$ with error at most $\varepsilon < 1$ has $d \leq k/(1 - \varepsilon)$. Both are conditional on an accuracy hypothesis; neither identifies a register. No Page curve.

Hawk07-A/B/C-CERT-v3. (sha256 1e91c9fb7bb97c16d9b4b0d48e821d511badea6e4ace77fc6643a61c76eee776, 2639d64e7ec3340017a91ed62e487b857317f2cef51966a2086fd016ffcf4176, 4352ee5e6a754a153631ab755a44e16ea9273fb1b6b9950e8fc29711b983a416.) The new Hawk07-A-CERT-v3 prop:v3-drule (identical in B and C) records that $1 - 1/\Im(\sigma)$ takes exactly the values $[1 - 1/d^2, 1]$, and that a rule given only d has worst-case error at least $1/(2d^2)$. The range agrees with Theorem (d) and the bound $S_{\sigma} \geq d^2$ proved there; the rule bound is half the length of that range. The Hawk07 sheets cite lem:cert-sandwich of I-C-CERT-v2 and related labels as source records; those readings match the statements here (checked).

En08-ST-DSS-CERT-v4. (sha256 98c42ee7397263bc694ebed7d66c682b090df437a3230f294b3245f4bd01bee9.) doc:EC-icsandwich reads lem:cert-sandwich of the I-C certificate sheets (version 2) as a feasible-lower/feasible-upper bracket and refuses any identification of targets; that is an accurate reading of Lemma . Its lem:EC-replacement proves decoder-optimized replacement recovery $1 - d^{-2}$ by a Weyl average, the value of PG-OB-1-CERT-v3 cor:v3-decoder. Its doc:EC-icreorg reads thm:kdreorg of I-C-Ex2-CERT-v2 as closure of the cuts of block

dimension at most k under reorganization, with value ceiling $v \le k/d$ (attained) and error floor $\varepsilon \ge 1 - k/d$, and warns that the word ``ceiling'' does not turn the error floor into an upper bound; that matches Theorem (checked; the theorem's title ``The access floor is a ceiling under reorganization'' is retained and is to be read with that warning). No Page curve or interior on either side.

Version 6: the fidelity primal, generic non-exactness, identity–pinching mixtures, and the companion readings

This section is new in version 6 (request t268u; no plan was attached). Everything in it is proved locally from the earlier sections of this sheet, including Sections and ; computer-assisted checks are labelled as such, and standard facts of real analysis and of linear algebra are named where they are used. The newly delivered companions (DS-, VC- and PG-OB-1-CERT-v4 and v5, Hawk07-*-CERT-v4 and v5, and the DSS-CERT-v5 and v6 sheets) are cited by sha256 as source records only, never as premises; I-C-Ex2-CERT-v5 is the previous version of this sheet. The companion files were verified before use against MANIFEST-CertPlan-S2v4-OB.sha256 and MANIFEST-CertPlan-S2v4-Hawk07.sha256, and, for the second wave read in this version, MANIFEST-CertPlan-S2v5-OB.sha256 and MANIFEST-CertPlan-S2v5-Hawk07.sha256 (in CertPlan/deliver), and MANIFEST-DSS-CERT-v5.sha256 and MANIFEST-DSS-CERT-v6.sha256 (in DSS/deliver); every listed hash matched.

The fidelity primal and exactness as a convex program

For a channel $\mathcal{T}$ on $\mathbb{C}^d$ with $J = J(\mathcal{T})$ put

$$L_F(\mathcal{T}) = 1 - \min \{\langle \mathrm{vec}\,(R) \mathrm{rvertJ}\, |\mathrm{vec}\,(R)\rangle : R \text{ a state on} E\},$$

with *vec* as in Section . The minimum exists, since the states form a compact set and the function is continuous.

Proposition.

• $1 - F_e(\mathcal{T}) \le L_F(\mathcal{T}) \le \varepsilon(\mathcal{T}) \le 1 - t_{*}(\mathcal{T})$.

• $t_{*}(\mathcal{T}) = inf\{\langle v|J|v\rangle: v \in E \otimes Q, \langle \Omega, v\rangle = 1\}$, the infimum over all vectors, not only over vectors $vec\ (R)$ of states.

• $L_F(\mathcal{T}) = 1 - t_{*}(\mathcal{T})$ if and only if $\varepsilon(\mathcal{T}) = 1 - t_{*}(\mathcal{T})$, if and only if some state R satisfies $(J - t_{*}\Omega)vec\ (R) = 0$.

• $R \mapsto \langle vec\ (R) rvertJ\ |vec\ (R)\rangle$ is a convex quadratic function on the convex set of states, so L_F is the value of a convex program, and (c) decides exactness of the best rank-one dual by that program.

Proof. (a) Let R be a state, $A = R^{1/2}$ and $\psi = (A \otimes I)|\Omega\rangle$, a unit vector since $\|\psi\|^2 = Tr\ A^{\dagger}A = 1$. Then $(id \otimes \mathcal{T})(proj\ \psi) = (A \otimes I)J(A \otimes I)^{\dagger}$ and $(A \otimes I)^{\dagger}\psi = (R \otimes I)|\Omega\rangle = vec\ (R)$, so the primal value of Lemma (a) at ψ is $1 - \langle vec\ (R) rvertJ\ |vec\ (R)\rangle$. Hence $L_F \le \varepsilon$. The state $R = I/d$ has $vec\ (R) = |\Omega\rangle/d$ and gives $1 - F_e$, so $1 - F_e \le L_F$. The last inequality is Theorem (b). (b) If $|\Omega\rangle \notin ran\ J$, then $|\Omega\rangle$ is not orthogonal to $ker\ J = (ran\ J)^{\perp}$, so some $v \in ker\ J$ has $\langle \Omega, v\rangle = 1$ and $\langle v|J|v\rangle = 0 = t_{*}$ (Theorem (a)). If $|\Omega\rangle \in ran\ J$, then $|\Omega\rangle = J^{1/2}(J^{+})^{1/2}|\Omega\rangle$, and for $\langle \Omega, v\rangle = 1$ the Cauchy–Schwarz inequality gives $1 = |\langle (J^{+})^{1/2}\Omega, J^{1/2}v\rangle|^2 \le \langle \Omega|J^{+}|\Omega\rangle\ \langle v|J|v\rangle$, so $\langle v|J|v\rangle \ge t_{*}$, with equality at $v = J^{+}|\Omega\rangle/\langle \Omega|J^{+}|\Omega\rangle$. (c) Put $K_0 = J - t_{*}\Omega \ge 0$. For a state R, $\langle vec\ (R)|\Omega|vec\ (R)\rangle = |Tr\ R|^2 = 1$, so $\langle vec\ (R) rvertJ\ |vec\ (R)\rangle = t_{*} + \langle vec\ (R) rvertK_0\ |vec\ (R)\rangle \ge t_{*}$. Thus $L_F = 1 - t_{*}$ if and only if the minimum of $\langle vec\ (R) rvertK_0\ |vec\ (R)\rangle$ over states is 0, if and only if (as $K_0 \ge 0$) some state has $K_0 vec\ (R) = 0$. By Theorem (a) this is equivalent to $\varepsilon = 1 - t_{*}$. (d) $J \ge 0$ and *vec* is linear.

par *Does not imply*. The proposition brackets the error by two programs and identifies when the outer bracket is tight. It does not say that $\varepsilon = L_F$ in general: check C30 (computer-assisted) finds $L_F < \varepsilon$ strictly on most random instances and equality for replacement channels. A closed error formula for channels that fail the criterion stays *open*. par

Depolarized unitaries: an explicit exactness boundary

For a unitary U on $\mathbb{C}^d$, $d \ge 2$, and $0 < p < 1$ let $\mathcal{T}_{U, p} = p\ Ad_U + (1 - p)\mathcal{R}_{I/d}$, and put $c = (1 - p)/d$ and $\tau = Tr\ U$.

Proposition.

• $J(\mathcal{T}_{U,p}) = c\,I + p\,vv^{\dagger}$ with $v = vec\,(U^{T})$, $\|v\|^2 = d$ and $\langle v, \Omega\rangle = \bar{\tau}$. In particular $J > 0$ and $F_e(\mathcal{T}_{U,p}) = (1 - p + p|\tau|^2)/d^2$.

• $M_{\mathcal{T}_{U,p}} = 1/c(I - (p\bar{\tau})/(c + pd)U^{T})$ and $t_{*}(\mathcal{T}_{U,p}) = (c(c + pd))/(d(c + pd) - p|\tau|^2)$.

• $\varepsilon(\mathcal{T}_{U,p}) = 1 - t_{*}(\mathcal{T}_{U,p})$ if and only if $\tau = 0$ or $e^{-i\arg\tau}\,U$ is Hermitian. For $\tau = 0$ the Bell witness is exact and $t_{*} = F_e$.

• For $U = diag\,(1, i, 1, \ldots, 1)$ the criterion fails, so $\varepsilon(\mathcal{T}_{U,p}) < 1 - t_{*}(\mathcal{T}_{U,p})$ for every $0 < p < 1$. For $d = 2$, $p = 1/2$: $t_{*} = 5/24$, so $\varepsilon < 19/24$.

Proof. (a) $J(Ad_U) = (I \otimes U)\Omega(I \otimes U)^{\dagger}$ and $(I \otimes U)vec\,(I) = vec\,(U^{T})$; $J(\mathcal{R}_{I/d}) = I \otimes I/d$. Also $\|v\|^2 = Tr\,(\bar{U}U^{T}) = d$ and $\langle v, \Omega\rangle = Tr\,\bar{U} = \bar{\tau}$. Since $c > 0$, $J \geq cI > 0$, and $\langle\Omega|J|\Omega\rangle = cd + p|\tau|^2$. (b) By the Sherman–Morrison formula, $J^{-1} = 1/c(I - (p\,vv^{\dagger})/(c + pd))$, so $J^{-1}|\Omega\rangle = 1/c(|\Omega\rangle - (p\bar{\tau})/(c + pd)v) = vec\,(1/c(I - (p\bar{\tau})/(c + pd)U^{T}))$. Then $1/t_{*} = Tr\,M_{\mathcal{T}_{U,p}} = 1/c(d - (p|\tau|^2)/(c + pd))$ by Theorem (c). (c) By Theorem (c), exactness holds if and only if $M = M_{\mathcal{T}_{U,p}}$ is Hermitian and positive semidefinite. M is Hermitian if and only if $X = \bar{\tau}U^{T}$ is, if and only if $\bar{\tau}U$ is (a matrix is Hermitian if and only if its transpose is), that is, $\tau = 0$ or $e^{-i\arg\tau}U$ Hermitian. If X is Hermitian, then $X^2 = XX^{\dagger} = |\tau|^2 I$, so its eigenvalues are $\pm|\tau|$, and those of M are at least $1/c(1 - (p|\tau|)/(c + pd)) > 0$, because $|\tau| \leq d$ and $c > 0$; so $M > 0$ automatically. If $\tau = 0$, then $J|\Omega\rangle = c|\Omega\rangle$, and with $F_e = c/d$ this is the Bell condition of Theorem ; Theorem (c) gives $t_{*} = F_e$. (d) $\tau = d - 1 + i \neq 0$, and $e^{-i\arg\tau}U$ has the eigenvalues $e^{-i\arg\tau}$ and $ie^{-i\arg\tau}$, which cannot both be real; so (c) fails, and $\varepsilon < 1 - t_{*}$ by Theorem (a) and Theorem (b). For $d = 2$, $p = 1/2$: $c = 1/4$, $c + pd = 5/4$, $|\tau|^2 = 2$, and (b) gives $t_{*} = (5/16)/(3/2) = 5/24$.

par *Does not imply*. The proposition decides exactness on one explicit family; it does not give the error when the criterion fails. Check C32 (computer-assisted: exact sympy for M, semidefinite programming for the error) confirms (b)–(d) at $p = 1/2$ for $U = Z, X, e^{7i/10}diag\,(1, 1, -1)$ (exact) and $diag\,(1, i)$, $diag\,(1, i, 1)$ (not exact; for $d = 2$ the solver value is $0.\,68402$). par

Non-exactness is generic

Let $A = \{J = J^{\dagger} on E \otimes Q : Tr_Q\,J = I_E\}$, a real affine space of dimension $d^4 - d^2$, let $C = \{J \in A : J \geq 0\}$ be the set of Choi operators of channels on $\mathbb{C}^d$ (Choi's theorem), and $C^{\circ} = \{J \in A : J > 0\}$.

Theorem. Let $d \geq 2$, and let $N \subseteq C^{\circ}$ be the set of J for which $M_{\mathcal{T}}$ is not Hermitian.

• C° is convex, open in A and dense in C.

• N is open in A and dense in C, and $C \setminus N$ has Lebesgue measure zero in A.

• Every channel with $J \in N$ has $\varepsilon(\mathcal{T}) < 1 - t_{*}(\mathcal{T})$ strictly. Hence the channels for which the best rank-one dual is exact form a subset of $C \setminus N$, which is closed and nowhere dense in C and has measure zero.

Proof. (a) Openness and convexity are clear, and $J_0 = I \otimes I/d \in C^{\circ}$. For $J \in C$ and $0 < s \leq 1$, $(1 - s)J + sJ_0 \in C^{\circ}$ and tends to J as $s \to 0$. (b) On C° the map $J \mapsto M_{\mathcal{T}} - M_{\mathcal{T}}^{\dagger}$, with $vec\,(M_{\mathcal{T}}) = J^{-1}|\Omega\rangle$, is real-analytic in affine coordinates of A (the entries of J^{-1} are rational functions with nonvanishing denominator $det\,J$). It is not identically zero: the depolarized unitary of Proposition (d) lies in C° and has non-Hermitian $M_{\mathcal{T}}$. So some real or imaginary part f of one entry is a real-analytic function on the connected open set C° that is not identically zero. By the standard fact that the zero set of such a function is closed in C°, has empty interior and has Lebesgue measure zero, $C^{\circ} \setminus N \subseteq \{f = 0\}$ is a closed null set with empty interior in C°. Hence N is open and dense in C°, so dense in C by (a). Finally $C \setminus C^{\circ}$ lies in the boundary of the convex set C, which has nonempty interior in A; such a boundary has measure zero. So $C \setminus N$ is null. (c) If $J \in N$, then $M_{\mathcal{T}}$ is not Hermitian, and Theorem (c) and Theorem (b) give $\varepsilon < 1 - t_{*}$. The set $C \setminus N$ is closed in C (as N is open) and has empty interior in C (as N is dense).

par *Does not imply*. This settles the version-5 item ``generic non-exactness is not proved''. The measure is Lebesgue measure on the affine hull of the channel set, not a physical prior; the theorem says nothing about any particular channel beyond Theorem , and it does not compute the error of non-exact channels, which stays the value of a finite semidefinite program. The exact channels include the identity, the replacement channels and the

examples of Section . Check C33 (computer-assisted) finds no exact channel among *30* random full-rank channels, and a non-Hermitian $M_{\mathcal{T}}$ along the open segment from a replacement channel to the example of Proposition (d). par

Mixtures of the identity and a pinching

Let $P_1, \dots, P_m$ ($m \geq 2$) be orthogonal projections with $\sum_i P_i = I$ and ranks $k_i \geq 1$, and for $0 \leq p \leq 1$ let $\mathscr{P}_p(X) = pX + (1-p)\sum_i P_i X P_i$.

Proposition. $\varepsilon(\mathscr{P}_p) = 1 - t_*(\mathscr{P}_p) = (1-p)(1-1/m)$, with $t_* = p + (1-p)/m$ and the exact witness $R = 1/m \sum_i P_i^T/k_i$ of Proposition , independent of p. Moreover $F_e(\mathscr{P}_p) = p + (1-p)\sum_i k_i^2/d^2$, and the Bell witness is exact if and only if $p = 1$ or all k_i are equal.

Proof. Put $e_i = \mathrm{vec}(P_i^T) = (I \otimes P_i)|\Omega\rangle$. Then $\langle e_i, e_j\rangle = \mathrm{Tr}(P_i^T P_j^T) = \delta_{ij} k_i$, $|\Omega\rangle = \sum_i e_i$, and $J(\mathscr{P}_p) = p\,\Omega + (1-p)\sum_i |e_i\rangle\langle e_i|$. For $t_0 = p + (1-p)/m$, $J - t_0\Omega = (1-p)(\sum_i |e_i\rangle\langle e_i| - 1/m \mathrm{proj}\,\Omega) \geq 0$, because $|\langle \Omega, w\rangle|^2 = |\sum_i \langle e_i, w\rangle|^2 \leq m \sum_i |\langle e_i, w\rangle|^2$ (Cauchy–Schwarz). Let $w = \sum_i e_i/k_i = \mathrm{vec}(mR)$, so $\langle e_i, w\rangle = 1$ and $\langle \Omega, w\rangle = m$. Then $\sum_i |e_i\rangle\langle e_i \mathrm{rvert} w = \sum_i e_i = |\Omega\rangle$ and $1/m \mathrm{proj}\,\Omega w = |\Omega\rangle$, so $(J - t_0\Omega)w = 0$. For $t > t_0$, $\langle w|(J - t\Omega)|w\rangle = -(t - t_0)m^2 < 0$; hence $t_* = t_0$. R is a state, and Theorem (a) gives $\varepsilon = 1 - t_0$. Next $\langle \Omega|J|\Omega\rangle = pd^2 + (1-p)\sum_i k_i^2$. Finally $J|\Omega\rangle = pd|\Omega\rangle + (1-p)\sum_i k_i e_i$ is a multiple of $|\Omega\rangle = \sum_i e_i$ if and only if $p = 1$ or all k_i are equal (the e_i are linearly independent), which is the Bell condition of Theorem .

par *Does not imply.* For $p = 0$ this is Proposition . The proposition is about this explicit family; it says nothing about other mixtures, and it identifies no physical decoherence model. Check C34 (computer-assisted) confirms the error, the witness, the optimality of t_* and F_e for block sizes $(1, 2)$, $(1, 1, 2)$, $(2, 2)$ at $p = 0.3$ and 0.7. par

Documentary readings of newly delivered companions

PG-OB-1-CERT-v4. (sha256 3dc921457bd781606e9e673231a8c832be104435950913521fdfb2ed737fc1c1.) PG-OB-1-CERT-v4 thm:v4-PG-floor: through an m-dimensional register, $F_e(\mathscr{D}_{\mathcal{T}}) \leq \min\{1, m^2/d^2\}$, so the error is at least $1 - m^2/d^2$, attained for $m \leq d$; PG-OB-1-CERT-v4 cor:v4-PG-register turns this into $m \geq d\sqrt{(1-\varepsilon)}$. The register bound concerns the dimension of an intermediate quantum register. The cuts of this sheet (Definition) are described by a classical record and blocks, not by one register, so the two bounds are not compared here. Its documentary reading of thm:v4-rankone(d) and cor:v4-translate of I-C-CERT-v4 ($t_*(\mathscr{R}_\sigma) = 1/\Xi(\sigma)$; corresponding witnesses, not the same operator) is accurate; those statements are identical in Theorem and Corollary of this sheet.

Hawk07-A-CERT-v4. (sha256 a88a2430a9e3672a9cbdb7c661dc021154dd3ac4237a1040b4eab097194fceb5.) Hawk07-A-CERT-v4 prop:v4-rankone(b),(c) are Lemma and Corollary written output-first, proved there independently. Its reading says that the same labels appear with the same statements in I-C-, I-C-Ex- and I-C-Ex2-CERT-v4; that is correct (the statements of lem:v4-convention, thm:v4-rankone and cor:v4-translate are character-identical in the three sheets, checked). Hawk07-A-CERT-v4 cor:v4-dim (decoder errors after a replacement channel fill $[1 - d^{-2}, 1]$) concerns optimization over decoders, a different task from the fixed composites here.

En08-ST-DSS-CERT-v5. (sha256 9be4fe4fdd9cf1164a75f0218035f0ee3a4c340ed7f15787551de3997fe5af64.) En08-ST-DSS-CERT-v5 cor:EC-decoders(a): after a replacement channel, the optimal decoders are exactly those with output I/d, and a decoder with output σ has error at least $1 - 1/S_\sigma$. En08-ST-DSS-CERT-v5 prop:EC-regfloor is the register floor of PG-OB-1-CERT-v4, proved there separately. En08-ST-DSS-CERT-v5 doc:EC-pgic5 reads lem:v4-convention and cor:v4-translate of I-C-CERT-v4 as ``the same values with different operators''; that is accurate. Its doc:EC-icrow5 is treated in Section (E6.2).

Red06-G-DSS-CERT-v5 and V-DST-DSS-CERT-v5. (sha256 e4a1b02552c138b8f9f009f2e71e391825ad0014f717b43f96883c445605213a, 4133961f566c059c22054b5922423d1ff94cb0d1c888ca353a7c143bba1541be.) Red06-G-DSS-CERT-v5 doc:RC-icrow5 and V-DST-DSS-CERT-v5 doc:VDC-icrow5 repeat the qualification of the version-4 plan-verification row; the errata they cite were confirmed in the version-4 error lists and are recorded in Section . Their other new items (prop:RC-ends, prop:VDC-odd, prop:VDC-E1) concern the de Sitter source map and are

not used.

VC-OB-1-CERT-v4 and DS-OB-1-CERT-v4. (sha256 f9802da66896159503132b736164016c1d06d2a6530bbbc06cb4ad3af0256387, 0b5b86f87f91503ba9e96cf221b2ef96a4b4e96e3f27489336eb2b89848463c6.) Both cite lem:v4-convention, cor:v4-translate and thm:v4-rankone of I-C-CERT-v4 as statements about finite channels that do not bear on their sheets; that is accurate. DS-OB-1-CERT-v4 notes that the radius lemma is lem:v4-radius of I-C-Th-CERT-v4, not of I-C-CERT-v4; that is correct. Their curvature results are read in I-C-Th-CERT-v6.

Documentary readings of the second wave of companions

The following files were delivered during the preparation of this version and verified against their manifests (Section , opening paragraph). They are source records only.

PG-OB-1-CERT-v5. (sha256 c96ea4631de51c635521ca42fd24337728591676ee9019a24f3ad9fcdcdeb8a2.) It reads I-C-Ex-CERT-v5 prop:v5-pinch as block pinchings with exact error $1 - 1/m$ in a class with a classical cut and branches of dimension at most k, and states that this class and its floor differ from its register model; the reading is accurate (Proposition , whose statement is the same in I-C-Ex-CERT-v5 and I-C-Ex2-CERT-v5). Its register results (PG-OB-1-CERT-v5 thm:v5-PG-shared, PG-OB-1-CERT-v5 prop:v5-PG-bracket) are not compared with the cuts of this sheet.

Hawk07-A-CERT-v5. (sha256 9db43c8772a84fcf2fcf8d3674b86781ab407b93de05d370740e89f3f00b7add.) Hawk07-A-CERT-v5 prop:v5-kernel describes, output factor first, the kernel of the block dual $\sigma \otimes I - \Xi^{-1}\Omega$ of a replacement channel: for $\sigma > 0$ it is spanned by $vec(\sigma^{-1})$, and for singular σ the operator is $\sigma \otimes I$. It cites I-C-CERT-v5, not this sheet; the replacement case is Corollary (b) here, with the same statement, and its reading is accurate.

En08-ST-DSS-CERT-v6. (sha256 7d821d99ced3a9d265d5437f5c1466d8f524a09884730d13662278c31b1da086.) En08-ST-DSS-CERT-v6 doc:EC-v6read reads I-C-Ex-CERT-v5 thm:v5-exact and I-C-Ex-CERT-v5 prop:v5-pinch as concerning cuts of bounded quantum block dimension and not used there; that is accurate. Its En08-ST-DSS-CERT-v6 prop:EC-assisted is not used here.

DS-OB-1-CERT-v5, VC-OB-1-CERT-v5, Hawk07-F/H-CERT-v5, Red06-G-DSS-CERT-v6 and V-DST-DSS-CERT-v6. (sha256 a0805f8eb081a250f78e8074b9de5986ccd23106749df9e0fe5fac2f71e157f9, d3a93a3afb0e8803d18f230ce824f78c19253f6a4bc841f8ceef763ce14f7cac, 2cfa6282f30e25ac57188298f2e16d3b05a66fff4440204e6f11acce352f5bc3, 74a70015a03906e76cac8fde88a082da9789ea6e863d987be1dbba67f08bb5dd, c3f04faab2c76fb661054f2c1393bd71f484a1bfdfeeace52e5c612a34fc4599, 710c656210aab6dd708cd9f830ab7bf59df592f0887ab168b1e7fa7be2df95f9.) The two OB sheets cite thm:v5-exact of I-C-CERT-v5 (Theorem here has the same statement) as a finite-channel criterion that does not bear on their labels; that is accurate. The other files do not cite this sheet; their curvature results are read in I-C-Th-CERT-v6. None of the second-wave files records an error in version 5 of this sheet, and none was found in their readings.

Verification of plan t256u against the sheets (version 1, retained)

toprule Plan statement Finding Status

midrule Resource law: fidelity $\le k/d$, error $\ge 1 - k/d$ I-C-Ex2 thm:sharp; re-proved as Lemma . Big01-A records the bound only (Big01-A prop:Bblocks). confirmed

Exactness Already proved in the parent (I-C-Ex2 thm:sharp: attainment by $T_{d, k}$, upper bound by the triangle inequality). Here the dual operator is added (Theorem , Theorem). confirmed; dual added

Error of every such channel equals formula False (Proposition). True: lower bound for every object; equality $1 - F_e$ on the covariant class (Theorem). corrected

Dual witness written $Z_{d, k} = (1 - k/d)\Omega$ (Theorem), and $(1 - 1/q)\Omega$ for the memory model. done

Exact recovery needs accessible dimension d (``already in I-C-Ex2") Confirmed in the parent: I-C-Ex2 cor:soft and I-C-Ex2 thm:complement. Corollary gives the k-form. confirmed

bottomrule

Plan corrections of version 1 (retained)

• ``Error of every such channel equals formula" is false for every $1 \le k \le d$: $Ad_X \circ \Delta$ is a classical-cut composite with error 1 (Proposition). Proved instead: $\varepsilon \ge 1 - F_e \ge 1 - k/d$ for every object; equality $\varepsilon = 1 - F_e$ on the covariant class; exact optimum $1 - k/d$ with dual operator $(1 - k/d)\Omega$; the set of errors is $[1 - k/d, 1]$.

• ``Exactness" was already proved in the parent by a primal construction; this sheet adds the dual operator only and changes no value.

• (Amendment t257u) The original ``new version of the existing sheet" is superseded: this is a new standalone sheet, and I-C-Ex2 is not given a new version.

Verification of plan t258u against the sources (version 2, retained)

Every label named by plan t258u was searched in the read-only copies (automated log plan_label_audit.json of this run). For this sheet: toprule Plan statement Finding Status

midrule Soft-memory dual in both sheets Kept here (Theorem); proved also in I-C-Ex-CERT-v2. done

Twirl as a reduction to the spectral law The twirl is covariant with exact error $1 - F_e$ (already in version 1, Theorem (c)). It is a replacement only when $F_e = 1/d^2$; then it is $\mathscr{R}_{I/d}$ and the spectral law agrees (Theorem). In general the twirl is not a replacement, so ``the spectral formula applies to the twirl" holds only in that case. proved (scoped); PC3.5

Reorganization reading via Big01-A def:reorg The label exists; pattern only. Theorem . proved

OpenAI isotropic-channel entries for lem:attainer(b) The catalogue (372 families) contains no Choi-operator, isotropic-channel, isotropic-twirl, dual-certificate or coefficient-certificate entry; ``commutant" occurs only in operator-algebra entries. Not cited; local argument (Remark). corrected; PC3.6

Sandwich lemma in three sheets; cross-reference Confirmed: the label exists in I-C-CERT-v1, I-C-Ex-CERT-v1, I-C-Ex2-CERT-v1; cross-reference added (Section). done

bottomrule

Plan corrections of version 2 (retained)

• ``Move soft-memory dual into I-C-Ex-CERT": implemented as a second local proof in I-C-Ex-CERT-v2; it stays in this sheet (no file of version 1 is edited).

• ``The spectral formula of thm:replacement applies to the twirl when the twirl is a replacement": correct, but the twirl is a replacement only when $F_e = 1/d^2$, and then it is always $\mathscr{R}_{I/d}$; for every other channel the reduction goes to the covariant certificate, not to the spectral law.

• ``OpenAI isotropic-channel entries can be cited as source records for the commutant argument": the fetched OpenAI catalogue has no such entries. The commutant step is proved locally.

Verification of plan t262u against the sources (version 3, retained)

Every label named by plan t262u was searched in the delivered read-only files, pinned by sha256 (automated log plan_label_audit.json of this run). The plan says it does not authenticate the other sheets; here they were checked against the delivered files. Findings for this sheet: toprule Plan statement Finding Status

midrule Local rank-one certificate Already Corollary (a),(b) (version 1). not new; PC3.7

Complete Bell-saturation criterion True; proved (Theorem), including the sufficiency of covariance, which the plan asserts without proof. proved

All channels attaining the access floor True; proved (Theorem), with the identification of the class with Kraus rank at most k. proved

Memory-model and covariant attainers in one class True (Corollary); the covariant weights are the Carathéodory weights of Lemma (c) scaled by d/k. proved

Hawk07-A/B/C, PG thm:s2-exact: dual operators of the common form $(1 - t)\Omega$ The label exists in all four sheets

(v1 and v2) and is a replacement-recovery statement. Its dual is a block operator $Y \geq \sigma \otimes I - \Omega$ (output-first, Choi operator of $\mathscr{R}_{\sigma} - id$: opposite sign convention), not $(1 - t)\Omega$. corrected; PC3.8

prop:s2-ceiling: fixed-resource bounds, not identical ceilings Confirmed: it is an entropy bound $S(\rho_R) \leq \min(\log d_R, \log d_{R^c} + S(\rho))$. confirmed

En08-ST-DSS-CERT-v2 thm:EC-dual Confirmed: matching witnesses for a hull deficit. confirmed

Big01-A def:reorg, thm:tick Exist (Big01-A v5); patterns only. confirmed

OpenAI catalogue: no Choi or isotropic-channel entries Re-checked on the same copy: confirmed. confirmed

Non-covariant ending Corrected wording adopted (Section). done

bottomrule

Plan corrections of version 3 (retained)

• The plan's Lemma ``local rank-one certificate'' restates Corollary (a),(b) of version 1; not restated.

• ``The dual operators have the common form $(1 - t)\Omega$'' is false for Hawk07-A-CERT-v2 thm:s2-exact and its copies in Hawk07-B/C-CERT and PG-OB-1-CERT: their dual Y is a block operator for the opposite sign convention (Section). Their value $1 - 1/\Xi$ and their Schmidt primal agree with the replacement law. That this sheet's convention is opposite to Hawk07's (reported by earlier streams) is confirmed.

Errors found in version 2. None. The open item ``an exact error formula for non-covariant objects'' of version 2 is narrowed by Theorem ; it was not false.

Verification of plan t266u against the sources

Every label named by plan t266u (sha256 39aea6f1117318f9517712567fc87db46dd188176498fd71ff637076175f0621) was searched in the delivered read-only files, pinned by sha256 (automated log plan_label_audit.json of this run: Big01-A v5; v1 and v2 of DS-OB-1-CERT, VC-OB-1-CERT and PG-OB-1-CERT; v2 and v3 of Hawk07-A/B/C/F/G/H-CERT; v2, v3 and v4 of the three DSS CERT sheets; v3 of the five sheets of this family; the OpenAI catalogue). The OB sheets' version 3, in preparation elsewhere, was not read. Findings for this sheet: toprule Plan statement Finding Status

midrule Big01-A: thm:Hcert, prop:Dyncert, def:reorg, thm:tick, thm:Bthermal, thm:Bdephase; also prop:shredsharp, thm:D, thm:D4 All present in Big01-A v5 (sha256 e1160613…). confirmed

I-C family v3: lem:cert-sandwich, thm:bell-iff, thm:v3-floor The first two are in all three v3 sheets; thm:v3-floor (and thm:kdreorg, since v2) only in I-C-Ex-CERT and I-C-Ex2-CERT, not in I-C-CERT. corrected; T266.1

PG-OB-1-CERT-v2: thm:s2-exact, prop:s2-ceiling, lem:v2-sandwich; second witness $Z_{\sigma} = (1 - 1/\Xi)\Omega$ All present in v2 (lem:v2-sandwich not in v1). Z_{σ} is in v2 as thm:v2-Z, a label the plan does not name; it has the shape of this sheet's rank-one dual. confirmed

PG reading: exact dual for replacement recovery; block dual in the opposite sign convention True. Made precise here: the block dual minus the Choi difference is exactly Z_{σ} (Corollary , Lemma). proved

Hawk07 thm:s2-exact of A/B/C gives ``the same dual certificate'' Qualified: same value and primal; the dual operators differ in form (block versus rank one) and sign convention, and correspond exactly under the convention translation. corrected; T266.2

Hawk07 CERT: thm:s2-exact, prop:s2-ceiling, prop:s2-vac, thm:s2-sep, thm:s2-cas A union, not one sheet's list: thm:s2-exact, prop:s2-ceiling only in A/B/C; prop:s2-vac, thm:s2-sep only in C/F/G/H; thm:s2-cas only in F/G/H. All cited statements are identical in v2 and v3; v3 (newest) is cited. corrected; T266.3

prop:s2-ceiling ``the same hard ceiling'' False: an entropy bound $S(\rho_R) \leq \min(\log d_R, \log d_{R^c} + S(\rho))$, a fixed-resource bound of a different quantity (as recorded in version 3). corrected; T266.4

En08-ST-DSS-CERT-v3 lem:EC-sandwich ``re-proves the primal/dual bracket τ_j'' Present in v3 and, unchanged, in v4; absent in v2. A bracket for the lattice hull deficit τ_j, exact when both witnesses are optimal; not a channel statement. confirmed, typed

DSS CERT-v3 labels (thm:EC-dual, thm:EC-reorg, thm:RC-coef, prop:RC-dyn, thm:VDC-dyn, prop:VDC-coef)

All present in v3; statements unchanged in the newer v4. confirmed

OpenAI catalogue: no isotropic-channel or Choi entries Re-searched: 0 whole-word ``Choi", 0 ``isotropic channel". confirmed

thm:kdreorg ``records the class closed under the monoid of def:reorg" thm:kdreorg exists (since v2). It records closure under the local moves of Definition , not under Big01-A's def:reorg, whose maps include the dimension-growing maps of thm:D4. corrected; T266.5

thm:D fibre-supremum ceiling; thm:D4 multiplies a declared dimension, excluded by hypothesis Confirmed; the effect of admitting such a move on a cut is computed in Proposition . confirmed, proved

bottomrule

Plan corrections of version 4, and errors found in version 3

T266.1 The plan's ``I-C family v3: thm:v3-floor" holds for this sheet and its companion, not for I-C-CERT.

T266.2 ``thm:s2-exact of A/B/C gives the same dual certificate": same value and primal; the duals differ in form and sign convention and correspond under $W \mapsto W - J(\mathcal{F}) + \Omega$ (Corollary).

T266.3 The Hawk07 label list is a union over sheets (see the table).

T266.4 prop:s2-ceiling is not ``the same hard ceiling" as $1 - k/d$: it bounds an entropy.

T266.5 thm:kdreorg records closure under Definition , not under the monoid of Big01-A def:reorg, which contains block-enlarging maps (Proposition).

Errors found in version 3.

V3E.1 Version 3 said that the PG replacement certificate's dual is not of the form $(1 - t)\Omega$. That is true of PG-OB-1-CERT-v2 thm:s2-exact, but the same sheet contains the rank-one witness Z_{σ} (PG-OB-1-CERT-v2 thm:v2-Z), which version 3 omitted. The version-3 file is not edited.

Version 5: the items of request t267u, and errors found in version 4

No plan was attached to request t267u, so version 5 has no plan audit. The request asked that the items left open by the version-4 report be addressed honestly; this table records where each is treated and with what status (proved, partial, open, or not applicable to this sheet). Missing is never zero. toprule Item Treatment here Status

midrule Exact error of non-classical decoders outside the eigenvector condition A statement of I-C-CERT (basis-measurement interface); treated there n/a here

Channels with an exact best rank-one dual beyond the two proved cases Criterion for every channel (Theorem); unitary orbits (Corollary); block pinchings of unequal sizes (Proposition) proved

VC-OB-1-CERT fibre statements with corrected tensors Treated in I-C-Th-CERT-v5 n/a here

I-B-G-Q-C: Δ_4 and the conical running constant Treated in I-B-G-Q-C-CERT-v5 n/a here

Readings: Hawk07 CERT v3, PG-OB-1-CERT-v3, En08-ST-DSS-CERT-v4 Section , documentary done

bottomrule

Errors found in version 4.

V4E.1 The non-claim of Theorem said ``for generic channels $1 - t_{*}$ is strictly above the error; check C23". Check C23 tested only $\varepsilon \le 1 - t_{*}$ (the strict gaps are visible in its recorded values but were not part of its pass criterion), and genericity was not proved. Version 5 replaces the statement by the exact criterion of Theorem , adds the strict test C26, and leaves generic non-exactness unproved. The version-4 file is not edited.

Version 6: the items of request t268u, and errors of earlier versions

No plan was attached to request t268u, so version 6 has no plan audit. The request asked that the items left open by version 5 be targeted and that the newest companion sheets be read; this table records where each item is treated and with what status (proved, partial, open, conditional, or not applicable to this sheet). Missing is never zero. toprule Item Treatment here Status

midrule Generic non-exactness of the best rank-one dual (version 5: not proved) Theorem , with the explicit

non-exact family of Proposition proved

Exact error of channels failing the criterion New lower program L_F with $1 - F_e \le L_F \le \varepsilon$ and the exactness test $L_F = 1 - t_{*}$ (Proposition); $L_F < \varepsilon$ in general (C30) *open* (partial: bracket)

Further exact families Depolarized unitaries (Proposition); identity–pinching mixtures (Proposition) proved

Conservation of $E^{(2)}$; (Hdil); singular basis outputs Treated in I-C-Th-CERT-v6, I-B-G-Q-C-CERT-v6 and I-C-CERT-v6 n/a here

Readings: OB CERT v4, Hawk07-A-CERT-v4, DSS CERT v5 Section , documentary; E6.1 below done

Readings, second wave: OB-1-CERT-v5, Hawk07-*-CERT-v5, DSS CERT v6 Section , documentary done

bottomrule

Errors of earlier versions found in version 6.

E6.1 The plan-verification table of version 4 (retained) records the DSS version-3 labels thm:EC-dual, thm:EC-reorg, thm:RC-coef, prop:RC-dyn, thm:VDC-dyn, prop:VDC-coef as ``all present in v3; statements unchanged in the newer v4". That is true of the text, but it omits the errata of DSS CERT version 4, which qualify how three of them are read: prop:RC-dyn(d) needs the normalization $\varphi(0) = 0$ or the stated rescaling (Red06-G-DSS-CERT-v4 app:RC-errors); thm:VDC-dyn(c) fails at a prescribed event without the pointwise qualification, and prop:VDC-coef(a) needs a non-zero component (V-DST-DSS-CERT-v4 app:VDC-errors). The omission was pointed out by Red06-G-DSS-CERT-v5 doc:RC-icrow5 (identical text in En08-ST-DSS-CERT-v5 doc:EC-icrow5 and V-DST-DSS-CERT-v5 doc:VDC-icrow5) and confirmed here against the version-4 error lists. The row is to be read together with those errata. No statement of this sheet uses the three labels as premises. No other statement of versions 1–5 was found false. Resolved rather than corrected: the version-5 non-claim ``generic non-exactness is not proved" (Theorem). The second-wave companions (OB-1-CERT-v5, Hawk07-*-CERT-v5, DSS CERT v6) record no error in version 5 of this sheet; their readings of it were checked and found accurate. The earlier files are not edited.

Audit of the parent, checks, and non-claims

Audit of the parent. I-C-Ex2 thm:sharp, I-C-Ex2 thm:complement and their proofs were re-read; no mathematical error was found. One source-level defect was observed: the parent's preamble defines verb | AA | with verb | newcommand |, which raises the LaTeX error ``Command verb | AA | already defined" in a pdfLaTeX run. This sheet does not revise the parent and records the defect only.

Computer-assisted checks. Appendix lists floating-point and semidefinite-programming checks. They are computer-assisted, corroborate the proofs, and no proof depends on them. No independent review has been performed. Checks C21–C23 are new in version 4 (convention translation, the Hawk07/PG block dual, the best rank-one dual). Checks C26 and C29 are new in version 5 (exactness criterion; unequal block pinchings). Version 6: checks C30, C32, C33 and C34 are new (fidelity primal; depolarized unitaries; generic non-exactness; identity–pinching mixtures).

Open. A closed error formula for channels outside the Bell-saturation criterion of Theorem (each finite instance is a semidefinite program [Watrous2013]); the error of non-attaining composites; an efficient synthesis of $T_{d, k}$; any physical identification of a cut. O.1, Gate 0, XV-B, J2, U6-sel and Host C are unchanged. Version 4: a characterization of the channels whose best rank-one dual is exact beyond Theorem (c),(d); no Page curve is claimed. Version 5: a closed error formula for channels failing the criterion of Theorem ; generic non-exactness of the rank-one dual (not proved). Version 6: a closed error formula for channels failing the criterion of Theorem (generic, by Theorem); whether $L_F = \varepsilon$ on any larger class.

Non-claims. This sheet does not imply that every channel of the class has error $1 - k/d$. The dual certificates do not imply an efficient or physical implementation, a horizon, a soft-hair sector, a temperature, a finite Hilbert space of the host, Einstein dynamics, or any quantum coefficient. Nothing here closes an open item of the parent or any standing gate. Host stays prescribed, labels stay named.

References

J. Watrous, *The Theory of Quantum Information*, Cambridge University Press, 2018.

B. M. Terhal and P. Horodecki, Schmidt number for density matrices, *Phys. Rev. A* 61 (2000) 040301.

J. Watrous, Simpler semidefinite programs for completely bounded norms, *Chicago J. Theor. Comput. Sci.* (2013), Article 8; arXiv:1207.5726.

I-C-Ex2, *Eight Further Extensions of the Shredder Cut*; title page names Ngo Cheung. Parent sheet; source record only.

Big01-A (version 5), *Horizon-Bounded Capacity on a Prescribed de Sitter Host*; title page names Ngo Cheung (authorship requires confirmation). Source record only.

appendix

Label audit

Every cited label of a parent or source sheet was found as a verb || in the read-only copy listed in Appendix (automated; 51 labels, 0 missing). Labels that the plan names but that do not exist are recorded as plan corrections. begingroup toprule Sheet Label Found

midrule endhead Big01-A def:reorg yes

Big01-A prop:Bblocks yes

Big01-A thm:D yes

Big01-A thm:D4 yes

Big01-A thm:tick yes

En08-ST-DSS-CERT-v2 thm:EC-dual yes

En08-ST-DSS-CERT-v3 lem:EC-sandwich yes

En08-ST-DSS-CERT-v5 cor:EC-decoders yes

En08-ST-DSS-CERT-v5 doc:EC-icrow5 yes

En08-ST-DSS-CERT-v5 doc:EC-pgic5 yes

En08-ST-DSS-CERT-v5 prop:EC-regfloor yes

En08-ST-DSS-CERT-v6 doc:EC-v6read yes

En08-ST-DSS-CERT-v6 prop:EC-assisted yes

Hawk07-A-CERT-v2 prop:s2-ceiling yes

Hawk07-A-CERT-v2 thm:s2-exact yes

Hawk07-A-CERT-v3 prop:s2-ceiling yes

Hawk07-A-CERT-v3 prop:v3-drule yes

Hawk07-A-CERT-v3 thm:s2-exact yes

Hawk07-A-CERT-v4 cor:v4-dim yes

Hawk07-A-CERT-v4 prop:v4-rankone yes

Hawk07-A-CERT-v5 prop:v5-kernel yes

Hawk07-B-CERT-v2 thm:s2-exact yes

Hawk07-B-CERT-v3 thm:s2-exact yes

Hawk07-C-CERT-v2 thm:s2-exact yes

Hawk07-C-CERT-v3 thm:s2-exact yes

I-C-CERT-v1 cor:cert-bell yes

I-C-CERT-v1 lem:cert-sandwich yes

I-C-CERT-v1 thm:replacement yes

I-C-Ex-CERT-v1 cor:cert-bell yes

I-C-Ex-CERT-v1 lem:cert-sandwich yes

I-C-Ex-CERT-v5 prop:v5-pinch yes

I-C-Ex-CERT-v5 thm:v5-exact yes

I-C-Ex2 cor:soft yes

I-C-Ex2 eq:cut yes

I-C-Ex2 eq:softmodel yes

I-C-Ex2 thm:complement yes

I-C-Ex2 thm:sharp yes

I-C-Ex2 thm:softhair yes

PG-OB-1-CERT-v2 thm:s2-exact yes

PG-OB-1-CERT-v2 thm:v2-Z yes

PG-OB-1-CERT-v3 cor:v3-decoder yes

PG-OB-1-CERT-v3 cor:v3-dimension yes

PG-OB-1-CERT-v3 prop:v3-Z yes

PG-OB-1-CERT-v4 cor:v4-PG-register yes

PG-OB-1-CERT-v4 thm:v4-PG-floor yes

PG-OB-1-CERT-v5 prop:v5-PG-bracket yes

PG-OB-1-CERT-v5 thm:v5-PG-shared yes

Red06-G-DSS-CERT-v4 app:RC-errors yes

Red06-G-DSS-CERT-v5 doc:RC-icrow5 yes

V-DST-DSS-CERT-v4 app:VDC-errors yes

V-DST-DSS-CERT-v5 doc:VDC-icrow5 yes

bottomrule endgroup

Computer-assisted checks

All checks in this appendix are computer-assisted (floating point, mpmath, sympy, scipy). They corroborate the proofs, and no proof depends on them. The selection is made by C-common/code/ic_select.py from C-common/code/ic_checks.py (C1–C8, version 1), ic_checks2.py (C9–C13, version 2), ic_checks3.py (C14–C19, version 3) and ic_checks4.py with ic_checks4_curv.py (C20–C24, version 4) and ic_checks5.py with ic_checks5_curv.py (C25–C29, version 5) and ic_checks6.py with ic_checks6_curv.py (C30–C35, new in version 6). Script code/checks.py, output logs/checks.json. begingroup toprule Id Content Result

midrule endhead C2 J(pinching)-Omega/d >= 0 (d=2,3,4); SDP errors of pinching 1-1/d and of X o pinching 1 (d=2,3); equal output spectra OK ({})

C3 errors along (1-s) pinching + s X o pinching, d=3, s=0,0.2,...,1: from 2/3 to 1, monotone (SDP) OK ({"values": [0.6666666668480365, 0.7333333337900749, 0.8000000000750112, 0.8666666668298688, 0.9333333342242476, 1.0000000004932152]})

C4 isotropic T_{d,k}: J-(k/d)Omega >= 0 and J >= 0 for (d,k) in (2,1),(3,1),(3,2),(4,2),(4,3); SDP error 1-k/d for d <=3 OK ({})

C5 random isotropic channels, d=3: SDP error equals 1-F OK ({"max_dev": 7.77567898957443e-10})

C6 soft-memory model q=k=2, random unitary U: J(D_U N_mem)-Omega/q >= 0 OK ({"min_eig": -6.046194757103579e-16})

C7 random channels, d=2,3: SDP error >= 1-F_e OK ({"min_gap": 0.07354203921983427})

C12 single-qubit Clifford group (24 elements mod phase, a unitary 2-design): twirl of 3 random channels has

isotropic Choi with the same F_e and SDP error 1-F_e <= error of the channel; twirl of a replacement channel is R_{I/2} OK ({"group_order": 24, "rep_twirl_dev": 1.6664148027291862e-16})

C14 Bell-saturation criterion, d=2,3, 12 channels (6 non-covariant projection channels from two random bases, 6 generic random channels): SDP error equals 1-F_e exactly when J Omega = d F_e Omega; error >= 1-F_e always OK ({"rows": 12, "sample": [[2, 0, 0.5, 0.5, 0.0], [2, 1, 0.5, 0.5, 0.0], [2, 2, 0.5, 0.5, 0.0], [2, 3, 0.947168, 0.816969, 0.58751141]]})

C15 channels sum_i lambda_i P_i X P_i with rank-k projections from two random bases, (d,k)=(2,1),(3,1),(4,2): SDP error 1-k/d and J-(k/d)Omega >= 0; a rank-2 Kraus channel P X P + V(I-P)X(I-P)V\{}dagger at (4,2) lies strictly above 1-k/d OK ({"excess_rank2": [0.49880290939854477], "max_dev": 1.0847994169616015e-09, "min_eig": -8.96367795816165e-16})

C21 random channels, d=2,3: W -> W-J_Delta maps witnesses W>=0, W>=J_Delta onto I-C witnesses Z>=0, Z >=Omega-J with the same Tr_Q; the two SDP optima agree with the SDP error OK ({"max_dev": 8.88262416449235e-16, "max_gap_optima": 3.513781043906761e-08})

C22 d=2,3,4, regular and singular sigma: the block operator Y of the Hawk07/PG replacement certificate, in input-first order, satisfies Y-J_Delta=(1-1/S)Omega, Y>=0, Y>=J_Delta, Tr_Q Y=(1-1/S)I OK ({"max_dev": 1.1102230246251565e-16, "min_eig": -5.857851386599364e-16})

C23 best rank-one dual t*=1/<Omega|J\{}+|Omega>: J-t*Omega>=0 and J-(t*+1e-5)Omega not, t*<=F_e, SDP error <= 1-t* (generic channels d=2,3); t*=1/S for replacement; t*=1/sum_x <x|s_x\{}-1|x> for basis composites; t*=0 for the shifted relabelling OK ({"sample": [[2, "gen", 0.14276, 0.2642, 0.78058], [2, "gen", 0.04157, 0.19296, 0.83706], [2, "gen", 0.13377, 0.34612, 0.73055], [2, "repl", 0.05943, 0.05943], [2, "basis", 0.1603, 0.1603, 0.81476], [2, "shift", 0.0], [3, "gen", 0.00206, 0.10585, 0.92403], [3, "gen", 0.02907, 0.10389, 0.93231]]})

C26 exactness of the best rank-one dual, d=2,3: for full-rank Choi operators M=mat(J\{}-1 Omega); generic channels: M not Hermitian PSD and SDP error < 1-t* strictly; replacement and isotropic channels: M Hermitian PSD and SDP error = 1-t*; unitary conjugation preserves error, t* and the criterion OK ({"sample": [[2, "gen", "False", 0.87, 0.98912], [2, "gen", "False", 0.80999, 0.95917], [2, "gen", "False", 0.84136, 0.99216], [2, "repl", "True", 0.79092, 0.79092], [2, "iso", "True", 0.3, 0.3], [2, "conj", 0.9129, 0.9129], [3, "gen", "False", 0.97269, 0.99797], [3, "gen", "False", 0.93915, 0.99108], [3, "gen", "False", 0.90122, 0.99625], [3, "repl", "True", 0.9938, 0.9938], [3, "iso", "True", 0.3...)

C29 pinching onto m orthogonal blocks of sizes (1,2),(1,3),(1,1,2),(2,3) in a random basis: SDP error 1-1/m, (J-Omega/m)vec R=0 for R=sum P_i\{}T/(m k_i), F_e=sum k_i\{}2/d\{}2 and 1-F_e<1-1/m OK ({"sample": [[[1, 2], 0.5, 0.5, 0.55556, true], [[1, 3], 0.5, 0.5, 0.625, true], [[1, 1, 2], 0.666667, 0.666667, 0.375, true], [[2, 3], 0.5, 0.5, 0.52, true]]})

C30 fidelity primal L_F=1-min_R <vec R|J|vec R> over states (convex program), d=2,3: random channels of Kraus rank d\{}2 and 2, basis composites and replacement channels satisfy 1-F_e <= L_F <= SDP error <= 1-t*; replacement: L_F = error = 1-t*; number of samples with L_F < error strictly (margin 1e-4) recorded OK ({"sample": [[2, "gen", 0.82969, 0.83872, 0.86287, 0.93016], [2, "gen", 0.67064, 0.70025, 0.7384, 0.93938], [2, "gen", 0.83836, 0.84574, 0.89111, 0.97477], [2, "gen2", 0.76736, 0.78948, 0.87041, 1.0], [2, "gen2", 0.61157, 0.8419, 0.85224, 1.0], [2, "basis", 0.76087, 0.91111, 0.91184, 0.9178], [2, "basis", 0.74406, 0.78708, 0.80939, 0.9551], [2, "repl", 0.75, 0.91332, 0.91332, 0.91332], [3, "gen", 0...)

C32 depolarized unitary T=p Ad_U+(1-p)R_{I/d}, p=1/2, exact sympy: M_T equals (I-p conj(Tr U) U\{}T/(c+pd))/c (c=(1-p)/d); Hermitian for U=Z, X, e\{}{7i/10}diag(1,1,-1) with SDP error = 1-t*; not Hermitian for diag(1,i), diag(1,i,1) with SDP error < 1-t* strictly; closed form of M_T on 6 random unitaries (d=2,3, random p) OK ({"p": "1/2"})

C33 30 random full-rank channels (d=2,3): none has M_T Hermitian PSD; on the segment from a replacement channel (exact) to the depolarized diag(1,i) channel the anti-Hermitian part of M_T vanishes only at the replacement end (11 points) OK ({"random_exact": "0/30", "segment_skew_norm": [0.0, 2.794142, 3.030872, 2.940796, 2.813619, 2.711787, 2.653379, 2.648762, 2.712698, 2.874594, 3.2]})

C34 T=p id+(1-p) pinching onto blocks of sizes (1,2),(1,1,2),(2,2) in a random basis, p=0.3,0.7: SDP error

(1-p)(1-1/m); (J-t Omega)vec R=0 for R=sum P_i\{}T/(m k_i) and t=p+(1-p)/m; J-t Omega >= 0 and J-(t+1e-4)Omega not; F_e=p+(1-p)sum k_i\{}2/d\{}2 OK ({"sample": [[[1, 2], 0.3, 0.35, 0.35, 0.68889], [[1, 2], 0.7, 0.15, 0.15, 0.86667], [[1, 1, 2], 0.3, 0.466667, 0.466667, 0.5625], [[1, 1, 2], 0.7, 0.2, 0.2, 0.8125], [[2, 2], 0.3, 0.35, 0.35, 0.65], [[2, 2], 0.7, 0.15, 0.15, 0.85]]})

bottomrule endgroup Result: 18 of 18 checks OK.

Artifact hashes (sha256)

begingroup toprule File sha256

midrule endhead inputs/Big01-A-v5.api.tex e1160613ea8e02b37fb45954fe677d89f5fa7cca90062d42b68973b03a2bff1d

inputs/DS-OB-1-CERT-v4.tex 0b5b86f87f91503ba9e96cf221b2ef96a4b4e96e3f27489336eb2b89848463c6

inputs/DS-OB-1-CERT-v5.tex a0805f8eb081a250f78e8074b9de5986ccd23106749df9e0fe5fac2f71e157f9

inputs/En08-ST-DSS-CERT-v2.tex 40d0dfb5adda8e3279bdb814d8d7912c0e94be51965927a057a5c5f63ac0ebc2

inputs/En08-ST-DSS-CERT-v3.tex 3882f4e56296b5fa8aecc5cb359bbba8e0b72dac7d2ccc6f2d040c631e7271ff

inputs/En08-ST-DSS-CERT-v4.tex 98c42ee7397263bc694ebed7d66c682b090df437a3230f294b3245f4bd01bee9

inputs/En08-ST-DSS-CERT-v5.tex 9be4fe4fdd9cf1164a75f0218035f0ee3a4c340ed7f15787551de3997fe5af64

inputs/En08-ST-DSS-CERT-v6.tex 7d821d99ced3a9d265d5437f5c1466d8f524a09884730d13662278c31b1da086

inputs/Hawk07-A-CERT-v2.tex 9f4a813e5ed721ac074c05dfd6316a591bac28ccb1be7e23c4972cc32cab0cca

inputs/Hawk07-A-CERT-v3.tex 1e91c9fb7bb97c16d9b4b0d48e821d511badea6e4ace77fc6643a61c76eee776

inputs/Hawk07-A-CERT-v4.tex a88a2430a9e3672a9cbdb7c661dc021154dd3ac4237a1040b4eab097194fceb5

inputs/Hawk07-A-CERT-v5.tex 9db43c8772a84fcf2fcf8d3674b86781ab407b93de05d370740e89f3f00b7add

inputs/Hawk07-B-CERT-v2.tex 47d9b5e886faf80023963a32953a899bba46df79f15ac5dbac33a51e41b5ba5b

inputs/Hawk07-B-CERT-v3.tex 2639d64e7ec3340017a91ed62e487b857317f2cef51966a2086fd016ffcf4176

inputs/Hawk07-C-CERT-v2.tex c54297c3f2e64ff35bf5261c259e9172140927502e5868f13a4e9f92a5e2ce25

inputs/Hawk07-C-CERT-v3.tex 4352ee5e6a754a153631ab755a44e16ea9273fb1b6b9950e8fc29711b983a416

inputs/Hawk07-F-CERT-v4.tex 3b4eef32040e7f3aad9766523b6c5deff83c9679fcd5924d3b93f5b110221e63

inputs/Hawk07-F-CERT-v5.tex 2cfa6282f30e25ac57188298f2e16d3b05a66fff4440204e6f11acce352f5bc3

inputs/Hawk07-H-CERT-v5.tex 74a70015a03906e76cac8fde88a082da9789ea6e863d987be1dbba67f08bb5dd

inputs/I-C-CERT-v1.tex 4f932078b3c653f94093b76ee99a5f78990027ff26d1da1b1914a6a12b4c3980

inputs/I-C-Ex-CERT-v1.tex 2b3e5ef48225271bbc8ad56bb798c7d138ecd5afd20fe31682b5f105f11f568f

inputs/I-C-Ex-CERT-v5.tex ba3babcc99d2b98ee9118fffc57d3e441dd1e7b4beecd49bab449b4be277b176

inputs/I-C-Ex2-CERT-v1.tex 8cc8832536c8cae006822f5f35e88c008a9b45dde9cb845c364fae3ec81c35b3

inputs/I-C-Ex2-CERT-v2.tex 76cb43695a7ca4e98333ccea318fb6a2b118246b1a52b830317427ab45ac256b

inputs/I-C-Ex2-CERT-v3.tex d3a5ddcf5dae6b48f5bdf031c5016e506466580629f0e7defd4003dca065a52b

inputs/I-C-Ex2-CERT-v4.tex 62dd45f881ff66fd6722b23c8e1cca42131d5ea5910fe7ccaccfc6d927806818

inputs/I-C-Ex2-CERT-v5.tex b147e794d9836bae2c7622bbdc0e07ccb6006adb3f755b3037ae0904525dd5b1

inputs/I-C-Ex2.tex 698337771275c0d804ce51948910abe930d1c30b65c10ad0d6e254ccd3d7d403

inputs/OpenAI.api.tex 76314b644e51864fa1818f28f49f32a2ce2e06bdd32364ad939595c792cbf99d

inputs/PG-OB-1-CERT-v2.tex 015f230247fc86885e278ecf2b796b14652f7742c57e200fde8e670fbaf09a40

inputs/PG-OB-1-CERT-v3.tex 8c0a7e2fdbb7e4558bad17bc35df10a66feea6ba4959af524921393a61eafbcc

inputs/PG-OB-1-CERT-v4.tex 3dc921457bd781606e9e673231a8c832be104435950913521fdfb2ed737fc1c1

inputs/PG-OB-1-CERT-v5.tex c96ea4631de51c635521ca42fd24337728591676ee9019a24f3ad9fcdcdeb8a2

inputs/PLAN_t258u.md a5984d52062bafe5e031047d509b7cbccf0f5ca537965d258c2ba1ca5edf6f49

inputs/PLAN_t262u.tex 630a938e80b215d75906abf685d62aa99ec48e89783a14395a3f25d3d5fdacbc
inputs/PLAN_t266u.md 39aea6f1117318f9517712567fc87db46dd188176498fd71ff637076175f0621
inputs/Red06-G-DSS-CERT-v4.tex 574d8abfdcd620476e8054092bce4323652afd03740ceb917a165798bc653ebc
inputs/Red06-G-DSS-CERT-v5.tex e4a1b02552c138b8f9f009f2e71e391825ad0014f717b43f96883c445605213a
inputs/Red06-G-DSS-CERT-v6.tex c3f04faab2c76fb661054f2c1393bd71f484a1bfdfeeace52e5c612a34fc4599
inputs/V-DST-DSS-CERT-v4.tex 550372abe2117364ec4e3f62e8840d66570c8845ad27e842d6ed1b74b5321302
inputs/V-DST-DSS-CERT-v5.tex 4133961f566c059c22054b5922423d1ff94cb0d1c888ca353a7c143bba1541be
inputs/V-DST-DSS-CERT-v6.tex 710c656210aab6dd708cd9f830ab7bf59df592f0887ab168b1e7fa7be2df95f9
inputs/VC-OB-1-CERT-v4.tex f9802da66896159503132b736164016c1d06d2a6530bbbc06cb4ad3af0256387
inputs/VC-OB-1-CERT-v5.tex d3a93a3afb0e8803d18f230ce824f78c19253f6a4bc841f8ceef763ce14f7cac
../C-common/code/preamble_template.tex
2b3b5de7495a5e175a4545b6c4a01927c9b879d373a0a3b14a0e17947cd9d3b5
../C-common/code/build.py 5dbc919f5773abb3bf12f3e56aacdab7a8161d61f05035945fd40204d536a88f
code/sheet.json 188d85a9736325da958aa357fb785c728f07d7786f2950489c8cc550931d930b
code/checks.py d49f9d472bdf5bf52478f7384d5f53a3e54bde0631619e5d268a017b7e4d85c8
parts/01_front.tex aed0427111a3f95b8930921cc05fc0f194ca5d4c3b395f7addd9e8ce3792bac3
parts/02_sandwich.tex 80339d2802add2fb52fb3f4fd8599bccba2478ab96bed50b7b4046c3c576b178
parts/03_setting.tex 1ebc27297f90eacbd7a548e80a09624a2ab1f98b5e4ac7d08ec3ba5db531ce01
parts/04_kd.tex 734c6162f7c24aab02d4d879e2485b2f9d76264bd1ac499402e42823af8273c9
parts/05_soft.tex a0f8e5351d862d07eb8e983ce94a53611932cb81853e81a2bc279287bad4bf21
parts/05b_v2.tex 04e4648632e1c20d6ca3cb5fe7389e93d591c59ac214b019e95a5c1ec759ae08
parts/05c_v3.tex 67c172935403c42076f1afb838101befbaac178dbb7afa2c0778bb214bd212fa
parts/05d_v4.tex a3838f705b4600e513664a8086b835f7e70e0946b6a3dfa8cabbc72605a70734
parts/05e_v5.tex 03081004cede58325be1ac5a21a29b6b4cc46710bdf6fac20d5bcf06fb6f6859
parts/05f_v6.tex ba687116da399907b8b15ee6d75025a1bd84bd09c820e0bb258f6501194b8f74
parts/06_plan.tex f24f98fd347cdfdce8d3a84c24a65e081ad60079401acacbd8f2b545844590db
logs/checks.json 587ce9a145b9ec00e127aa2cefc3ad9ab4ba6efd142120a26f1e00ddb29c584e
logs/inputs.sha256 5347ec8d0f7db97132c09e3427a2adb0e18114bf69b46454747d5e068130bcb8
../C-common/code/ic_checks.py 912c95e4b1db7998f312426da001034368de6c3dd64e0fb4a9a42b7e53021bbf
../C-common/code/ic_checks2.py 1f5f5c348b1c711e6466347e12904db433a889a4247c94d378fb9275e38c2da7
../C-common/code/ic_select.py 8bc20e667e8571b0f6f78b5e6ea3168fb9e815186613d2c07735690b4ec5729e
../logs/get_log_S1.txt f4ad94933a9086eed368e50d2b7a5d9ea1b22ded9a1a07ec55ad12d5fb4872ac
../logs/get_log.txt f639ab99a753eed5217b1d001fe5cfa88a1c4191f4c4c7b89decb05f71514108
../logs/plan_label_audit.json f59bffdb66c63c8e64734a5490205357ea03fe78ea6108abed0d05b9365ef86b
../C-common/code/ic_checks3.py d2661722e5dfa7a3493a3d1a7f7b15db2f20db64e6fca34eb6641d62d90b4fca
../logs/get_log_S1v2.txt 470a182c91cdb81697779de8edc5498d31b4806ce398ca65e9481e46d6214683
../C-common/code/ic_checks4.py 5e573a4fa9fdee85ae57225cad0d652b0ae87e26a0213902433278467228b204
../C-common/code/ic_checks4_curv.py
355487e6b9e2d97d373cef95ab4849b155069a05b6357c19427c8517adab0365
../C-common/code/ic_checks5.py fb661fa24dca783522dd2a86bd39a4b5d178f840d6539c2a865ba730e4e532bb
../C-common/code/ic_checks5_curv.py
3c08d86437024cb0ed1f1031fb7409b055de461517d0b92e21f16a27f484b13d

../C-common/code/ic_checks6.py d883c43acada82668fa5dc4f51d8f9437aafe1d701fa88b11855df075a7db81e

../C-common/code/ic_checks6_curv.py 5436c6c1011618c22e8ac3e57b13c865108b0a7def56cf1f842d8d3efbe1d7c1

midrule Frozen tree (before)

XVI-P ec965c15ae25cd3bb9cc2b3a134e966b19a1803c78d1fe5bf7554da2a8b3aaa6 OK

XVI-Q 0df9a0d10409d31ac0c2d0a47197e3f6e03c7ae258ec5fb775137f66cb0493c4 OK

XVI-R 57187bc6825f5c6ae88633b7279c6c0be57f7c4918fdbc525da4aaf958331edb OK

bottomrule endgroup The hashes of the delivered .tex and PDF are recorded in MANIFEST-CertPlan-S1v6.sha256, not here, because a file cannot contain its own hash.